The Frequency Component Structure of a 3-D Grinding Wheel Surface and its Effect on Ground Surface Texture

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ABSTRACT
In order to produce ground parts that have desirable surface properties, it is necessary to understand the evolution of these characteristics through surface generation mechanisms involved in the grinding process. Since the geometry of the wheel surface, in part, determines the final workpiece geometry, the influence of the 3-D structure of a wheel surface on the final workpiece geometry is studied. In this work, a wheel surface model is integrated with a surface grinding process model for simulating workpiece surface texture. The simulations utilizing the integrated model are used to study the workpiece surface roughness as a function of the frequency characteristics of the wheel surface. The 2-D Fourier forward and inverse transforms are employed to study and model the 3-D surface structure. In particular, the effect of specific frequency components in the wheel surface on the ground surface are analyzed. It is shown that workpiece surfaces resulting from wheel surfaces with dominant low frequency components have higher roughness, and that the low frequency components indicate a clustering of abrasive grains on the wheel surface.

INTRODUCTION
Grinding is a widely used machining operation that accounts for about 20-25% of the machining costs in industrialized countries (Malkin, 1987). The interaction kinematics of a grinding process are fairly complex due to the random distribution of uneven cutting edges on the wheel surface. The ability to control surfaces in grinding depends, to a large extent, on the knowledge of the wheel and workpiece surfaces and their interaction. The objective of this study, is to provide an understanding of the surface interactions in a grinding process. This study utilizes an integrated grinding wheel surface and grinding process simulation model.

This remainder of this paper consists of five sections. The first section provides a review of the relevant literature. Section 2 describes frequency decomposition of the grinding wheel surface. This section presents the theoretical background on the 2-D Fourier transforms necessary for the frequency decomposition, and, describes studies made to understand the structural composition of the wheel surface from its spectrum. In addition, a wheel surface is simulated with critical frequency components only. In section 3, the surface simulation model is integrated with a grinding surface texture model. The integrated model generates simulated ground surfaces. Simulation studies and their results are discussed in section 4. The research in this paper is summarized and relevant conclusions are presented in Section 5.

1. LITERATURE REVIEW
Research pertaining to this paper may be studied under two categories:
I) Grinding process modeling with topographic considerations,
II) Mathematical modeling of surfaces.

This paper is concerned with the kinematics of the grinding process, therefore, research involving only topographic considerations of grinding processes is discussed in this section. Topographic models of grinding processes have been developed by Nakayama and Shaw (1967), Reichenbach (1956), Chen et al. (1989) by considering either one or two-dimensional characteristics of the wheel. The surface generation mechanism in profile grinding of special purpose molds was investigated by Franse and de Jong (1987). Pawecct and Dow (1992) developed a model for the surface generated during contour grinding, by considering the process as a fly cutting operation using a single tool with radius equal to the nose radius of the wheel. Domala et al. (1995a) developed a model for workpiece surface texture using the initial workpiece geometry, cutting conditions and wheel topography as the inputs.

Mathematical modeling of surfaces involved description of surfaces either by time-series modelling or by using statistical techniques.
DeVor and Wu (1972) used time-series modelling for surface profile characterization. The profile was decomposed into periodic and non-periodic structures. Pandit and Wu (1973) characterized grinding wheel surface profiles using ARMA models. A two-wavelength model for grinding wheel profiles was proposed by Pandit and Satyanarayana (1982) after identifying the stochastic and deterministic components of a wheel profile using time-series models.

The use of statistics for describing grain positions was proposed by Orioka (1961). Peklenik (1964) used autocorrelation functions for describing wheel profiles. Renshaw and Ford (1983) proposed the use of spectral analysis in three-dimensions for surface characterization. Salisbury et al. (1994) used this analysis for characterizing precision-ground computer hard disks. You and Ehmann (1991) used the reconstruction property of the inverse Fourier transform to synthesize abrasive surfaces. Spectra of wheel profiles obtained from time-series models were imposed onto the continuous 2-D Fourier spectrum in the direction of the profile and inverse transformed. A different approach for simulating the spectrum is developed in this paper. Frequency characteristics are introduced into the surface by simple manipulations of a basic unit of the frequency spectrum. A few representative basic units are used to simulate the spectrum that comprises of several frequencies.

