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An Analysis of U.S. and World Oil Production Patterns Using Hubbert-Style Curves[1]

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A quantitative analytical method, using a spreadsheet, has been developed that allows the determination of values of the three parameters that characterize the Hubbert-style Gaussian error curve that best fits the conventional oil production data both for the U.S. and the world. The three parameters are the total area under the Gaussian, which represents the estimated ultimate (oil) recovery (EUR), the date of the maximum of the curve, and the half-width of the curve. The "best fit" is determined by adjusting the values of the three parameters to minimize the root mean square deviation (RMSD) between the data and the Gaussian. The sensitivity of the fit to changes in values of the parameters is indicated by an exploration of the rate at which the RMSD increases as values of the three parameters are varied from the values that give the best fit. The results of the analysis are as follows: (1) the size of the U.S. EUR of oil is suggested to be 0.222 × 10^{12} barrels (0.222 trillion bbl) of which approximately three-fourths appears to have been produced through 1995; (2) if the world EUR is 2.0 × 10^{12} bbl (2.0 trillion bbl), a little less than half of this oil has been produced through 1995, and the maximum of world oil production is indicated to be in 2004; (3) each increase of one billion barrels in the size of the world EUR beyond the value of 2.0 × 10^{12} bbl can be expected to result in a delay of approximately 5.5 days in the date of maximum production; (4) alternate production scenarios are presented for world EURs of 3.0 and 4.0 × 10^{12} bbl.

KEY WORDS: petroleum, energy, gaussian, logistic curve, peak production, estimated ultimate recovery, reserves-to-production ratio.

INTRODUCTION

One of the best known products of the work of M. King Hubbert is the "Hubbert curve" (Hubbert, 1974), which empirically approximates the full cycle of the growth, peaking, and subsequent decline to zero of the "production" [(quantity/ year) vs. year] of a finite, nonrenewable resource. The main central portion of a representative Hubbert-style curve is shown by the solid line of Figure 1.

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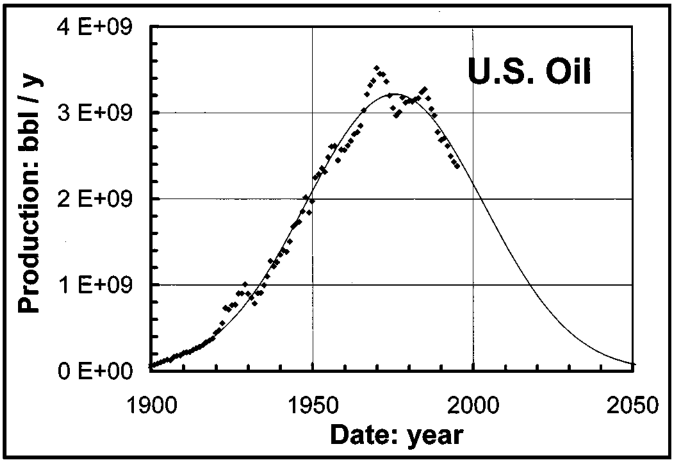

Figure 1. The data for the production of oil in the U.S. are shown, along with the primary Gaussian. This is the Gaussian that has the smallest RMSD and hence is the best fit to the data. Each major square has the units of 1×10^9 bbl/yr multiplied by 20 yr, equals 20×10^9 barrels of oil. The values of the three parameters that characterize this Gaussian are given in Table 1.

This analysis is the result of asking the following questions:

1. What is the maximum amount of information one can gain from analytical comparisons of a Hubbert-style curve with data on historical oil production for the U.S. and for the world?
2. How sensitive are the results of this analysis to changes in important parameters?
3. How do the results of the analysis compare with the results of geological studies of the probable estimated ultimate recovery (EUR)?

OTHER CURVES

In his original work, Hubbert fitted production data to the derivative of the Logistic curve, which is similar in shape to the Gaussian curve (Hubbert, 1982). The analysis described here was done with both curves to allow comparison of the results. The differences in the results were smaller than the root mean square deviations (RMSD) of the fits, so that the results did not indicate a clear preference for either curve. Both curves are widely understood, but the Gaussian curve was used because the analysis seemed simpler in execution and interpretation.

No attempt was made to explore other similar curves to see if one could find a curve that gave a significantly improved fit to the data, nor was there any attempt to find improved fits by using superposition of several Gaussian curves representing the production patterns of several different regions or provinces.

BACKGROUND FOR THE METHOD

An analytical method will be outlined that allows one to determine the values of the three parameters of the Hubbert curve that gives the "best fit" to the historical data of oil production in the U.S. and world.

This method assumes that the complete curve of production vs. time of a nonrenewable resource such as oil (the Hubbert curve) can be represented by a Gaussian error curve (Gaussian) that is characterized by three parameters:

1. The area under the Gaussian is the size Q_∞ of the EUR (U.S. or world), expressed in barrels (bbl).
2. The time t_M is the date (year) of the peak of the Gaussian.
3. The parameter S (years) is a measure of the width of the Gaussian.

