Modeling the Effects of Component Level Geometric and Form Deviations on Machine Tool Slideway Errors

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abstract

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MODELING THE EFFECTS OF COMPONENT LEVEL GEOMETRIC AND FORM DEVIATIONS ON MACHINE TOOL SLIDEWAY ERRORS

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ABSTRACT

Geometric inaccuracies of the guide rails and structural parts adversely affect the tracking accuracy of machine tool slideways. These deviations are introduced by factors such as manufacturing and assembly inaccuracies, thermal and structural distortions. A mapping of the effects of component level geometric and form deviations on the tracking errors of a machine slideway is presented. A numerical algorithm for contact deformation at the sliding joint, based on an elastic foundation model, is also included. Autocollimator experiments on a "T" slideway are used to investigate pitch and yaw errors and to validate the models. Measurements of pitch and yaw error made on a "T" slideway compare favorably with the model predictions.

INTRODUCTION

Dimensional accuracy of machined components c:ten depends on the accuracy of the machine tool used for the manufacture. Machine tool errors influence the relative position of the tool with respect to the workpiece. They may be classified into systematic errors (deterministic) and random errors (stochastic). Hocken [1980] emphasized the importance of developing a mechanistic understanding of the systematic errors of a machine tool. Error prediction models have been developed by a number of researchers over the past two decades [Portman, 1980; Ferreira & Liu, 1986; Kiridena & Ferreira, 1994; Shin and Wei [1994], and Lin & Ehmann, 1996]. To aid machine tool component selection and sizing however, a mapping between component level attributes (geometry and form deviations) and workspace error behavior is required. Reshetov [1988] estimates that up to 75% of initial errors of a new machine tool may be the result of inaccuracies introduced during manufacture and assembly. The focus of this paper therefore is characterizing the effects of component level geometric and form deviations on slideway errors.

There are six error motions associated with the six degrees of freedom possible for the saddle of a machine slideway. Three of these are translational errors δx, δy, and δz, and the remaining three are angular - pitch α (rotation about the Z axis), yaw β (rotation about the Y axis) and roll γ (rotation about the X axis) shown in Figure 1. The combined influence of these errors results in three components of the tracking error viz., e_x, e_y and e_z in the three axis directions at the saddle. The angular errors can potentially magnify the tracking error in a given direction by the lever arm effect. Slideway errors depend on a number of factors including profile and form error of the saddle, base and guide surfaces and thermal distortions. Additional factors such as wear and foundation effects may also influence the errors. A model of slideway errors is presented first, followed by two models to predict effects of component level geometric and form deviations on yaw and pitch errors.

SLIDEWAY ERROR MODEL

The components of the tracking error may be computed using coordinate transformations provided
the positional error \( \delta_x(x) \) and angular errors \( \alpha(x), \beta(x) \) and \( \gamma(x) \) are known at that point.

A mapping between geometric deviations of slideway parts such as saddle, base, guide rails, and the tracking error. The method uses homogenous transformation matrices (HTMs) to move from one coordinate frame to another. Figure 1 illustrates a slideway and the equivalent kinematic chain that may be used for analysis. Four coordinate frames are used to move through the chain. The reference frame \( F_1 \) is located on the seating face of the base with its origin at the center of the dowel hole locates the slideway on the machine tool structure. Two frames \( F_2 \) and \( F_3 \) are located at the sliding joint, one on the saddle and the other on the guide rail. The final frame \( F_4 \) is located on the upper surface of the saddle and constitutes the workpiece coordinate system. Positional and angular errors of both the joint and slideway parts may be described with the elements of homogenous transformation matrices that run through the kinematic chain. Three HTMs are used to transform vectors from frame \( F_1 \) to frame \( F_4 \). Using the approach presented by Ferreira & Liu [1986], the three HTMs are computed and the position vector of the origin of the saddle \( F_4 \) coordinate frame is transformed to the base \( F_1 \) coordinate frame by the equation:

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix} = [T]
\begin{bmatrix}
x + a_1 + a_2 \\
b_1 + b_2 \\
c_1 + c_2 \\
1
\end{bmatrix} + \begin{bmatrix}
e_x \\
e_y \\
e_z \\
0
\end{bmatrix}
\]

