A DYNAMIC MODEL OF THE CUTTING FORCE SYSTEM IN PERIPHERAL MILLING
CHARACTERIZING THE EFFECTS OF FLANK FACE INTERFERENCE

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ABSTRACT
An enhanced model for the dynamic behavior of the end milling process is described. The model predicts the cutting forces and cutter deflections by including the effects of the flank face interference mechanism in addition to the chip removal effects. The interference mechanism is accounted for by considering the flank interference forces to be proportional to the interference volume. The volume of interference is estimated numerically. The total force acting on the tool is a combination of the forces due to the cutting action and forces due to the interference. Experiments performed on 6061-T6 Aluminum validate the simulation results.

INTRODUCTION
Peripheral milling is a widely used machining process in the aerospace and automotive industries. Models that can accurately simulate the performance of this process can be used by these industries to achieve significant improvement in their productivity. Several models have been proposed to study the phenomenological aspects of the peripheral milling process. Devor et al. [1980], Kline et al. [1982, 1983] presented a static model to predict the cutting forces and surface errors in end milling. This model used the relationship between the chip load and the cutting forces developed by Martelli[1941, 1945]. The model predicted the instantaneous force system characteristics such as the force distribution along the axis of the cutter, the total force and associated force center and the surface error as a function of the angle of rotation of the end mill. Sutherland [1986, 1988] extended this static flexible end milling model to obtain improved chip thickness and force predictions by including the regeneration dynamics in the milling process. This two degree of freedom dynamic model described the transverse response of the end mill and chip load geometry and predicted the dynamic cutting forces using their dependence on the chip load. The transverse response was described with a distributed parameter model by considering the end mill to be a beam cantilevered at one end. However, anecdotal evidence suggests the model predicts forces that are unbounded due to the lack of a process damping mechanism.

Most machining system models in the past, however, have not characterized the effects of the interference mechanism between the flank face and the work cut surface. The interference mechanism indirectly results in damping at the tool workpiece interface which has a significant effect on the cutting dynamics in milling. Tusty and Ismail [1981, 1983] explained the role of cutting dynamics in machining chatter, and correlated the flank wear with the positive damping during machining. Smith and Tusty [1990] found that by incorporating the effect of cutting process damping in the chip formation, the stability of the machining system against chatter could be increased. Wu [1988, 1989] considered the effects of a sinusoidally varying cut surface on the cutting forces in orthogonal cutting. Elbestawi [1994] modified the Wu model to include the effect of tool flank wear and varied machined surface undulations (wavelength) on stability against chatter. The limitation in these models is the assumption that the work surface is a sinusoidal wave of certain amplitude. Montgomery et al. [1991], Altintas et al. [1992] presented a model for end milling that accounted for process damping by considering the ploughing force to be proportional to the material yield strength and the interference contact area. The most recent development in this area has been the Dual Mechanism Approach by Endres et al. [1995] which discusses a numerical method to compute the interference volume and its use in estimating the flank forces. Force coefficients are calibrated experimentally.

The present work focuses on a flank face interference model for peripheral milling. The flank interference forces are assumed to be proportional to this interference volume. As against previous models the interference region is computed for every angle of engagement using the current tool and worksurface positions. The model predicts these positions and hence the volume based on mechanisms like cutting conditions, dynamic response, trochoidal path of the cutting edge, and the cutter geometry. This volume is used to obtain the flank forces. An approach similar to Endres'[1995] in numerically estimating the volume of the interference region is used. The cutting forces are computed as in the Sutherland model[1988]. The cutting and flank interference forces are combined to obtain the resultant forces.
A DYNAMIC FORCE MODEL FOR END MILLING

The dynamic model for the computation of the cutting forces and the deflections in the end milling process developed by Sutherland [1988] has been enhanced to include the effects of flank face interference leading to a more accurate prediction of the cutting forces. Figure 1 illustrates the end milling operation (Down milling).

\[ R(i,k) = \left[ \frac{\rho^{2} + 2 \cdot \rho \cdot \cos(\lambda - (k-1) \cdot \frac{2\pi}{N_f}) \cdot T_{hx}}{1/2} \right] \]

where \( \rho \) is the helix angle of the end mill.

\[ \alpha_{hx} \] is the helix angle of the end mill.

\[ CDX(i,j) \] is the displacement in the X direction of the \( i \)-th axial disk at the \( j \)-th angular position of the end mill relative to the workpiece.

\[ CDY(i,j) \] is the displacement in the Y direction of the \( i \)-th axial disk at the \( j \)-th angular position of the end mill relative to the workpiece.

\[ R(i,k) \] is the radius of the \( k \)-th flute, given as

\[ a = j - (k-m)/(d\theta) \cdot (2\pi/N_f) \] represents an angular increment associated with a surface machined by the flute in the past.