Logic suggests that it is best to express quantities of oil in the SI units of cubic meters or joules of energy. However in the *lingua franca* of the world oil business, the "barrel" (bbl) is the standard unit of quantity. The following conversion factors may be used to convert barrels to cubic meters or to convert barrels of oil to joules of energy:

—One barrel has the volume of 0.15899 . . . cubic meters.
—One barrel of oil has an energy content of approximately 5.9×10^9 joules.

Production data have only approximately followed the Gaussian pattern in the past. However, as Hubbert pointed out, over the long run, production of oil started initially at zero, will rise to one or more maxima, and then, at some time in the future, will return to zero. Because of these characteristics, the Gaussian can always be used as an *approximate* representation of the curve of production vs. time of oil or of any nonrenewable resource. The actual production curves will be modified by economic, geological, political, technological, and other factors, which may result in a deterioration of the quality of the fit (an increase in the RMSD) between the data and the Gaussian, but the role of these important factors is limited to changing the quality of this fit.

As applied to oil, the method does the following:

1. Uses the following data:
   (a) The history of the production (bbl/yr) of oil vs. time.
   (b) The geologically estimated EUR, Q_∞.
2. Uses all of the available annual oil production data, or a subset of the data; the longer the time span covered by the data, the greater may be the precision of the fit between the Gaussian and the data.

3. Is quantitative and analytical, using the accepted mathematical criterion that the Gaussian that is the best fit to the data is the one for which the RMSD between the historical data and the Gaussian has a minimum value.
4. Is mathematically reproducible.
5. Is easily updated as more production data become available.
6. Produces numerical values (primary values) of the three parameters of the particular Gaussian that is the best fit to the data; this Gaussian is the "primary Gaussian."
7. Allows one to explore the "goodness" of the fit of the Gaussian to the data by determining how rapidly the RMSD of the best fit deteriorates (increases) with changes in any of the three parameters from their primary values.
8. Can be used to determine best fit values for two of the parameters of the Gaussian, along with the associated RMSD, when the third parameter is given an arbitrary numerical value other than its best fit (primary) value. The Gaussians calculated from these values of the three parameters are "secondary Gaussians."
9. Can be used to determine a best fit value for one of the parameters of the Gaussian, along with the value of the associated RMSD, when the second and third parameters are given arbitrary values other than their primary values.
10. (Except for the numerical value of Q_∞) is completely decoupled from theory, judgment, or speculation about the future consequences of complex geological, technological, economic, or political factors that can affect annual production.

This decoupling (10) need not be of concern because all of these factors were present and operating in the real-world data that Hubbert used when he recognized that the production curve had an approximately Gaussian shape. These factors affect the quality of the fit between the Gaussian and the historical data.

DEFINITIONS

Let us define quantities:

t = Date (year).
t_M = The date of the maximum of the Gaussian Hubbert curve.
P = The production of oil in barrels per year (bbl/yr).
Q = The estimated amount of oil remaining in the ground (bbl).
Q_∞ = The integrated total production (bbl) as the time t approaches infinity; this is the EUR.
W = The full width at half-height of the Gaussian.
$W = [(8 \ln 2)^{0.5} S] = 2.355\ldots S$, where S is a convenient width parameter.

GAUSSIAN CURVES

As applied to the oil analysis, the Gaussian curve for the annual production vs. time is given by

$$P = -dQ/dt = [Q_\infty/(S(2p)^{1/2})]\exp[-(t_M - t)^2/(2S^2)]$$

This equation for P contains the three parameters: Q_∞, t_M, and S.

METHOD OF ANALYSIS

One seeks the values of the three parameters that characterize the particular Gaussian that is the best fit to a set of historical oil production data. First, one assumes reasonable approximate values of the three parameters and uses the spreadsheet (Microsoft Excel 5.0) to calculate the year-by-year values of the Gaussian that is prescribed by these assumed values. The RMSD between the assumed Gaussian and the historical data points is then calculated and is displayed.

The values of the three parameters of the Gaussian are then varied systematically until the RMSD is found empirically to have a minimum value. The Gaussian characterized by this minimum RMSD is the "primary Gaussian" that gives the best fit to the data. The values of the three parameters that yield the primary Gaussian are then the "primary values" of these parameters.

THE GAUSSIAN APPLIED TO U.S. OIL PRODUCTION

Figure 1 shows the plot of the U.S. oil production data (U.S. Energy Information Administration, 1995) along with the primary Gaussian. Table 1 tabulates the results of the analysis.

Table 1.

