ABSTRACT

The current trend in the automotive industry is to minimize/eliminate cutting fluid use in most machining operations. Research is required prior to achieving dry or semi-dry machining. Issues such as heat generation and transfer, thermal deformation and fluid lubricity related effects on tool life and surface roughness determine the feasibility of dry machining. This paper discusses recent advances in achieving dry/semi-dry machining. As the first step, research has been conducted to investigate the actual role of fluids (if any) in various machining operations. A predictive heat generation model for orthogonal cutting of visco-plastic material was created. A control volume approach allowed development of a thermal model for convective heat transfer during machining. The heat transfer performance of an air jet in dry machining was explored. The influence of machining process variables and cutting fluid presence on chip morphology was investigated through designed experiments. The transport of chips along drill flutes was studied for both wet and dry drilling of Aluminum alloys. Finally a model for cutting fluid mist formation in wet machining is presented. These research findings are being used to gain a complete understanding of the mechanisms at work in dry and semi-dry machining.

INTRODUCTION

Cutting fluids have traditionally been used in machining operations to obtain higher tool life and better part quality. The cost of cutting fluid use in machining however, is on the rise due to environmental concerns in fluid disposal and higher treatment costs. A recent German study indicates that 16% of machining cost in the high volume manufacturing industry may be attributed to use of cutting fluids [Csele,1995] and only 4% are associated with cost of cutting tools.

Recent advances in tool materials and the capability of high speed machining have made it possible to perform some machining operations without cutting fluid use. Open faced machining operations such as turning and face milling are widely performed dry. Operations such as drilling, reaming and grinding however present a much bigger challenge to dry machining. There is a clear trend in industry to either minimize or eliminate cutting fluids from machining operations. There are many research issues to be addressed within the framework of dry and semi-dry machining. For example, the functions of cutting fluids have not been clearly established for many machining operations.

One of the functions of cutting fluids in machining is to cool the workpiece by efficiently removing heat away from the cutting zone. Cutting fluid heat transfer in machining depends on the nature of the fluid formulation and the application strategy chosen for the particular operation. These two factors determine the convective heat transfer coefficient for the operation. While information on fluid composition and recommended concentrations are available from vendors, process-focused data on the thermo-physical properties of cutting fluids are not available.

A number of researchers have investigated heat generation and transfer in machining. Analytical and experimental studies have been conducted to quantify temperatures at or near the cutting zone in metal cutting [Chao and Trigger, 1958, Strenkowski and Moon, 1990]. The functions of cutting fluids in metal machining have been extensively studied in the technical literature [Merchant, 1958, De Chiffre, 1988]. The influence of cutting fluid use on temperatures and heat transfer in machining has also been investigated by many researchers [Shaw et al., 1958, Childs et al., 1988]. Heat generation and transfer models are introduced in subsequent sections of this paper to better understand the underlying mechanisms of cutting fluids in machining.

A mist waste stream is generated through the use
of cutting fluids in machining. Oil mist formed and not sufficiently captured can remain suspended in air for many hours, and will eventually settle over large areas of the plant [Marano.1995]. To ensure that workers have minimal exposure to cutting fluid mist, machine enclosures and mist collectors may be used. However, the collectors and machine enclosures are expensive and their effectiveness may be somewhat suspect. A model for cutting fluid mist formation by atomization is presented.

Drilling is the most widely used machining operation in the automotive industry and presents unique challenges to elimination of cutting fluids. Lubricity and heat transfer issues influence drill life and hole quality. Experiments were conducted to investigate the true roles of fluid in drilling of aluminum alloys under a wide range of process conditions and the findings of this research are presented. The multifaceted nature of the research is presented in sections dealing with heat generation and transfer, chip formation and transport and fluid mist formation.