(2)

where \([T]\) is the product of the three HTMs. The system of equations is solved for the three unknowns \( e_x \), \( e_y \) and \( e_z \) [Daniel et al., 1997]. The solution accuracy depends on the order of the approximating polynomials \( \alpha(x), \beta(x), \gamma(x) \) and \( \delta(x) \). In many machine tool applications a first order polynomial is found sufficient to approximate the roll, pitch and yaw variations along the axis [Ferreira & Liu, 1986]. If the angular errors are modeled by linear equations, and if second order error terms are neglected, the tracking error components for the slideway are given by:

\[
e_x = -b_2(\alpha_1 + \alpha_2) + c_2(\beta_1 + \beta_2) -b_1\alpha_1 + c_1\beta_1 + \Delta a_1 + \Delta a_2 + x(-b_2A_1 + c_2B_1 + D_1)
\]

(3)

\[
e_y = a_2(\alpha_1 + \alpha_2) - c_2(\gamma_1 + \gamma_2) + a_1\alpha_1 + c_1\gamma_1 + \Delta b_1 + \Delta b_2 + x(a_1A_1 - c_1C_1 + \alpha_1) + x^2\left(\frac{1}{2}a_2\right)
\]

(4)

\[
e_z = -a_2(\beta_1 + \beta_2) + b_2(\gamma_1 + \gamma_2) -a_1\beta_1 + b_1\gamma_1 + \Delta c_1 + \Delta c_2 + x(-a_2B_1 + b_2C_1 + \beta_1) + x^2\left(\frac{1}{2}B_1\right)
\]

(5)

These errors vary with saddle position and may be approximated by polynomial equations in \( x \) [Kirdena & Ferreira, 1994]:

\[
\alpha(x) = \sum_{n=1}^{\infty} A_n x^n;
\beta(x) = \sum_{n=1}^{\infty} B_n x^n
\]

(1)

\[
\gamma(x) = \sum_{n=1}^{\infty} C_n x^n \quad \text{and} \quad \delta(x) = \sum_{n=1}^{\infty} D_n x^n
\]

(2)

Errors \( \delta_x \) and \( \delta_z \), related to slideway clearances, are assumed to be constant along the travel of the slide. Coordinate transformations may be used to obtain
The $\Delta$ and $\delta$ terms in the above equations relate deviations in slideway geometry and variations in angular errors to the tracking error in the three cartesian directions. Similar equations may be derived where the roll, pitch and yaw errors along the slide travel are defined by quadratic or higher order equations. Traditionally these angular errors are measured on a slideway once it is manufactured and assembled. However, it is desired to predict angular errors based on known component level deviations during the design of a machine tool.

SLIDEWAY GEOMETRY AND FORM DEVIATIONS

 Deviations of geometry and form of the slideway originate from manufacture and assembly inaccuracies. Further, thermal and structural deformations during use of the slideway affect these deviations. Figure 2 illustrates some common deviations of orientation (e.g., parallelism, perpendicularity), dimensions (e.g., clearance) and form (e.g., surface error) that are present on machine slideways. Parallelism deviations in the horizontal and vertical directions occur between the two guide rails of a slideway and also on opposite edges on the same guide rail. Typical tolerances set in the design for this deviation are of the order of 80 micron for a meter length of rail (0.001 inch/foot). Perpendicularity errors on the edges of slideway components cause a shift in the location of the coordinate frames in Figure 1. These in turn result in tracking errors.

 Surface form errors on guide rails is another common deviation. A representation of the form error may be obtained by fitting a curve through data points collected using a CMM. A Brown & Sharpe CMM is used for this purpose and surface form errors modeled as polynomial equations, cubic splines and bezier curves. Though similar results are obtained with each of these curve types, cubic splines are used here because of computational simplicity of the algorithm. Modeling the guide surfaces (3D) by curves (2D) is reasonable considering the small width compared to length of guide rail.