Analytically determined primary values of the three parameters that describe the Gaussian that is the best fit to the data on the production of oil in the U.S.	
Ultimate Resource Q_∞ (bbl)	222.2×10^9
Year (date) of maximum t_M	1975.6
S (width parameter), yr	27.56
Quantities that characterize the primary Gaussian	
RMSD between the data and the primary curve: bbl/yr	0.10293×10^9
Production through 1995: bbl	0.171×10^{12}
Percent of EUR produced by the end of 1995	76.8%
Full width at half-maximum of primary Gaussian: yr	64.9
Gaussian maximum (peak) production, bbl/yr	3.217×10^9
RMSD/maximum production	3.20%

The analysis suggests that approximately three fourths (77%) of the EUR, ($Q_\infty = 222.2 \times 10^9$ bbl) in the 50 states had been produced by the end of 1995.

This EUR gives a best fit that is significantly larger than the value found by Hubbert (1982), who based his analysis on U.S. production data for the lower 48 states through 1980. Hubbert's EUR was 161.8×10^9 bbl (p. 90).

UNCERTAINTY AND SENSITIVITY OF THE MODEL

Hubbert gave little detailed discussion of the magnitudes of the analytical uncertainties in the quantities he derived from his curve fitting. The method used here allows the quantitative exploration of these uncertainties.

The overall quality of the fit between the data and the primary Gaussian is indicated by the fact (Table 1) that the RMSD between the primary Gaussian and the data is 3.2% of the height of the Gaussian maximum.

To explore the sensitivity of the fit of the data to the primary Gaussian, one can give two of the three parameters their primary values of Table 1, and then one can systematically change the value of the third parameter to explore how the RMSD changes with changes in the third parameter. For example, when the parameters S and t_M have their primary values, how sensitive is the RMSD to changes in the parameter Q_∞ (the EUR)? The answer to this question is shown in the upper curve of Figure 2, where one sees that increasing Q_∞ by 8.1%, from its primary value of 222.2×10^9 bbl to 240×10^9 bbl, causes the RMSD to increase approximately quadratically from 0.103×10^9 to 0.175×10^9 bbl/yr, an increase of approximately 70%.

A second investigation is to give one parameter a value other than its primary value and then vary the values of the other two parameters until one finds a secondary minimum RMSD. To illustrate: If one changes the value of Q_∞ from its primary value to 240×10^9 bbl, what is the RMSD if S and t_M are then varied from their primary values until a new minimum RMSD is found? When this is done, the resulting secondary minimum is characterized by $t_M = 1977.5$ and $S = 29.7$ yr. The RMSD is 0.1183×10^9 bbl/yr, which is a point on the lower curve of Figure 2 for $Q_\infty = 240 \times 10^9$ bbl.

Figure 3 shows the data, the primary Gaussian, and the secondary Gaussian that is the best fit to the data for an assumed value of the EUR, of 250×10^9 bbl.

The results of a detailed exploration of the date of the peak of oil production in the U.S. as a function of the assumed size of Q_∞ is shown in Figure 4. The slope of a chord between the ends of the plotted curve suggests that the date of peak production in the U.S. is delayed about 39 days for every billion barrels of new oil that is added to the estimated size of the EUR of the U.S. The historical data show that the peak production of oil in the U.S. was in 1970 with a smaller peak following in 1984.

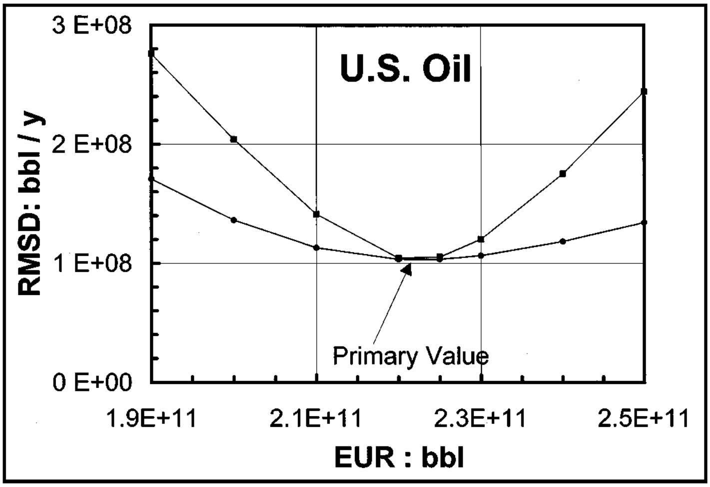

Figure 2. For U.S. oil, the upper curve shows the values of the RMSD when S and t_M have their primary values of 27.555 yr and 1975.6 respectively, and the assumed value of the EUR (Q_∞) of the U.S. is changed systematically about its primary value of 0.2222×10^{12} bbl (0.22 trillion bbl). The lower curve shows the values of the RMSD when, for each nonprimary value of the EUR, one systematically varies S and t_M until one locates a secondary minimum value of the RMSD. The quantities S and t_M have the same value (their primary values) for all points on the upper curve; on the lower curve their values change from point to point. The two curves share the same minimum at the primary value of (EUR $= 0.2222 \times 10^{12}$ bbl).