\( \rho, \lambda \) are the parallel axis runout parameters (offset and location angle respectively), and 

\( \tau, \phi \) are the cutter tilt parameters (cutter tilt angle and location angle respectively).

RAD is the instantaneous radius of the trochoidal path as given by Martello[1945].

The undeformed chip thickness at any tooth engagement angle \( \beta(i,j,k) \) can be described as the smallest radial distance between the path approximated as the circular arc described by the current tooth of interest on the \( i \)-th axial disk centered at the deflected position and the work surface path generated by the past teeth at the same tooth engagement angle \( \beta(i,j,k) \). The chip load equations use the difference in the present and past cutter deflection values in the X and Y directions. The regenerative nature of the chip load calculation accounts for the phase difference of the surfaces generated in the present and past cutter positions.

This chip model accurately characterizes the effects of cutter runout and system flexibilities on the chip thickness and thereby the cutting forces produced in the end milling process. The elemental tangential and radial forces on any flute for an arbitrary axial element of the end mill are computed next using the relations

\[ dF_{\text{tan}}(i,j,k) = K_{t} \cdot t_{c}(i,j,k) \cdot d\tau \]

\[ dF_{\text{rad}}(i,j,k) = K_{r} \cdot dF_{\text{tan}}(i,j,k) \]

where \( dF_{\text{tan}}(i,j,k) \) is the elemental tangential force,

\( dF_{\text{rad}}(i,j,k) \) is the elemental radial force, and,

\( K_{t}, K_{r} \) are the empirically determined tangential cutting pressures and radial to tangential cutting pressure ratio.

As mentioned earlier, a distributed parameter model characterizes the dynamic response of the end mill to the cutting forces. It essentially describes the displacement variation along the axis of the cutter. The end mill is assumed to be a cantilevered beam, fixed to the tool holder at one end and free at the other. The equation of motion for the transverse vibrations of a cantilevered beam (Fig. 2) is described by,
The flexural rigidity $EI(z)$ and mass $m(z)$ are assumed to be constant along the length of the beam for further analysis. The case of free vibration is considered first and the equation is assumed to be separable in time and space, i.e., $y(z,t) = Y(z) \cdot q(t)$, where $q(t)$ is harmonic and has a frequency $\omega$. The natural frequencies of the system are computed, and the boundary conditions for the fixed and free ends are applied to arrive at the system response $y(z,t)$.

Similarly, the dynamic displacement of the beam in the X-direction $x(z,t)$ are computed. The equations are solved numerically using a finite difference scheme to predict the displacement and velocity one time step into the future. The predicted position of the end mill is used to compute the forces acting on the end mill.

**Fig. 2: Transverse Vibrations of the End Mill**

**FLANK FACE INTERFERENCE**

Damping in the machine tool - workpiece system is attributed to two sources, namely, the structural damping from the machine tool structure and the damping arising from within the machining process itself. The structural damping component, that is composed of the energy dissipated by the workpiece, the tool-holder and such parts of the machine tool system, contributes only to a small part of the energy dissipation. Most of the energy is dissipated through the cutting process damping for low-to-medium speed machining. Attempts have been made by researchers to account for the cutting process damping by adjusting the structural damping value. But unreasonably high values for the structural damping are needed to obtain more realistic values of the cutting force system in these dynamic models.

Any machined surface with relative vibrations between the tool and the workpiece is wavy in nature. This is caused by deflections in the cutter-workpiece system or relative vibrations between the tool and workpiece. The presence of non-ideal geometric factors such as runout may also add to the waviness of the surface. The surface may be periodic in nature or random (stochastic behavior) depending on the cutting conditions and the cutter configuration. Regeneration of the surface waviness [Merritt, 1965] may also contribute to the waviness. When the cutting tool moves over an undulated surface, its flank 'rubs' or interferes with the work surface. This causes material below the tool flank to be compressed. This exerts a pressure on the tool flank. The force acting on the tool due to the interference or ploughing effect is dependent on the sharpness of the tool, the hardness of the work material and the instantaneous cutting conditions.