 Figure 3 shows surface form error measured on the guide surface of a ground “T” slide. A waviness of 15 micron amplitude is seen over 600 mm length of guide. Due to their long length and slender cross section, form errors are inevitable on guide rails. The occurrence of these form errors may be traced to manufacturing of the guide rails. The assembling of the guide rail to the base also contributes to form errors on guide surfaces due to forced compliance between surfaces fastened together. Additionally, thermal deformations and foundation effects influence surface form errors. Form errors potentially affect slideway roll, pitch and yaw behavior.

EFFECTS OF GEOMETRY AND FORM DEVIATIONS

 Parallelism errors of the guide rails may influence slideway errors [Slocum, 1992]. Horizontal parallelism errors (in the X-Z plane) between the two guide rails cause a variation of yaw along the travel of the slide, while vertical parallelism errors cause variation in roll and pitch along the trajectory of the axis. The variation of angular errors due to a known parallelism errors of the guide rails may be computed for various contact modes. The contact mode may be any of the cases shown in Figure 4 depending on the directions of forces and moments (clockwise / counterclockwise) acting on the saddle. The thick lines in Figure 4 represent the edges of physical parts (saddle, guide), while the thin lines represent position vectors.

 In Figure 4, a parallelism error between the two guide edges of a “T” slideway produces an inclination of $\theta$ between them. For a saddle of length, $l$, width, $w$, and distance between guide surfaces, $b$, the distance between contact points is $l_{y}$. A closed form equation of yaw error as a function of saddle position ‘x’ for the
contact case (a), is derived. The position vector for point B with respect to origin O may be written as:

$$ \mathbf{r}_B = (x + l_d \cos \beta) \hat{x} + (l_d \sin \beta) \hat{k} $$ (6)

and from coordinate geometry:

$$ l_d \sin \beta = \tan \theta (x + l_d \cos \beta) + b $$ (7)

Eq. (7) may now be rearranged to obtain:

$$ \beta(x) = \theta + \arcsin \left( \frac{x \sin \theta + b \cos \theta}{l_d} \right) $$ (8)

For a slideway with guide rail parallelism errors of 75, 150 and 300 micron / meter of guide length, the yaw error predicted by Eq. (8) are illustrated in Figure 5.

Form deviations affect slideway errors in two ways. On a macro level they affect the straightness of travel of the saddle. On a micro level they affect the contact stiffness of the sliding joint. These two effects need to be included within any predictive model of slideway errors. For any saddle position $$x$$, a curve representing the saddle may be overlaid on the curve representing guide surface form error. The effect of contact deformation at the joint should now be included.

Researchers Levin[1967], Furukawa & Moronuki[1987], and others have used the following equation to model contact deformation at slideway joints:

$$ \lambda = c \sigma^m $$ (9)

In Eq. (9), $$\lambda$$ is the deformation and $$\sigma$$ is the stress at the point. The coefficients 'c' and 'm' vary with material and surface preparation of the contacting surfaces.

Values for the coefficients exist in the literature for common slideway materials and finishing methods. Using Eq. (9) locally at every point of a joint, Levina[1967] and Tenner[1968] computed the load induced deflection of joints for cases with simple uniform form errors on the contacting surfaces (e.g., parabolic, triangular, sinusoidal). The inclusion of arbitrary form deviations of real surfaces in contact was not explicitly considered in the above models. The geometry of contacting surfaces in a machine slideway make it a non-Hertzian problem that is best modeled by numerical methods [Johnson, 1985]. Two possible methods for this study are a variational approach in which an appropriate energy function is minimized and an elastic foundation approach suitable for arbitrary contact geometries. Use of the variational approach for this problem is not suitable due to complexity in meshing an edge with arbitrary form error and excessive computation time to reach a solution.

A contact deformation algorithm including effects of arbitrary form error of the joint surfaces is developed in this work for a "T" slideway. This model uses Winkler's elastic foundation model which states that contact pressure at a point depends only on displacement at that point. The elastic deformation and pressure distribution at the joint are determined for contact of surfaces with arbitrary form error. Within the algorithm, the form errors on the contacting surfaces are represented by two cubic splines. The two curves have a common coordinate system and can be moved relative to each other using subroutines for translational and rotational transformations. The interference $$\lambda$$, between the curves at each contact point in the joint can be identified and Eq. (9) used to compute stress at that point. By integrating the point stresses over the contact length of the joint, the total reaction force and moment are obtained. Within the model, a numerical algorithm iteratively changes the position and inclination of the upper curve relative to
Figure 6: Pitch Error Computation from Contact Deformation Algorithm

Figure 7: Iterative Translations and Rotations for Force and Moment Balance

the lower curve until a force and moment balance with the loads applied on the joint is achieved.