SENSITIVITY OF THE RMSD TO CHANGES OF S AND t_M

If the three parameters are set at their primary values (minimum RMSD), and if then the assumed date of the peak (t_M) is moved from 1975.6 to 1980, the RMSD is found to increase by about 80%. If the three parameters are set at their primary values, and if then the assumed value of S is increased from 27.56 years to 30.00 years, the RMSD is found to increase by about 52%.

THE GAUSSIAN APPLIED TO WORLD OIL

The data for world oil production (U.S. Energy Information Administration, 1995) and the primary Gaussian that best fits the data are shown in Figure 5.

The value of the EUR that gives the minimum RMSD for world oil is 1.115×10^{12} bbl, which is much smaller than the value (2.0×10^{12} bbl) that Hubbert used in 1972. This discrepancy points out a limitation of this analysis. In contrast to the

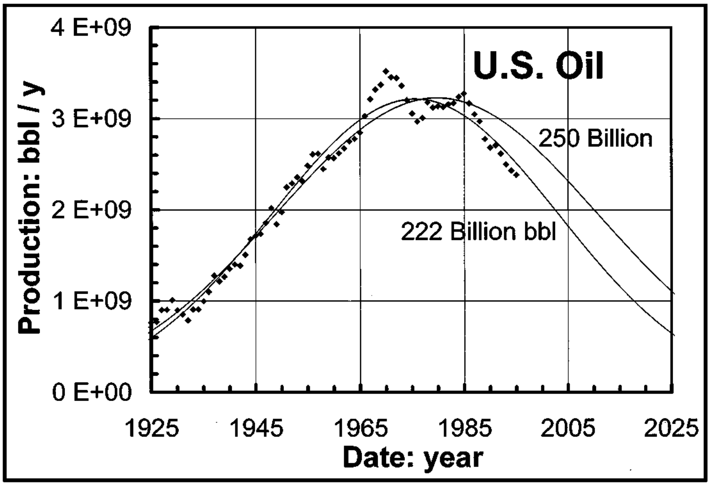

Figure 3. The data for U.S. oil production and two Gaussians are shown: the left one is the primary Gaussian in which all three parameters are adjusted to give the minimum RMSD between the data and the curve. The value of the EUR for this best fit is 0.2222×10^{12} bbl. The right curve is a secondary Gaussian for the case where the EUR is arbitrarily given the nonprimary value of 0.250×10^{12} bbl, and then the two parameters S and t_M were adjusted to find the values that gave the secondary minimum RMSD. The right curve represents a value of the EUR that is 12.5% higher than the primary value of the EUR. For the right curve, the RMSD of the fit of the Gaussian to the data is approximately 15% higher than it is for the fit of the primary Gaussian on the left.

case of U.S. oil, the world data do not yet show a long and persistent downturn in production. As a consequence, a wider range of values of the EUR can give plausible fits to the data.This is illustrated in Figure 6. For assumed values of the EUR that are less than the primary value, the RMSD rises very rapidly, but for values of the EUR that are greater than the primary value, the RMSD rises only slowly. If a production maximum has not been passed, this analysis tends strongly to reject assumed values of the EUR that are less than the primary value, but the analysis does not discriminate strongly among values of the EUR that are larger than the primary value.

If one traces out the date t_M of the peak of the secondary Gaussians corresponding to a series of increasing values assumed for the EUR , one gets the curve shown in Figure 7. Reading from Figure 7, it can be seen that for values of the EUR of 2.0×10^{12} bbl, 3.0×10^{12} bbl, and 4.0×10^{12} bbl, peak production is indicated for the years 2004, 2019, and 2030 respectively.

The average slope of the curve of Figure 7 shows that *for every new billion barrels of oil added to the estimate of the world's EUR, the date of the world peak*

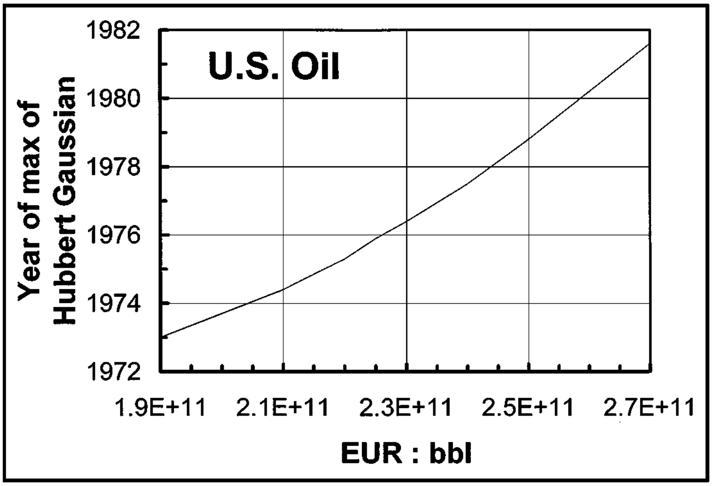

Figure 4. As one increases the assumed EUR for the U.S., the date of the peak of the secondary Gaussians moves to later times; the delay is approximately 39 days for each billion barrels of oil that are added to the estimate of the EUR of the U.S.