HEAT BALANCE IN MACHINING

Heat generation in machining occurs in two locations in the cutting zone. The primary heat source is located in the shear zone and generates heat due to plastic deformation of the work material. The secondary heat source lies along the chip tool interface and generates heat due to friction as the chip slides along the tool rake face. The generated heat is transferred into the workpiece, chip and tool. The equation for the heat balance is given by:

\[ q_g = q_w + q_c + q_t \]  

where \( q_g \) is the rate of heat generation and \( q_w \), \( q_c \) and \( q_t \) are heat transfer rates to the workpiece, chip and tool. The heat transfer rate to the workpiece, chip and tool, is made up of a heat storage rate and a heat transfer rate to the surrounding fluid. These may be represented by the following equations:

\[ q_w = q_{ws} + q_{wf} \]  
\[ q_c = q_{ct} + q_{cf} \]  
\[ q_t = q_{ts} + q_{tf} \]

where \( q_{ws} \) represents the rate of heat storage and \( q_{wf} \), the rate of heat lost to surrounding fluid for each component ‘a’ as illustrated in Figure 1. In considering the entire workpiece, for one machining cycle, all of the heat generated in the cutting process is either stored in the workpiece, chip and tool or transferred to the cutting fluid medium.

**FIGURE 1: HEAT BALANCE IN MACHINING**

Boring tests were performed on workpieces of Aluminum alloy 308 material on a Cincinnati Milacron CNC milling machine. To enable heat balance studies, a special work holding fixture was used in which all of the cutting fluid that contacts the work surface (inner diameter) could be collected in an insulated container for temperature measurements. Heat generation was computed using an analytical method based on the measured cutting force. At the end of the machining cycle, the total heat generated during the cutting process is either removed by the cutting fluid or stored in the workpiece to be released slowly into the environment.

\[ Q_{gen} = Q_{fluid} + Q_{workstored} + Q_{unaccounted} \]  

In a boring test with a spindle speed of 1000 rpm, feed of 0.01 ipr and depth of cut of 0.035 inch, the following heat balance results were obtained:

<table>
<thead>
<tr>
<th>Table 1: Heat Balance in Boring</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heat</strong></td>
</tr>
<tr>
<td><strong>Component</strong></td>
</tr>
<tr>
<td>( Q_{gen} )</td>
</tr>
<tr>
<td>( Q_{fluid} )</td>
</tr>
<tr>
<td>( Q_{ws} + Q_{unacc} )</td>
</tr>
</tbody>
</table>
A MODEL FOR HEAT GENERATION

A predictive heat generation model for orthogonal cutting of visco plastic material has been developed based on the principle of minimization of the total work done [Zheng et al, 1996]. Under an assumed streamline and shear plane curvilinear system, the deformation rate tensor in the primary zone is obtained analytically. The secondary zone is modeled as a plastic boundary layer in which the flow field of velocity and deformation rate is also obtained.

The overall work per unit width of cutting, $W$, done in machining is the sum of that done in the primary zone, secondary zone as well as at the chip/tool interface, and is given by:

$$W = \int \psi_p dq \cdot dq^2 + \int \psi_s dx dy + \int \psi_f dx$$  \hspace{1cm} (6)

where $J$ is the Jacobian determinant for transformation from the streamline coordinate system to global. The variables $\psi_p$, $\psi_s$, and $\psi_f$ refer to the power consumed for plastic deformation in the primary and secondary zones and $\psi_f$ refers to the frictional power in the rake face.

Temperature predictions from the heat generation model for orthogonal cutting of plain carbon steel are presented in Figures 2 and 3. The cutting velocity for the example is 100 m/min, the undeformed chip thickness and rake angle are 0.25 mm and $\alpha = 5$ deg.

It may be seen from the plots that maximum temperatures in the cutting zone occurs in the secondary zone behind the tool tip in the contact zone.

**MODELS FOR HEAT TRANSFER**

One of the modes of heat transfer in machining with cutting fluids is by forced convection to the fluid. Heat may also be transferred to work holding devices by conduction and a small part may be radiated into the surrounding environment. In dry machining, convective heat transfer to air occurs but is much less intense. A model for heat transfer requires determination of thermophysical properties of the media (cutting fluid or air) and estimations of heat transfer coefficients and wall temperatures.