2. FREQUENCY COMPONENTS IN A WHEEL SURFACE

A wheel surface may be considered to be a two-dimensional spatial function. Fourier transform techniques applied to spatial data, will yield the following relation for the Fourier coefficient \( F(\omega_p, \omega_q) \) for a continuous function:

\[
F(\omega_p, \omega_q) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(x, y) e^{-j2\pi(\omega_p x + \omega_q y)} dx dy
\]

where \( S(x, y) \) is the continuous surface. Its equivalent in the discrete case is as follows:

\[
F(\omega_p, \omega_q) = \frac{1}{M} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} S(x, y) e^{-j2\pi\left(\frac{p}{M} x + \frac{q}{N} y\right)}
\]

where \( p=0,1,...,M-1 \); \( q=0,1,...,N-1 \); and \( S(x, y) \) is an array of surface heights. \( M \) and \( N \) are integers relating to the number of sampling points in the \( X \) and \( Y \) directions. Correspondingly \( F(\omega_p, \omega_q) \) is a matrix of Fourier coefficients, \( \omega_p \) and \( \omega_q \) are the angular frequencies in the two orthogonal directions. Also,

\[
\omega_p = \frac{p}{\Delta x M} \quad \text{and} \quad \omega_q = \frac{q}{\Delta y N}.
\]

The Fourier coefficients are complex numbers of the form \( a_{pq} + ib_{pq} \), where \( p \) and \( q \) are the indices associated with the \( X \) and \( Y \) directions.

The surface is decomposed into a set of cosine waves of the form:

\[
S(x, y) = A \cos \left[ 2\pi \left( \frac{p x}{M} \right) + 2\pi \left( \frac{q y}{N} \right) \right]
\]

where each wave results in a spike at point \( (p, 2q) \) in the spectrum. The amplitude of a wave can be found from the amplitude of its Fourier coefficient. The wavelength of each wave \( W(\omega_p, \omega_q) \) is:

\[
W(\omega_p, \omega_q) = \frac{M N}{\sqrt{\omega_p^2 + \omega_q^2}}
\]

or the frequency in the direction of travel \( \omega_{pq} \) is:

\[
\omega_{pq} = \sqrt{\omega_p^2 + \omega_q^2}
\]

The direction in which the wave traverses or angle \( \theta(\omega_p, \omega_q) \) is:

\[
\theta(\omega_p, \omega_q) = \tan^{-1} \left( \frac{N \omega_q}{M \omega_p} \right)
\]

where \( 0 < \theta(\omega_p, \omega_q) < \pi \). The phase \( \phi \) associated with the wave is given by:

\[
\phi(\omega_p, \omega_q) = \tan^{-1} \left( \frac{\omega_p}{\omega_q} \right)
\]

The power spectral density or the power associated with each frequency, \( P(\omega_p, \omega_q) \), can be obtained by:

\[
P(\omega_p, \omega_q) = \mathcal{F}(\omega_p, \omega_q) \cdot \mathcal{F}^*(\omega_p, \omega_q)
\]

where \( \mathcal{F}^* \) is the complex conjugate of \( \mathcal{F} \). The magnitude of the power at a particular frequency is the measure of its relative importance.

The Fourier amplitude spectrum for the wheel surface shown in Fig. 1, calculated using equations (1-8), is shown in Fig 2. Figure 2 shows numerous peaks indicating that the wheel has a complex spatial structure. Further, the symmetric distribution of peaks about the origin suggests that the wheel surface has similar frequency content in all directions. In other words, the surface is geographically isotropic. Analysis of the structural composition of the wheel can be performed to understand the role of certain frequencies on the geometry of the wheel surface.