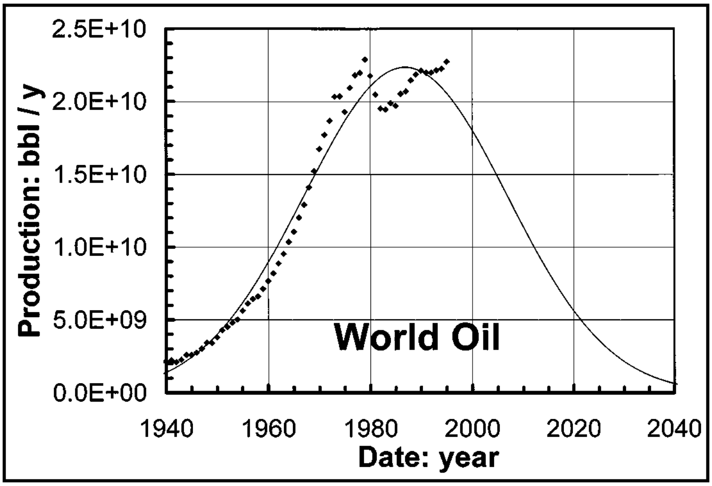

Figure 5. The data for world oil production are compared with the best-fit primary Gaussian. Because the world data do not yet show a prolonged downturn, this analysis is very insensitive to the size of the parameter Q_∞ (the EUR) that, from this fit, is lower than many geological estimates. Curves for more widely accepted geological estimates of the EUR are shown in Figure 8. The large fluctuations in the recent data are due to political and economic factors.

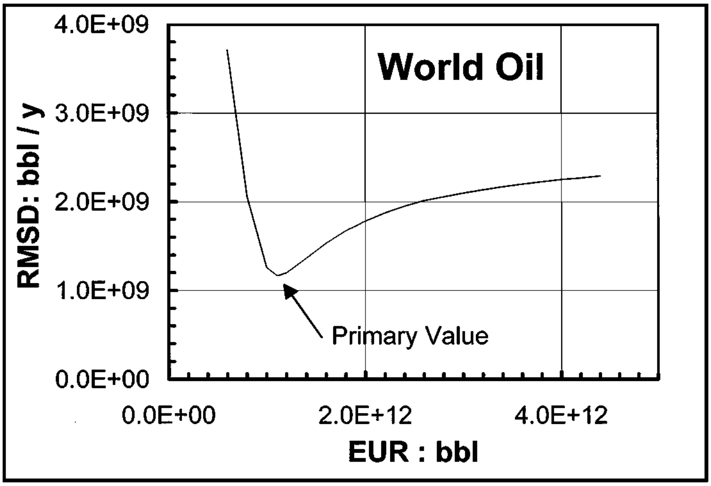

Figure 6. The RMSDs of the secondary Gaussians for world oil are shown as a function of the assumed values of the world EUR. The primary Gaussian that is shown in Figure 5 is characterized by values of the EUR and the RMSD at the minimum of this curve. For assumed values of the EUR that are less than the minimum, the RMSD deteriorates (rises) rapidly, while for values larger than the minimum, the RMSD is seen to rise more slowly. The reason for this assymetry is that the world oil production data have not yet shown any prolonged downturn. This should be compared with the lower curve of Figure 2, which is the same plot for U.S. oil where there has been a long downturn in production and where the RMSD rise around the minimum is more symmetrical.

production is delayed approximately 5.5 days! Doubling the world EUR moves the date of the maximum back by about 26 years!

Figure 8 shows the data and the best-fit secondary Gaussians for these three assumed values of the EUR. Three different values of the EUR are listed in the upper left and the years of the corresponding peak production are given. It should be noted that the highest curve in Figure 8 assumes not only that the EUR is 4.0×10^{12} bbl, but it implies that the world production capability and world demand can rise to 39×10^9 bbl/yr by the peak year 2030.

PER CAPITA OIL PRODUCTION

In Figure 9 one sees two curves of world daily *per capita* production of oil which are normalized to have the same value in the year 1920. The upper curve assumes that the world population has not changed since 1920, while the lower curve takes account of the growth of world population since 1920, and so it shows the actual *per capita* oil production. At the end of 1995, world *per capita* oil production was *less than 2 L per person per day*! The world population is