**Determination of Fluid Properties**

Cutting fluid formulations vary in their thermophysical properties and hence in their heat transfer characteristics. While water is an excellent heat transfer medium, water miscible cutting fluids and straight oils may exhibit considerably poorer heat transfer performance. The thermo-physical fluid properties influencing heat transfer are dynamic viscosity $\mu$, specific heat capacity $c_p$, mass density $\rho$, and thermal conductivity $k$.

A search for fluid properties reveals that the thermo-physical properties of many commonly used cutting fluid emulsions are not available. The thermophysical properties of common cutting fluids were experimentally determined and used in the heat transfer calculations. Cutting fluid viscosities were measured using a Canon Fenske viscometer equipped with a constant temperature bath. Fluid specific heat was measured using an electrically heated calorimeter. Figures 4 and 5 show measured fluid viscosities and specific heat as a function of temperature and oil concentration respectively.
The traditional correlations for forced convection heat transfer use a heat transfer coefficient (film coefficient) of the form:

\[ h = f(Nu) = f(Re, Pr, geometry) \]  

(8)

The generic forms of the Nusselt number Nu, Reynolds number Re and Prandtl number Pr, may be found in standard text books on heat transfer [Holman, 1966]. For cylindrical workpieces, these numbers are given below:

\[ Nu = \frac{hD}{k} \]  

(9)

which relates to the Reynolds number Re, and Prandtl number Pr, through a power law equation. The Reynolds and Prandtl numbers are given by:

\[ Re = \frac{\rho VD}{\mu} \]  

(10)

\[ Pr = \frac{\mu c_p}{k} \]  

(11)

The Reynolds number, depends on both the type of fluid and application method and the Prandtl number, depends only on the fluid properties. The Nusselt number for a given set of conditions may be calculated using empirical correlations available in the heat transfer literature. From the Nusselt number, the heat transfer coefficient may then be computed using Eq. (9).

Estimation of Heat Transfer Coefficient

In machining operations, the heat transfer from the workpiece to cutting fluid is primarily by forced convection with either laminar or turbulent boundary layers depending on the relative velocities of the work surface and the fluid flow. The heat transfer rate is given by the following equation:

\[ q = hA(T_w - T_f) \]  

(7)
The selection of an empirical correlation for the Nusselt number depends on the nature of the machining operation, the generic shape of the workpiece and the method of application of the fluid.

![Velocity Vectors in Jet Application of Fluid](image)

\textbf{FIGURE 7: VELOCITY VECTORS IN JET APPLICATION OF FLUID}

A model for fluid application in a turning operation flooded with cutting fluid as shown in Figure 6, requires a correlation for forced convective heat transfer from a rotating cylindrical surface with transverse fluid flow. A Nusselt number correlation [General Electric, 1985] with these effects is shown in equation (12) and has been used in the heat transfer model for turning. The equation involves two Reynolds numbers, one ($Re_t$) that takes into account the effect of workpiece rotation and the other ($Re_I$) that accounts for the effect of transverse fluid flow. The effect of natural convection is modeled in the correlation using the Grashoff number ($Gr$).

\begin{equation}
N_u = 0.1075 Pr^{0.35} \left(0.5 Re_t^2 + Re_I^2 + Gr\right)^{0.35}
\end{equation}  \hspace{1cm} (12)

Using equation (12), the heat transfer in a turning operation with pure water as cutting fluid is found to lie between 9000 to 12000 W/m²K depending on the cutting speed and the fluid velocity.