It was observed that in any frequency band of Fig. 2, the number of peaks \( n \) in the frequency band, is always a multiple of four. In the limiting case, i.e., when the frequency band is narrowed down to a single frequency, \( n \) was either four or zero depending on whether that frequency existed in the spectrum or not. Figure 3 (a) shows the spectrum at a spatial frequency of 419.9 m\(^{-1}\). The four peaks in the limiting case (width of the spatial frequency band is zero) are distributed in the four quadrants and are separated exactly by 90° as can be seen from fig. 3 (b). In other words, every peak in a certain quadrant has three other peaks associated with it in the remaining three quadrants, the angle between any two successive peaks being 90°. The topographic and contour maps of the surface reconstructed from this set of four Fourier coefficients are shown in Figs. 4 (a) and (b) respectively. These figures clearly show the presence of distinct peaks, and also the texture at the angles at which the peaks occur (0°, 90°). The characteristics associated with these Fourier coefficients are listed in Table 1. From the table, it can be observed that the Fourier coefficients corresponding to the peaks that are separated by 180° are complex conjugate pairs. These co-existing set of four Fourier coefficients at a certain frequency may be utilized for understanding a grinding wheel surface. Such a quartet (a pair of complex conjugate pairs) essentially forms the basic element of the frequency components.
in a wheel surface. A basic set occurs at a certain frequency and has phases and amplitudes associated with it (the phases and amplitudes being determined by the complex numbers in the basic set). Therefore, a basic set is completely defined if the orientation, frequency, amplitudes and phases are specified. In short, the frequency components of a wheel surface consist of numerous basic sets. This knowledge can be used to simulate the grinding wheel surface texture through the use of the Fourier inverse transform, which is defined by Eq.(10). A two-dimensional discrete spatial function \( S(x,y) \), representing surface heights can be represented as a linear combination of exponentials as:

\[
S(x,y) = \sum_{q \neq 0} \sum_{p \neq 0} F(p,q) \exp(-j2\pi \left( \frac{p x + q y}{M,N} \right))
\]  

Reconstruction through inverse Fourier transformations is, simply the conversion of the complex Fourier coefficients obtained by Eq.(2), back to real numbers using Eq.(10). The reconstructions illustrates how the superposition of spectra with different frequency ranges is reflected in the spatial domain, and explains the physical significance of such a superposition. Table 2 lists the generated frequencies of the selected spatial frequency bands, amplitudes, and orientation associated with four of the twenty sets used for simulation. A complete listing of the generated basic sets used for simulation can be found in Domala (1995), but is not included here for space considerations.

The spectrum generated using the Fourier coefficients listed in the table is displayed in Fig. 5(a). The peaks present in the spectrum are shown in Fig. 5(b). The topographic and contour maps resulting from this reconstruction are shown in Figs. 6(a) and 6(b) respectively. These figures show that the peaks are randomly distributed, indicating no spatial trend, therefore, the surface is near isotropic. The discrete topographic and contour maps of the real surface reconstructed using the above spatial frequency bands are shown in Figs. 7(a) and 7(b). The figures clearly indicate that the contours due to longer wavelengths surround the ones due to shorter wavelengths, indicating that some of the grits are being grouped together. Therefore, the presence of long wavelengths in the surface suggests that a grain clustering phenomenon is occurring in the wheel surface.

The \( R_s, R_p \) and \( R_t \) of the real surface were found to be 27, 33 and 176 microns respectively, while the \( R_s, R_p \) and \( R_t \) of the simulated surface were 19, 24 and 163 microns, respectively. Also, the contour map of the simulated surface compares well with the real surface. Thus, the simulated surface can be a good representation of the real surface, even though only a few frequencies have been utilized.

<table>
<thead>
<tr>
<th>Peak</th>
<th>( \theta ) degrees</th>
<th>( \varphi ) degrees</th>
<th>Fourier Coefficient</th>
<th>Amplitude ( F(q) )</th>
<th>Frequency m (^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>80.55</td>
<td>-3163-19014</td>
<td>19275.6</td>
<td>4199.5</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>13.26</td>
<td>14503-34914</td>
<td>14900.8</td>
<td>4199.5</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>-80.55</td>
<td>3164+19014</td>
<td>19275.6</td>
<td>4199.5</td>
</tr>
<tr>
<td>4</td>
<td>270</td>
<td>-13.26</td>
<td>14503-34914</td>
<td>14900.8</td>
<td>4199.5</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel</td>
<td>Work</td>
<td>Wheel</td>
</tr>
<tr>
<td>$R_s$ (µm)</td>
<td>18</td>
<td>10</td>
</tr>
<tr>
<td>$R_m$ (µm)</td>
<td>25</td>
<td>13</td>
</tr>
<tr>
<td>$R_m$ (µm)</td>
<td>178</td>
<td>56</td>
</tr>
</tbody>
</table>
Table 3: Simulated Parameters (Cases 1-3)

<table>
<thead>
<tr>
<th>Orientation, degrees</th>
<th>Frequency, $\omega_{\text{m}} \text{ rad s}^{-1}$</th>
<th>Amplitude, $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>Set 1 45/225</td>
<td>1671436</td>
<td>0</td>
</tr>
<tr>
<td>135/315</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set 2 18/198</td>
<td>1671436</td>
<td>159184</td>
</tr>
<tr>
<td>108/288</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set 3 76/536</td>
<td>1671436</td>
<td>318369</td>
</tr>
<tr>
<td>166/346</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set 4 27/207</td>
<td>1671436</td>
<td>477553</td>
</tr>
<tr>
<td>117/297</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

manufacture these types of grinding wheel surfaces, but they are seldom obtained.