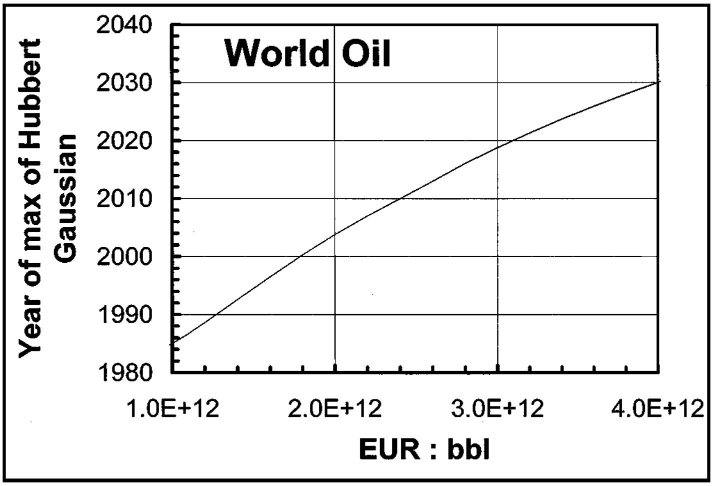

Figure 7. As one follows the secondary Gaussians for world oil for increasing assumed values of the EUR , the location of the peaks of the best-fit secondary Gaussians move to later times at a rate of approximately 5.5 days for each billion barrels of oil added to the EUR. For assumed values of the EUR of 2.0×10^{12} bbl, 3.0×10^{12} bbl, and 4.0×10^{12} bbl, this analysis suggests that the peaks would occur respectively in the years 2004, 2019, and 2030, and the respective peak productions would be 26.5×10^9, 33×10^9, and 39.5×10^9 bbl per year.

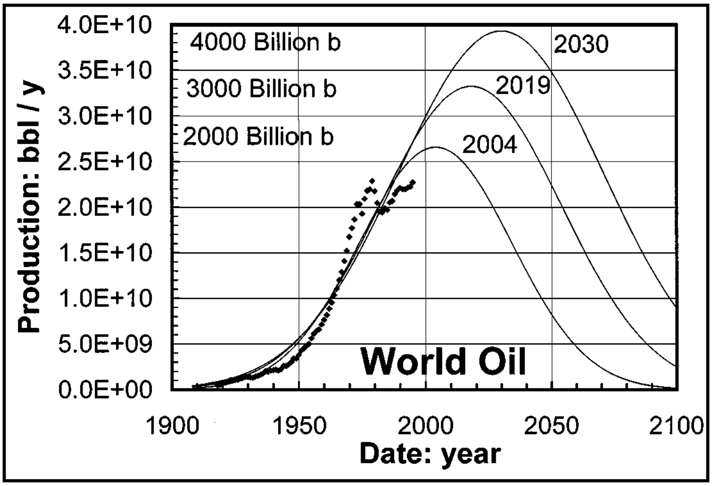

Figure 8. The world oil production data are shown, along with three best-fit secondary Gaussians corresponding to values of the EUR of 2.0×10^{12} bbl, 3.0×10^{12} bbl, and 4.0×10^{12} bbl, with respective dates of peak production of 2004, 2019, and 2030. The number of days required to produce one billion barrels of oil at each of the three peaks are, respectively, 13.8, 11.0, and 9.2.

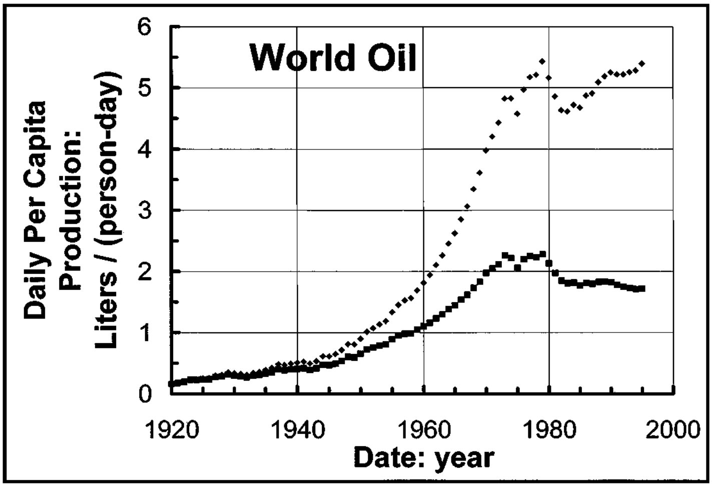

Figure 9. The lower curve shows the recent history of the *per capita* world production of oil, which had its largest value of approximately 2.2 L per (person-day) in the 1970s and which has fallen to approximately 1.7 L per (person-day) by 1995. The upper curve shows what the recent history would have been if the world population had not changed since 1920. In the period from 1920 to 1995 the world population has had an average growth rate of approximately 1.5% per year, which has resulted in the population tripling in these 75 years.

increasing (1996) by about 1.5% per year (~90 million per year), so world oil production will have to climb by 1.5% per year just in order to keep the *per capita* world oil production constant with no further decline.

For the U.S., the maximum *per capita* oil production was approximately 7 L per day in 1970, which has declined to approximately 4 liters per day in 1995.