The proposed model computes the instantaneous volume of the interference region for every angular increment. To determine this region at every angular position requires the tool position and worksurface profile at that position. Several factors influence the tool path and worksurface waviness, some of them having been mentioned earlier. Models by Wu[1988], Elbestawi[1994] have assumed the worksurface to be a sinusoidal wave of specific amplitude and frequency. Endres et al.[1995] have computed the volume using an empirically determined penetration depth. The present research does not make any assumptions on the surface waviness nor does it use any empirical coefficients in the volume calculation. It specifically considers the effects of the following mechanisms: cutting conditions, the current as well as past deflections, static runout, the trochoidal trajectory and the cutter geometry in obtaining the worksurface and tool position. The actual wavy surface constructed based on these factors is now used in the volume and force computations. Once the instantaneous tool and work profiles are estimated, they have to satisfy certain conditions that check if the occurrence of interference is possible or not. The model does not preclude the possibility of intermittent occurrences of interference during cutting.

The flank forces are considered to be proportional to the interference volume. This concept has been used previously by Wu[1988], Elbestawi et al.[1994], Endres[1995]. The calibration of the flank interference force coefficients is done based on the elasto-plasticindentation theory. On the other hand, the cutting forces are computed using the instantaneous chip load values. The present model thus computes the cutting and flank forces as an instant of cut independently. The radial and tangential components of the cutting and flank forces are respectively added. These resultant forces are finally resolved into the external coordinate system to obtain the machining forces in the X- and Y- directions. A sharp tool is considered for the computation of the interference volume. The model assumes complete elastic recovery of the material once the tool pressure is removed.

**Geometrical Representation of the Interference Region.**

Figure 3 shows the interference geometry in down milling. The point $(x_p, y_p)$ represents the coordinates of the leading edge of the sharp tool at any tooth engagement angle $\beta(i,j,k)$. $(x_p, y_p)$ and $(x_p, y_p)$ represent the co-ordinates of the past positions of the workpiece and the tool flank respectively. $m_f$ and $m_f$ represent the slopes of the work surface and the tool respectively for the present position of the cutting edge. The instantaneous interference area is determined by approximating it as a series of triangles and trapezoids. The first and the last elemental areas of the interference region are approximated as triangles and the other elemental areas as trapezoids. For example, $A(i, j, k)$ represents the area of the interference region computed for the trapezoid whose corners are represented by the points $[1, 2, 3, 4]$ (refer to Fig. 4).
Calculation of the Interference Volume.

The interference volume is the product of the sum of all elemental areas and the thickness of the axial disks. It is given by the following equation,

\[ V(i, j, k) = \sum_{p=1}^{l} A(i, j-p, k) \cdot dz \]  

(7)

where

\[ V(i, j, k) \] is the volume of interference for the \( j^{th} \) axial disk, \( j^{th} \) angular position and \( k^{th} \) flute looking at \( i-1 \) past interference points,

\[ A(i, j-p, k) \] represents the interference area for the element at the \( (j-p)^{th} \) angular step.

Since the interference volumes are very small, the assumption of a circular tool sweep trajectory may lead to significant differences in the computation of the interference volume. To account for this the nominal radius in eqn. 3 is replaced by the trochoidal path radius \( \text{RAD}_t \) as given in Martelloti [1945] as

\[ \text{RAD}_t = \left[ \text{RAD}^2 + (\text{it} \cdot N_t \cdot (2 \cdot \pi))^2 \right]^{1.5} \]

(8)

\[ \left( 2 \cdot \left( \text{it} \cdot N_t / (2 \cdot \pi) \right) \cdot (\text{RAD} - y) \right) \]

where

\( y \) is the instantaneous depth of cut again computed using Martelloti’s equations.

‘-’ is for down milling and ‘+’ for up milling.

The position of the leading edge is obtained from the current deflections and instantaneous radius. The leading edge of the tool that is also a point on the workpiece surface, is given by the coordinates,

\[ x_0(i, j, k) = CDX(i, j) + R(i, k) \cdot \sin \beta(i, j, k) \]

(9)

\[ y_0(i, j, k) = CDY(i, j) - R(i, k) \cdot \cos \beta(i, j, k) \]

(10)

where

\[ CDX(i, j), CDY(i, j) \] are the deflections of the cutter center at that instant.