The ground surface that resulted from Case 3 has the best roughness values. Manufacturing a wheel that has the spectral properties as prescribed by Case 1, is also difficult to implement in reality. Case 3, hence presents a more viable solution. In order to achieve this, it may be recommended that the dispersion of grains in the bonding medium should be such that individual grains are bonded to the medium and not to each other. Such dispersions would eliminate the detrimental characteristics of the wheel due to the long wavelength components.

4. SUMMARY AND CONCLUSIONS

A model for simulating the wheel surface has been presented. The wheel surface was decomposed, and characteristic features were identified which were later used for simulating a wheel surface. Comparison with a real surface showed good agreement. Wheel surfaces with required frequency characteristics were generated and used for generating workpiece surfaces using a workpiece surface model. The following conclusions may be drawn from the studies made in this paper:

1) Spectral decompositions of the wheel surface indicate that the clustering of grits result in long wavelengths. Further, surfaces having dominant low frequency components yield surfaces with higher roughnesses. Therefore low frequency components with high power are undesirable.

II) The basic structure of the Fourier spectrum was determined to be a set of four complex conjugate numbers. These basic sets in the spectrum may be used as building blocks for simulating spectra. By assigning real and imaginary coefficients to these Fourier coefficients at different frequencies and orientations, the desired frequency-domain properties may be introduced.

REFERENCES


Figure 9: A Flow Chart of Integrated Grinding Process Model

\( \mu m \ R_c = 292 \mu m \) (compared to \( R_w = 18 \mu m \), \( R_p = 25 \mu m \), \( R_z = 178 \mu m \) for Case 1), which lead to higher surface roughness values of \( R_p = 27 \mu m \), \( R_g = 35 \mu m \), \( R_z = 128 \mu m \) (compared to \( R_w = 10 \mu m \), \( R_y = 13 \mu m \), \( R_z = 56 \mu m \) obtained in Case 1). The higher roughness of the surfaces is also evident from Figs. 13(b) and (d) for Case 2, and Figs. 14(b) and (d) for Case 3.

In Case 3, since the power assigned to lower spatial frequencies is lower, and that for the frequencies near the grid size being higher, the variance in the surface is distributed mainly among the grids composing the wheel. The simulated wheel surface in Fig. 14(b) shows distinct peaks, which is also clear from its contour map in Fig. 14(c). The grits appear to be randomly distributed in these figures, while the roughnesses of the wheel surface obtained are: \( R_w = 15 \mu m \), \( R_p = 21 \mu m \), \( R_z = 118 \mu m \). The result of grinding with this simulated wheel surface produces the workpiece surface that is shown in Fig. 14(d). The roughnesses obtained in this grinding operation are \( R_w = 8 \mu m \), \( R_y = 10 \mu m \), \( R_z = 49 \mu m \), and are comparable to those obtained in Case 1. A high spatial frequency implies more closely packed grits on the wheel surface. Since the number of cutting edges is increased, more grains are involved in the material removal process. It is typically the intent to
Table 2: Characteristics of Basic Sets used in Simulation

<table>
<thead>
<tr>
<th>Set</th>
<th>Orientation, degrees</th>
<th>Fourier Coefficients</th>
<th>Amplitude $A$</th>
<th>Frequency $m^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76/166</td>
<td>-1292459 +j1447720</td>
<td>1940707</td>
<td>750</td>
</tr>
<tr>
<td>2</td>
<td>256/346</td>
<td>22377 +j940579</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>45/135</td>
<td>-1121879 +j856377</td>
<td>1411380</td>
<td>1000</td>
</tr>
<tr>
<td>4</td>
<td>225/315</td>
<td>91664 +j1168445</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>72/162</td>
<td>410378 +j361176</td>
<td>1421693</td>
<td>1250</td>
</tr>
<tr>
<td>6</td>
<td>252/342</td>
<td>-590799 +j293123</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>207/297</td>
<td>765758 +j225946</td>
<td>1488094</td>
<td>1500</td>
</tr>
</tbody>
</table>