For a jet application of cutting fluid on the workpiece as shown in Figure 7, an appropriate correlation for the Nusselt number [Martin, 1977] is given by equation (13).

\begin{equation}
\frac{\overline{N_u}}{Pr^{0.42}} = 2 Gr Re^{0.5} (1 + 0.005 Re^{0.55})^{0.5}
\end{equation} \hspace{1cm} (13)

where $G$ is given by

\begin{equation}
G = \frac{D_s}{R \left(1 + 0.1 \left(\frac{H}{D_s} - 0.6\right)D_s/R \right)}
\end{equation} \hspace{1cm} (14)

Fluid heat transfer in a cylinder boring operation flooded with cutting fluid, may be modeled using correlations for convective heat transfer from a vertical flat plate to fluid in parallel flow. The effect of surface curvature on the convective heat transfer should be included in the model. A suitable correlation for the Nusselt number for a boring operation [Churchill and Ozoe, 1973] involving laminar fluid flow is given by equation (15).

\begin{equation}
N_u = \frac{0.3387 Re_t^{0.5} Pr^{0.33}}{(1 + (0.0468/Pr)^{0.66})^{0.25}}
\end{equation} \hspace{1cm} (15)

Once the Nusselt number for a given operation is determined, the heat transfer coefficient may then be calculated using equation (9). The heat transfer rate may then be computed using equation (7) provided the wall temperature is known.

\textbf{Wall Temperature Model}

The wall temperature is a function of both time and position and needs to be determined. The transient temperature distribution in the workpiece may be determined by applying the energy balance on a differential element as shown in Figure 8. For a ring shaped control volume in the thin walled cylinder, the heat balance involves terms for heat generation, heat storage, heat conduction across the boundaries and heat convection from the walls. The governing equation and boundary conditions [Cozzens, 1995] are given in Eqs. (16-19).

![Control Volume for Heat Transfer Model](image)

\textbf{FIGURE 8: CONTROL VOLUME FOR HEAT TRANSFER MODEL}
\[ 2\pi r \Delta T \left[ \frac{\partial T}{\partial x} \right] - k2\pi r \Delta x \frac{\partial T}{\partial x} \bigg|_{x+\Delta x} - 4h\pi r \Delta x (T - T_f) + \]

\[ 2\pi r \Delta x \Delta g(x, t) = \rho 2\pi r \Delta x \Delta c \frac{\partial T}{\partial t} \]

(16)

with the boundary conditions:

\[ -k2\pi r \frac{\partial T}{\partial x} = h2\pi r (T - T_f) \quad \text{at } x=0 \]

(17)

\[ k2\pi r \frac{\partial T}{\partial x} = h2\pi r (T - T_f) \quad \text{at } x=L \]

(18)

\[ T - T_f = 0 \quad \text{at } t=0 \]

(19)

The governing equation may then be solved to obtain the temperature field along the work surface at any time. The solution for temperature in Eq. (20) in terms of the kernel \( K_n \) of the transform for the eigenvalues \( \lambda_n \) was developed by Cozzens [1995].

\[ T(x, t) = \sum_{n=0}^\infty K_n(\lambda_n, x)\tilde{\phi}(\lambda_n, t) + T_f \]

(20)

Figures 9 and 10 illustrate the predicted wall temperature histories obtained for a case study involving a turning operation performed without and with cutting fluid use. The temperature histories plotted, are for two locations, one near the start of the workpiece (position 1) and the other further into the workpiece (position 2).

\[ dQ = 2\pi rh dL \int_c (T_w - T_f) \, dt \]

(21)

The heat transfer from the entire workpiece may be obtained by integrating the zonal heat transfer given by equation (21) over the length of the workpiece.

\[ Q = 2\pi rh \int_0^L (T_w - T_f) \, dL \]

(22)
Heat Transfer Model Results

A Matlab program was developed to compute heat transfer coefficients and total workpiece heat transfer for turning and boring operations. The program outputs for different fluids and application variables are shown in Table 2 and Figures 12 and 13.