RESERVES-TO-PRODUCTION RATIOS

The ratio of current reserves (bbl) to current annual production (bbl/yr) is the number of years the current reserves would last if the current annual production continued unchanged. This number is widely quoted as an approximate indication of the future of oil production. If the ratio has the value of 40 yr, it means that the reserves would last 40 years "at the present rate of production" (Bartlett, 1978) This suggests to some that the world production might remain constant for 40 yr and then abruptly drop to zero. Rates of production are not constant over long periods, so this widely quoted ratio is a meaningless indicator of the future course of oil production.

Theoretical curves that suggest the future path of the reserves-to-production ratio can be calculated for each of the best-fit Gaussians. Figure 10 shows the

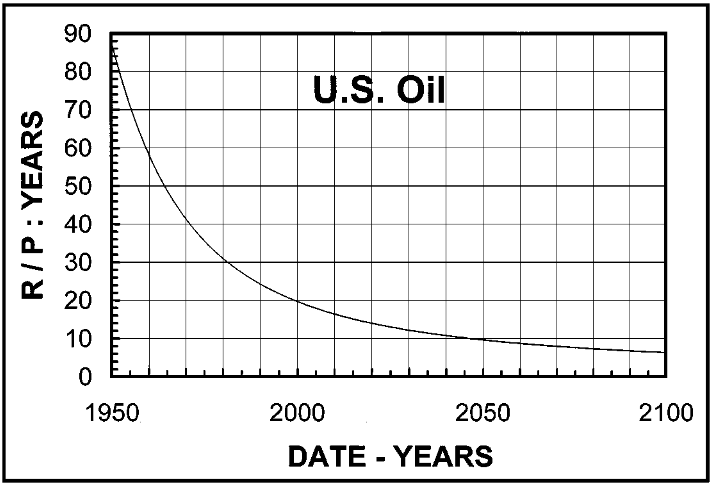

Figure 10. This curve shows the reserves-to-production ratio for U.S. oil, and it is derived from the primary Gaussian for U.S. oil. A point on the curve shows, for that date, how many years U.S. oil would last if production were held constant at the value it had on that date. For example, in the year 1980, the remaining reserves of U.S. oil would last approximately 30 years if production remained unchanged from its 1980 value: production would then drop abruptly to zero.

predicted reserves-to-production ratio as a function of time for the primary Gaussian of Figure 1 for U.S. oil production. Figure 11 shows three curves of the predicted reserves-to-production ratios for world oil production that correspond to the three Gaussians of Figure 8.

One notes that for a fixed value of the EUR, the reserves-to-production ratio decreases monotonically but at a rate that is less rapid than one year each year. New enlarged estimates of the value of the EUR could slow or temporarily reverse the decline in the actual ratio.

In Figure 11 one can see that for EUR $= 2.0 \times 10^{12}$ bbl, the world reserves-to-production ratio in the year 2000 can be estimated to be approximately 42 years.

SUSTAINABILITY

The term "sustainability" is frequently invoked to describe the conditions that will allow a society to continue many generations into the future (Bartlett, 1997–98). Figures 1 and 8 suggest that current rates of consumption of oil cannot continue for many generations in the future, so that present U.S. and world rates of consumption of oil are not sustainable. In general, a society cannot be sustainable as long as it remains vitally dependent on oil.

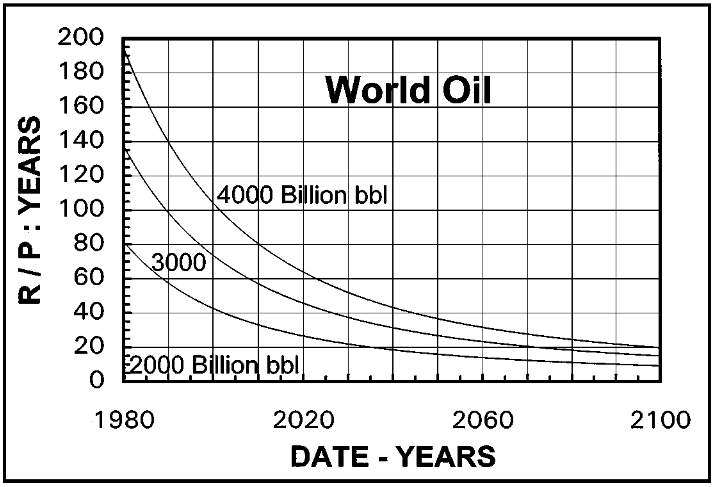

Figure 11. These curves show how the reserves-to-production ratios vary with time for each of the three Gaussians of Figure 8, which correspond to world EURs of 2.0, 3.0, and 4.0 $\times$ 10^{12} bbl. For example, if the world EUR is 3000 billion barrels, the reserves-to-production ratio in the year 2000 would be expected to be approximately 74 years.

COMPARISONS WITH OTHER ANALYSES

Because of the critical importance of oil to modern society, many studies have yielded estimates of the size of the remaining oil resources, and of the probable future paths of U.S. and world oil production. Only a few are cited here.