Similarly, the past positions are computed as follows:

\[ x_0(i, j-p, k) = CDX(i, j-p) + R(i, k) \cdot \sin \beta(i, j-p, k) - p_i N_t (d \theta / 2 \pi) \]

(11)

\[ y_0(i, j-p, k) = CDY(i, j-p) - R(i, k) \cdot \cos \beta(i, j-p, k) \]

(12)

where

\[ CDX(i, j-p), CDY(i, j-p) \] are the deflections of the cutter center in the past, and

\( p = 2, 3, 4 \ldots l \)

The instantaneous slopes of the workpiece surface and that of the tool face are given as:

\[ m_w, p = \frac{y_0(i, j, k) - y_0(i, j-p, k)}{x_0(i, j, k) - x_0(i, j-p, k)} \] and \[ m_t = \tan \beta(i, j, k) + \gamma_c \]

(13)

where

\( \gamma_c \) is the clearance angle of the tool, and

\[ m_{w, 1} > m_t \]

The corresponding condition for up milling is \( m_{w, 1} < m_t \).

If the above condition is satisfied, the first elemental area is approximated by a triangle (represented by the points \( \{5, 1, 2\} \)) and is given by:

\[ A(i, j-1, k) = \frac{1}{2} [(y_0(i, j-1, k) - y_0(i, j-1, k)) \cdot (x_0(i, j-1, k) - x_0(i, j, k))] \]

(16)
Past positions on the work surface are successively considered and the elemental areas are calculated as given in Eq. 17 till the end of interference zone is reached.

\[ A(i, j, p, k) = \frac{1}{2} \left[ (y(i, j, p, k) - y(i, j, p - 1, k)) + (y(i, j, p + 1, k) - y(i, j, p + 1, k)) \right] \cdot \left( x(i, j, p + 1, k) - x(i, j, p, k) \right) \]  

(17)

The end of the interference zone is indicated by either of the two conditions given below:

\[ y(i, j, p, k) < y(i, j, p, k) \quad \text{or} \quad \theta > L_f / R(i, k) \]  

(18)

(19)

where

\[ \theta = (p + 1) \cdot d \]  

\[ L_f \] is the length of the primary clearance face.

The area of the final element is again a triangle (in Fig. 4, points 6, 7, 8) and is calculated as follows:

\[ A(i, j, -l, k) = \frac{1}{2} \left[ (y(i, j, -l, k) - y(i, j, -l, k)) + (x(i, j, -l + 1, k) - x(i, j, -l, k)) \right] \]  

(20)

The interference volume is given by,

\[ V(i, j, k) = \sum_{p=1}^{P} A(i, j, p, k) \cdot dz \]  

(21)

The summation is taken over all the elements where the triangular elements are simply considered as trapezoids with one base length of zero. The volume given above represents the interference volume for one axial disk of a specific flute of the cutter. This procedure is repeated for each axial disk and each flute of the cutter and the volumes are accordingly generated.

**Force Relations**

The flank interference forces in the normal and tangential directions are given by the relations

\[ dF_{\text{rad}}(i, j, k) = V(i, j, k) \cdot K_1 \]  

\[ dF_{\text{tan}}(i, j, k) = V(i, j, k) \cdot \mu \]  

(22)

(23)

where

\[ K_1 \] is the specific ploughing force whose units are \( N/\text{mm}^3 \), and

\[ \mu \] is the frictional coefficient.

The force computations require the knowledge of four coefficients \( K_1, K_2, K_3, \mu \). The cutting force coefficients are calibrated from the actual cutting tests and expressed in a power law form as discussed in the next section. The flank interference force coefficients are computed using the theory of elastoplastic indentation as described below.

Shaw and DeSalvo [1970] examined a two-dimensional punch problem and described the relation of the displaced volume of the material with the force (or the load) applied to the punch in terms of the depth of the elastoplastic zone. The specific cutting force (which is the ratio of the load to the displaced volume) is given by

\[ K_1 = P/V = E/(1 - 2\nu) \cdot (1/8) \]  

(24)

where

\[ V \] is the displaced volume,

\[ P \] is the load on the punch,

\[ E \] and \( \nu \) are the Young's modulus of elasticity and Poisson's ratio respectively of the work material. Their values are taken as \( E = 7.3 \times 10^5 \text{ N/mm}^2 \), Poisson's ratio \( \nu = 0.33 \) for Al 6061 and assumed constant with respect to temperature variations in cutting.