Table 2: Calculated Heat Transfer Coefficients (Flood Application)

<table>
<thead>
<tr>
<th>Fluid Type</th>
<th>Heat Transfer Coefficient (W/m²·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>11830</td>
</tr>
<tr>
<td>Air Stream</td>
<td>238</td>
</tr>
<tr>
<td>3% C225 Soluble Oil</td>
<td>11240</td>
</tr>
<tr>
<td>10% C225 Soluble Oil</td>
<td>9933</td>
</tr>
<tr>
<td>10% SS-405L Semi-Synthetic Fluid</td>
<td>10368.4</td>
</tr>
<tr>
<td>10% Vytron-N Synthetic Fluid</td>
<td>10807.6</td>
</tr>
</tbody>
</table>

A set of calculated values of heat transfer coefficients are shown in Table 2. The workpiece diameter used in the calculation is 30 mm and the spindle speed and fluid flow velocity are 2000 rpm and 1 m/sec. It may be seen that water has the best heat removal properties among the liquids considered in the computation. In dry machining, an air flow may be used as a heat transfer medium but with a much inferior cooling performance due to the low specific heat capacity and mass density of air. Higher air stream velocities may be used to improve the cooling performance.

Fluid application parameters also play an important role in determining the heat transfer rate from workpiece to the cutting fluid. The influence of spindle speed and fluid velocity on the heat transfer coefficient for the above turning operation is shown in Figures 12 and 13.

A Model for Workpiece Thermal Deformation

Temperature predictions from Eq. 20 may be used to compute thermal deformation of the workpiece. The thermal deformations are the result of stresses occurring within the workpiece as a result of temperature gradients. For a hollow cylindrical workpiece, the radial, hoop and axial components of the stresses may be computed if the temperature distribution is known. Azimuthal symmetry may be assumed for the analysis based on the high temperature gradient in the radial direction as compared to the other two directions. The analysis simplifies to a plane strain problem and the displacements as a result of thermal expansion is described by the differential equation below.

\[
\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} = (1 + \nu) \frac{\beta d\theta}{dr}
\]

The surface error due to thermal stresses in a hollow cylinder is the negative of the displacement of the work surface which is computed using the following equation [C

Figure 12: Effect of Spindle Speed on Heat Transfer Coefficient in Turning (Flood)

Figure 13: Effect of Fluid Velocity on Heat Transfer Coefficient
The inner and outer radii of the cylinder are denoted by 'a' and 'b' respectively.

\[ SE_z(z_t(t)) = \frac{2\pi a}{a^2 - b^2} \int_{-a}^{b} \Theta(r, z_t(t)) r dr \]  \hspace{1cm} (24)

**A MODEL FOR FLUID MIST GENERATION**

One of the harmful waste streams generated in conventional wet machining is cutting fluid mist. Oil mist can remain suspended in air for many hours and inhalation by workers may cause serious respiratory illnesses. Mist collection systems have been widely used in industry to ensure that workers have minimal exposure to cutting fluid mist. The Occupational Safety and Health Administration (OSHA) standard for airborne particulates is 5 mg/m³, and the UAW has proposed to reduce the standard to 0.5 mg/m³.

![Mechanisms of Mist Formation](image1)

**FIGURE 14: MECHANISMS OF MIST FORMATION**

During a machining process, the heat generated may vaporize the cutting fluids and subsequently form the droplets by condensation of vapor around tiny, spontaneously generated liquid nuclei, or other foreign particles such as dust. Additionally as shown in Figure 14, fluid atomization occurs when cutting fluid is used in machining. The mechanism of cutting fluid mist formation via atomization has been investigated. As the cutting fluid impacts a flat workpiece surface, a liquid film develops over the surface and disintegrates into droplets. For a cutting fluid jet impinging on a rotating workpiece, three different disintegration modes (drop mode, ligament formation mode and film formation mode) have been considered in the research [Yue et al., 1996]. For a rotating workpiece as in a turning operation, Figure 15 illustrates mist droplet formation by drop and ligament formation modes. Using the drop mode of atomization (valid for low fluid flow rates) the mean mist drop diameter has been computed using Eq. 25. For the ligament mode of atomization (valid for higher fluid flow rates) however, Eq. 26 provides an estimate of the mean mist drop diameter.