This modeling approach is illustrated for the case of pitch computation for a "T" slideway. Using Eq. (9) with values of 'c' and 'm' from Furukawa & Moronuki [1987] for a cast iron slideway joint, the iterative algorithm was used to compute pitch error subject to form error of the guide surface shown in Figure 3, and contact deformation at the joint due to the weight 'W' of the saddle. These are illustrated in Figures 6 and 7. For a guide width b, and a small joint length dx, the reaction force dF, and moment dMx0, are computed about the saddle center point x0 using the following equations:

\[
dF = b\sigma_d dx = b\left(\frac{\lambda}{c}\right)^{1/m} dx
\]

\[
dM_{x_0} = xb\sigma_d dx = xb\left(\frac{\lambda}{c}\right)^{1/m} dx
\]

To achieve static equilibrium, the forces and moments must be balanced over the entire joint length:

\[
W = b\int_{-L}^{L} x\left(\frac{\lambda}{c}\right)^{1/m} dx
\]

\[
\int_{-L}^{L} x\left(\frac{\lambda}{c}\right)^{1/m} dx = 0
\]

For various positions of the saddle (x0), the vertical displacement and orientation of the saddle are iteratively changed and the total reaction force and moment from the elastic deformation computed. The iteration step size is modified within the algorithm, depending on the reaction forces and moments computed until a balance with the external loads is achieved. At this point, the pitch error is computed. The pitch error depends on both the form error on the guide surface and the normal load applied at the sliding joint.

**MODEL PREDICTIONS AND MEASUREMENTS**

The yaw predictions obtained using Eq. (8) for known parallelism error of guide rail are compared with measurements made on a "T" slideway. Experiments were conducted on a special slideway on which a horizontal parallelism error of 250 micron / 300 mm (~0.01 inch per foot) between guide edges was introduced by a grinding operation. Yaw measurements were then made along the travel of the slide for contact cases (a) and (b) shown in Figure 4. Figure 8 compares predicted yaw behavior made using Eq. (8) with measured values of yaw on the slideway for contact case (a). It is observed that the yaw change along the travel is close to linear for small values of parallelism errors typically encountered in manufacture of machine guide rails. This may explain the linear variation of angular errors along machine axis reported by previous researchers Ferreira & Liu [1986].

To verify the contact deformation algorithm and to evaluate its prediction of pitch error, a set of
measurements were made on a "T" slideway. For the slideway studied, the pitch error predicted by the contact deformation algorithm using Eqs. (10-13) is compared to autocollimator readings of pitch error in Figure 9. A Nikon 6D autocollimator with a resolution of 0.5 arc sec. was used for this purpose. It is observed that the trend in pitch error is well characterized by the model predictions. Abrupt variations in the pitch prediction occur when contact deformation is not included in the model (W=0). The inclusion of contact deformation through the proposed algorithm results in a smoother variation of pitch error.

CONCLUSIONS

Models have been developed and experimentally evaluated for estimation of slideway yaw and pitch errors due to guide rail parallelism and form deviation respectively. An algorithm for prediction of the contact deformation of surfaces with arbitrary form error has also been developed based on an elastic foundation model. Specific findings include:

- For a "T" slideway, guide parallelism deviation affects yaw error and guide form deviation affects pitch error
- A 2-D representation of the form error on guide surfaces may be adequate for modeling pitch and yaw errors
- An elastic foundation model is useful for prediction of slideway joint contact deformation
- The inclusion of contact deformation at the slideway joint is important for achieving smooth variation of predicted pitch error of a "T" slideway

The inclusion of contact deformation of surfaces with arbitrary form errors provides a foundation for the analysis of slideway joint deformation.

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