\[
y = y_{\omega ij} - f\Delta t + (R + h_{ij}) \sin (\theta_{\omega ij} - \omega \Delta t) \tag{12}
\]

\[
z = z_{\omega ij} - d - (R + h_{ij}) \cos (\theta_{\omega ij} - \omega \Delta t) \tag{13}
\]

where,
- $i$ = index of the point along the $X$-axis during data collection;
- $j$ = index of the point along the $Y$-axis during data collection;
- $h_{ij}$ = height of the point under consideration;
- $\theta_{\omega ij}$ = initial angle of a point with respect to $(O)$;
- $\omega = \pi/2$ radians for $i=1, j=1$;
- and $y_{\omega ij}$ and $z_{\omega ij}$ are constants.

A program coded in FORTRAN was used for simulations. The program uses the model Eqs. (11-13) according to the flowchart shown in Fig. 9.

4. SIMULATION STUDIES

This section involves studies made by integrating the wheel surface model with a grinding surface texture model. The process model for a workpiece surface texture generation in a surface grinding process is based on the initial workpiece geometry, wheel topography and cutting conditions that have already been verified individually. Different kinds of model-simulated wheel surfaces shall be used as an input to the process model, to study how the frequency structure of the grinding wheel surface affects the workpiece surface texture. Evaluation of the wheel surfaces in a design sense is possible through such simulation studies.

Three different relationships between the frequency and amplitude are considered, and their effect on the ground surfaces is investigated.

The different cases considered are listed below:

**Case 1:** Initially, the amplitude is assumed to be a constant function, i.e. the same amplitude is used for a number of basic sets at different orientations and frequencies. The amplitude $A$, is maintained a constant at $1.67 \times 10^6$ (the average of the band amplitudes corresponding to spatial frequency bands of width 50 $m^{-1}$ in the range 0-1100 $m^{-1}$ for the real wheel surface).

**Case 2:** Secondly, it is assumed that the magnitude decreases linearly along the spatial frequency axis, the total...
power remaining the same. The relationship between the amplitude and spatial frequency may be summarized according to the following equation:

\[ A(\omega_{pq}) = -636.74 \cdot \omega_{pq} + 3502056.58 \]  \hspace{1cm} (14)

Case 3: Finally, it is assumed that the magnitude increases along the spatial frequency axis, again with the same total power. The following equation was used to represent the relationship between frequency, \( \omega_{xy} \), and amplitude \( A \):

\[ A(\omega_{pq}) = 636.74 \cdot \omega_{pq} - 159184.39 \]  \hspace{1cm} (15)

A plot depicting the various relationships used in the simulations is shown in Fig. 10. Figure 11 shows the basic sets used in the wheel surface simulations. The values assigned to the basic sets in each of these cases are listed in Table 3. This table lists the amplitude that corresponds to the sum of the power in a selected frequency band. Again, only four of the twenty-two basic sets used for simulations, have been included in the table for space considerations.

The simulated Fourier spectrum for Case 1 is shown in Fig. 12(a). This figure shows that all the peaks have the same amplitude. The topographic and contour maps of the surface obtained from inverse transforming this spectrum are shown in Figs. 12(b) and (c) respectively. The simulated wheel surface shown in Fig. 12(d) is now used as an input to the process model. A depth of cut of 150 microns, feed of 0.0127 m/sec (30 IPM), and a wheel speed of 900 rpm were used as cutting conditions. The resulting workpiece surface is shown in Fig. 12(d). The results of simulation in Case 2 are shown in Figs. 13(a)-(d), while the results of simulation in Case 3 are shown in Figs. 14(a)-(d). The total power assigned to the basic sets is the same in all the three cases.

For Case 1, the variance at all angles and frequencies is the same. If the entire range of frequencies and angles is covered, the reconstructed surface would exhibit what may be regarded as an extreme case of isotropy, analogous to white noise in random signals. The different roughness parameters associated with the simulated wheel and ground surfaces generated in each of these cases is documented in Table 4.

The influence of higher amplitude associated with low frequency components can be understood with the simulations in Case 2. The high amplitude of the low frequency components leads to greater deviation of the surface height from a mean plane and, hence, a higher roughness in the wheel surface. This translates to a higher roughness of the workpiece surface. The results from simulations indicate a similar trend, the wheel roughness values being higher at \( R_a = 51 \mu m \), \( R_q = 66 \mu m \).