\[ D = \frac{\varepsilon}{\omega} \left( \frac{r}{\rho} \right)^{\frac{1}{2}} \]  \hspace{1cm} (25)

\[ D = \left( \frac{3\pi}{\sqrt{2}} \right)^{\frac{1}{6}} \left[ 1 + \frac{3\mu}{(\rho \gamma \dot{d})^{1/2}} \right] \]  \hspace{1cm} (26)

![Atomization Modes](image2)

**FIGURE 15: ATOMIZATION MODES - (A) DROP MODE AND (B) LIGAMENT MODE**

![Comparison of Mist Droplet Size](image3)

**FIGURE 16: COMPARISON OF MIST DROPLET SIZE**

A comparison of the mist droplet size for different cutting fluids based on their physical properties and application strategies is being developed. Figure 16
shows the comparison of predicted mist drop size as a function of spindle speed for a turning case study.

**CUTTING FLUID EFFECTS IN DRILLING**

Experiments were designed and conducted on aluminum alloys and gray cast iron to determine the function of cutting fluid in drilling. The variables studied were the speed, feed, hole depth, drill type and geometry, work material, fluid conditions, and workpiece temperature. The results indicated that cutting fluid has a significant effect on hole surface finish and that it interacts with other process variables to have an effect on hole quality [Haan et al., 1995]. Figure 17 shows a normal probability plot for drilling torque clearly identifying the significant variables. The x-axis refers to estimates of the effects of changing each parameter and the y-axis is the cumulative probability in terms of standard normal variable.

![Figure 17: Factors Influencing Drilling Torque](image)

**FIGURE 17: FACTORS INFLUENCING DRILLING TORQUE**

The effects of dry and wet machining on the chip morphology in drilling of cast aluminum alloys has also been investigated [Batzler et al., 1996]. Statistically designed experiments were conducted to identify significant process variables that affected the size and shape of the generated chips. Various types of chips were collected and identified as shown in Figure 18, and from the data on chip shape and size, normal probability plots were constructed to determine significant effects. It was found that the feed, alloy type, drill type and cutting fluid presence were significant in influencing chip morphology.

Chip transport away from the cutting zone is influenced by cutting fluid presence particularly in closed face cutting operations such as drilling. Lubricity at the drill margins provided by the cutting fluid is currently being investigated as part of an effort toward modeling chip transport along the flutes of a drill.

**FIG. 18: GENERATED CHIP FORMS (A: CONICAL, B: FAN SHAPED, C: CHISEL EDGED, D: AMORPHOUS, E: NEEDLE, F: IMPACTED).**

The effects of dry and wet machining on the chip morphology in drilling of cast aluminum alloys has also been investigated [Batzler et al., 1996]. Statistically designed experiments were conducted to identify significant process variables that affected the size and shape of the generated chips. Various types of chips were collected and identified as shown in Figure 18, and from the data on chip shape and size, normal probability plots were constructed to determine significant effects. It was found that the feed, alloy type, drill type and cutting fluid presence were significant in influencing chip morphology.

Chip transport away from the cutting zone is influenced by cutting fluid presence particularly in closed face cutting operations such as drilling. Lubricity at the drill margins provided by the cutting fluid is currently being investigated as part of an effort toward modeling chip transport along the flutes of a drill.

**SUMMARY AND CONCLUSIONS**

Both experimental and analytical research are being performed to address various issues pertaining to dry and semi-dry machining. Analytical models have been developed to predict the phenomenon of heat generation, transfer, mist formation and chip formation in machining with and without cutting fluid use. Using appropriate correlations for the Nusselt number, the heat transfer coefficient for turning and boring operations with flood and jet application of fluid have been calculated. Workpiece wall temperature has been modeled using a energy balance involving heat generation, storage, conduction and convection for a
control volume. The thermo-physical properties of soluble oil emulsions, semi-synthetic and synthetic cutting fluids were experimentally determined. Designed experiments were conducted on drilling of aluminum alloys to investigate the effects of cutting fluid on hole quality, surface finish and chip morphology.