\[ \delta \] is the depth of the elastoplastic deformation zone as mentioned in Shaw's work. Its values are obtained by extrapolation from Wur's model[1988] by the procedure suggested by Elbestawi et al.[1994]. \( K_1 \) is computed as \( 2.23 \times 10^5 \text{ N/mm}^2 \) for Al 6061, using a value of \( \delta = 1.016 \text{ mm} \). The value of \( \mu = 0.03 \) was taken from Elbestawi et al.[1994] and assumed same for Al 6061. These coefficients are assumed constant for the feed range used in the simulations.

The flank interference forces are combined with the radial and tangential chip removal forces. The consolidated radial and tangential forces acting on the tool at a particular angular position are given as:

\[ dF_{\text{rad}}(i, j, k) = dF_{\text{rad}}(i, j, k) + dF_{\text{rad}}(i, j, k) \cdot \cos \gamma_c + dF_{\text{rad}}(i, j, k) \cdot \sin \gamma_c \]  

\[ dF_{\text{tan}}(i, j, k) = dF_{\text{tan}}(i, j, k) + dF_{\text{tan}}(i, j, k) \cdot \cos \gamma_c - dF_{\text{tan}}(i, j, k) \cdot \sin \gamma_c \]  

(25)

(26)

Resolving the radial and the tangential components into the external coordinate system, the X- and Y- forces are obtained.

\[ F_x(i) = \sum_{j=1}^{N_j} \sum_{k=1}^{N_k} \left[ -dF_{\text{rad}}(i, j, k) \cdot \sin \beta(i, j, k) \right] \]  

\[ \pm dF_{\text{tan}}(i, j, k) \cdot \cos \beta(i, j, k) \]  

\[ F_y(i) = \sum_{j=1}^{N_j} \sum_{k=1}^{N_k} \left[ dF_{\text{rad}}(i, j, k) \cdot \cos \beta(i, j, k) \right] \]  

\[ \pm dF_{\text{tan}}(i, j, k) \cdot \sin \beta(i, j, k) \]  

(27)

(28)

where

`\pm` is for down milling and `\mp` for up milling.

The dynamic model described above has been implemented with a computer program. The code computes the cutting forces in the X- and Y- directions.

**MODEL VERIFICATION AND RESULTS**

A series of end milling experiments were performed on a Cincinnati Milacron vertical milling machine using a 12.5 mm diameter, 30° helix, four fluted, HSS end mill. The experiments were based on a 2^4 factorial design with two levels of each variable, namely, axial depth (AD), radial depth (RD), feed per tooth (fz), spindle speed (N). The cutting conditions have been listed in Table 1 for the various tests. The workpiece material was 6061-T6 Aluminum and the workpiece was 'rigidly' mounted on the dynamometer. The dynamometer was clamped firmly to the machine tool table. During milling, the cutting forces in the X- and Y- directions were measured using a Kistler 9257A dynamometer. The static calibration of the dynamometer was done with
weights and its dynamic range was obtained from the bandwidth specifications. The force signals were collected by the data acquisition system, digitized (sampling rate of 10000 Hz for the three channels) and stored using the Lab Windows software. The coefficients \( K_t \) and \( K_r \) are estimated from the force data that was collected. The calibration was performed using the tests with axial depth of 25 mm. The other portion of the data were used to validate the simulation results. The cutting pressures are related to the average chip thickness by a power law function of the form:

\[
K_t = 32386.75 (t_2)^{-0.509} \\
K_r = 0.31763 \cdot (t_2)^{-0.089}
\]  

(29) (30)

**Cutter Runout Measurement**

A brief discussion of the methodology of cutter runout measurement is presented below. The attempt is to measure three types of runout, namely, the parallel axis offset, cutter tilt and cutter grind. The radius measurement is taken with a linear displacement indicator. The end mill is mounted on the spindle and rotated slowly. The radius of a flute and its locating angle are measured. Thus at a particular position along the Z-axis, four measurements are taken. The measurements are repeated for several Z-positions starting from the bottom (free end) of the cutter up to the end of the fluted region. The parallel axis offset \( \rho \) is the radial distance of the cutter center from the spindle axis and the cutter tilt \( \tau \), the slope of the line fitted through the cutter centers. The measured runout parameters are \( \rho = 0.00925 \text{mm} \), \( \lambda = 0^\circ \), \( \tau = 0.0332^\circ \) and \( \phi = 20^\circ \). To account for cutter grind, the measured flute radii are used to compute the forces in the simulation.