Campbell and Laherrere (1998) report that “Global production of conventional oil will begin to decline sooner than most people think, probably within 10 years” (p. 78). “Using several different techniques to estimate the current reserves of conventional oil and the amount still left to be discovered, we conclude that the [peak will be reached and the] decline will begin before 2010” (p. 79). From Figure 7 one sees that if the world EUR is 2.4 $\times$ 10^{12} bbl, the year of the maximum of the Hubbert Gaussian is indicated to be in 2010.

Edwards (1997) has given an extensive summary of the works of others and has added his own detailed analysis of the long-term situations in the U.S. and the world in regard to all fossil fuels. For U.S. oil, he cites (his Table 4) three estimates of the EUR whose average is 277 $\times$ 10^9 bbl. This is considerably higher than the 222.2 $\times$ 10^9 bbl that is yielded by this analysis. The secondary curve for 277 $\times$ 10^9 bbl would be some distance to the right of the secondary curve shown in Figure 3.

Edwards' Table 1 lists 14 estimates of the EUR for world oil ranging from a low of 1.65×10^{12} bbl to a high of 3.2×10^{12} bbl, along with 11 predictions of the date of the peak of world oil production. The mean values of the EUR and their corresponding peak dates, along with the RMSD of the EUR and the peak dates, derived from the spread of the tabluated values, are: EUR $= (2.4 \pm 0.4) \times 10^{12}$ bbl, and Peak Date $=$ (2010 $\pm$ 11 yr). Perhaps it is only fortuitous, but these two numbers are the coordinates of a point on the line of Figure 7, and hence they are in agreement with the analysis given here.

Ivanhoe (1995) shows Hubbert curves for discoveries and production for both the U.S. and the world, and then shows graphically the probable production scenarios for the future. In his Figure 3 he shows curves from Hubbert that he has reworked for world oil production based on EURs of 1.5×10^{12} and 2.0×10^{12} bbl. The two peaks are shown in 1988 and 1996, respectively. Ivanhoe's peaks thus show that the delay in the date of the peak is approximately 5.8 days per billion barrels of oil added to the world EUR.

Ivanhoe has written (1997) that the critical date when global oil demand will exceed world production will fall sometime between 2000 and 2010.

MacKenzie (1996) has done a computer analysis of world oil that he has combined with a review of published estimates of oil reserves. He concludes that

> At the low end, for EUR oil equal to 1.8×10^{12} bbl, peaking could occur as early as 2007; at the high end (2.3×10^{12} bbl), peaking could occur around 2014. (An implausibly high 2.6×10^{12} bbl for EUR would postpone peaking only another five years—to 2019).

MacKenzie's computer-generated estimates can be compared with the estimates read from Figure 7 where, for EURs of 1.8×10^{12}, 2.3×10^{12}, and 2.6×10^{12} bbl, the predicted peak dates are in the years 2000, 2009, and 2013.

From his analysis, MacKenzie has produced his estimate of the date of peak world production vs. EUR, which is given in his Figure 13. His predicted peak dates are 6–8 years later than those given here in Figure 7, but his curve has the same slope as Figure 7, namely 5.5 days delay per billion barrels added to the estimate of the EUR.

Masters, Attanasi, and Root (1994) have estimated the world EUR of petroleum to be 2.3×10^{12} bbl. They note that this value

> . . . is limited by our concepts of world petroleum geology and our understanding of specific basins; nonetheless, continued expansion of exploration activity, around the world, has resulted in only minimal adjustments to our quantitative understanding of ultimate resources. . . .

They also indicate that

> Unconventional resources are present in large quantities, in particular in the Western Hemisphere, and are of a dimension to substantially contribute to world reserves should economic conditions permit.

Adelman and Lynch (1997) point out that even as oil is produced and used, estimates of reserves of oil generally to rise with time so that a "fixed view of resource limits creates undue pessimism." Their optimism is based on the past history of increases in the value of the world EUR. Because of the increases in reserves that they see, they indicate their belief that it is misleading to think of the EUR as a fixed quantity, so that analyses such as the one presented here are seriously misleading.

We can note that according to Figure 7, an increase in the world EUR of approximately 66 billion bbl is necessary to delay the date of the maximum of the Gaussian by one year.

CONCLUSION

The work reported here is an analytical study of the data on U.S. and world production of oil. The study has no geological content beyond that of the values of the EUR. The results are internally precise and self-consistent, and hence are reproducible, and they are consistent with the results of a number of other studies. Studies based on assumed fixed values of the EURs are often criticized by noting that values of the EURs tend to increase with time. Increasing estimates of the EUR of the U.S. can be accomodated in this analysis by refering to Figures 3 and 4. The consequences of increasing estimates of the world EUR can be evaluated by examination of Figures 7 and 8.

Prices, politics, and the consequences of the law of supply and demand will be significant short-term determinants of the course of oil production in the future. The effects of these economic factors are not modeled in this analysis.

Only time will tell the degree to which the results of this analysis may or may not be reasonable.

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