From the experiments on cutting fluid properties, it is clear that the heat transfer performance of water is superior to the cutting fluids used in this study. Synthetic cutting fluid is closest to water in heat transfer performance while semi-synthetic and soluble oil fluids fare considerably poorer. When using soluble oil emulsions, heat transfer performance deteriorates with higher oil concentration in the fluid formulation. Cutting fluid heat transfer increases with cutting speed and velocity of fluid flow. Jet application of cutting fluid is effective in achieving rapid heat transfer from a small area of the workpiece. An air jet used for cooling the cutting zone in dry machining removes only a fraction of the heat that a jet of cutting fluid would. Efforts are underway to improve the cooling achievable with an air jet using high speed jets and vortex tubes.

Results from drilling tests indicate that cutting fluid has a significant effect on hole surface finish and that it interacts with other process variables to have an effect on hole quality. There is evidence that cutting fluid lubricates the drilling process at the margins of the drill and influences the morphology of drill chips. In order to achieve dry drilling, efforts are presently underway to characterize and model chip transport along the drill flutes.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge funding assistance for this research from the NSF-ARPA Machine Tool Agile Manufacturing Research Institute (MT-AMRI), the NSF under DMI-9502109, the State of Michigan REF and the Ford Motor Company.

REFERENCES


DEFINITIONS, ACRONYMS, ABBREVIATIONS

β - coefficient of thermal expansion

Cp - specific heat
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>temperature at any point on workpiece</td>
</tr>
<tr>
<td>$T_w$</td>
<td>wall temperature of the workpiece</td>
</tr>
<tr>
<td>$g(x,t)$</td>
<td>heat generation per unit volume</td>
</tr>
<tr>
<td>$h$</td>
<td>heat transfer coefficient</td>
</tr>
<tr>
<td>$A$</td>
<td>surface area</td>
</tr>
<tr>
<td>$T_f$</td>
<td>fluid temperature</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity of workpiece material</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$Gr$</td>
<td>Grashoff number</td>
</tr>
<tr>
<td>$q$</td>
<td>heat transfer rate</td>
</tr>
<tr>
<td>$Q$</td>
<td>heat transfer from entire work surface</td>
</tr>
<tr>
<td>$q^1, q^2$</td>
<td>coordinates in the streamline system</td>
</tr>
<tr>
<td>$W$</td>
<td>work done</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>mass density of work material</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity (generic)</td>
</tr>
<tr>
<td>$Nu_x$</td>
<td>local Nusselt number</td>
</tr>
<tr>
<td>$\bar{Nu}$</td>
<td>Average Nusselt number</td>
</tr>
<tr>
<td>$D_n$</td>
<td>jet nozzle diameter</td>
</tr>
<tr>
<td>$R$</td>
<td>radial distance from jet stagnation point</td>
</tr>
<tr>
<td>$r$</td>
<td>workpiece radius</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of workpiece</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$L$</td>
<td>length of workpiece</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>thermal diffusivity of work material</td>
</tr>
<tr>
<td>$\theta$</td>
<td>temperature gradient</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>integral transform of $\theta$, where $\theta = T - T_f$</td>
</tr>
<tr>
<td>$K_n$</td>
<td>kernel of Fourier transform</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>eigenvalues of the governing equation</td>
</tr>
<tr>
<td>$\phi$</td>
<td>mean droplet diameter</td>
</tr>
<tr>
<td>$\omega$</td>
<td>workpiece angular velocity</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>fluid surface tension</td>
</tr>
<tr>
<td>$d_l$</td>
<td>diameter of ligament</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson's ratio</td>
</tr>
</tbody>
</table>