**Table 1: Cutting Conditions for Experiments**

<table>
<thead>
<tr>
<th>Test #</th>
<th>AD (mm)</th>
<th>RD (mm)</th>
<th>( f_t ) (mm/tooth)</th>
<th>( N_e ) (rpm)</th>
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</thead>
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<tr>
<td>1</td>
<td>12.5</td>
<td>2.5</td>
<td>0.05</td>
<td>1000</td>
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<tr>
<td>2</td>
<td>12.5</td>
<td>2.5</td>
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<td>0.1</td>
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<td>2.5</td>
<td>0.1</td>
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<td>25.0</td>
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<td>1000</td>
</tr>
</tbody>
</table>

**Comparison of the Predicted Forces with and without the Inclusion of Flank Face Interference**

Fig. 5 shows a comparison between the simulated forces with and without the effects of flank face interference. The test case shown represents cutting well within the chatter free zone. The dynamic model, with only the structural damping element, predicts a force system which seems unbounded. But the new flank interference model provides the necessary damping to the system resulting in more realistic force predictions.

Figure 6 shows a model predicted flank interference for down milling test 4 at a particular angular position \( \beta(i,j,k) \). The model uses equations - to compute the tool position and worksurface profile at this position. These equations include the effects of mechanisms like the cutting conditions, process dynamics and, the cutter geometry in computing the tool position and worksurface waviness. When the tool and work paths are plotted for a specific \( \beta(i,j,k) \), they may interfere provided the interference checks are satisfied. If no interference occurs at this instant then the flank forces are zero at that instant.
Comparison of Model Predicted and Measured Force Profiles

Figure 7 shows a comparison of the model predicted and the experimental forces for down milling test 1. The force predictions over three revolutions are shown in the figure. The model predicted force signatures in the X- and Y- directions match very well with the corresponding measured force signatures. The forces generated for the static case (i.e., tool is ‘rigidly’ held to prevent deflections) are also included for comparison purposes. The table gives the cutting conditions for the various tests performed and figures 9 and 10 give a comparison of the average and peak forces in the X- and Y- directions for the tests conducted. For most of the tests, the percent deviation of the predictions from the measured forces are within 20%. Plots of the frequency spectrum for the same cutting conditions are given in Fig. 8. The two peaks with highest power correspond to the spindle frequency (16.67 Hz) and the tooth passing frequency (66.67 Hz). Results from the spectrum analysis further validate the model predicted forces.

CONCLUSIONS

A dynamic model that incorporates the effects of flank face interference has been presented. The flank interference mechanism has been described by modeling the flank interference forces arising from the ‘rubbing’ to be proportional to the interference volume. The interference volume is calculated by numerically approximating the interference region with a series of triangles and trapezoids.

The positions of the leading edge of the tool and points on the workpiece surface are determined by keeping track of the past and present deflections of the cutter relative to the workpiece. The flank interference model was implemented with a computer program. The code computes the cutting and flank forces in the end milling process. The force predictions of this model are much better than predictions of existing dynamic model that does not account for flank interference.

The model has been verified through machining experiments conducted over a range of cutting conditions. The forces predicted by the model have been compared to the forces measured from the experiments. Plots of the forces indicate that the simulation results match very well with the experimental results. An analysis of the power spectrum of the model predicted and experimental forces further validate the experimental results.

While the enhanced dynamic model provides improved results over the existing model, there are a few issues that can be resolved to further improve the model. A simple method to calibrate the interference coefficients should be developed. Cutter runout measurements can further be improved by just characterizing the flute radii variation along the axial length of the cutter instead of considering the parallel axis offset, cutter tilt and cutter grind as three separate types of runout. This will provide for the true flute radii being used for the simulation of the end milling process. This may be a much simpler way to account for runout.
Table 2: Average Predicted & Measured Milling Forces

<table>
<thead>
<tr>
<th>Test #</th>
<th>Average Measured Force, (N)</th>
<th>Average Predicted Force, (N)</th>
<th>X (%) Error</th>
<th>Y (%) Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Y</td>
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Table 3: Peak Predicted & Measured Milling Forces

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ACKNOWLEDGEMENTS

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