PHASE SHIFT ESTIMATION: A METHOD FOR IMPROVING THE ACCURACY
OF PHASE SHIFT INTERFEROMETERS

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ABSTRACT
The increasing use of precision manufacturing processes in both research and industry has precipitated the need for new measurement tools that can be used to assess the results of these processes. In the area of precision surface measurement, one such tool is the phase shift interferometer (PSI). Unfortunately, PSI devices have a limited accuracy due to a number of systematic error sources, most significantly inaccuracies in the phase shift. In this paper, a new technique for the compensation of phase shift errors in PSI systems is presented. The technique includes a phase shift estimation algorithm, and an algorithm that utilizes the estimated phase shift to generate a new surface reconstruction equation. The algorithms were validated using a simulation example, and then an actual experimental test was run. Both the simulation and experimental results indicate that the algorithms yield a significant reduction in measurement error and a corresponding increase in accuracy.

INTRODUCTION
Forecasts indicate that machining accuracy will continue to improve reaching nanometer levels near the year 2000 (Taniguchi, 1983). The advent of new precision manufacturing processes such as precision grinding and turning, electrolytic machining, lithography, etc. have provided a variety of new products. The process dependent characteristics of these products, such as surface texture, strength and chemical composition must be studied to assess the functionality of these products.

A key product characteristic that directly results from the manufacturing process is the surface texture. The surface texture is important for many reasons including friction, wear, lubrication, contact with other parts, and the ability to hold a coating or other finish. Therefore, it is imperative that manufacturers of precision products have measurement technologies available to measure precision surface texture.

Many studies utilizing optical methods for surface assessment have been performed, some of which are described by Yoo, et al. (1990). One optical method for surface assessment that appears to hold great promise for precision manufacturing applications is interferometry. Interferometry produces an interference fringe pattern when two or more waves of light interact with each other. Traditionally, interferometry has been used to determine surface roughness (Bennett, 1976) and construct the topography of a part.

The reconstruction of a specimen’s surface topography from fringe patterns has been accomplished by several different methods. Perry, et al. (1983) created a topographic map of the specimen using the undulations of the fringe lines. Bruning, et al. (1974) introduced Phase Shift Interferometry (PSI) which is currently the most used surface reconstruction method. PSI involves moving the reference mirror in small linear increments and storing the fringe pattern at each step. The phase shift of each point on the fringe relative to a central point is calculated and converted to a height, thus creating a topographic map of the surface. Montgomery, et al. (1992) and numerous others have expanded on this work to develop simple, fast algorithms.

The existence of systematic error sources such as intensity noise, mechanical drifts, vibrations, and most importantly, inaccuracies in the reference phase shifter can significantly reduce the accuracy of the PSI surface reconstruction (Kinnstetter, 1988). It has been shown that phase shift errors of as little as 3nm can cause significant measurement errors. The actuators used to obtain these precise phase shifts usually contain non-linearities and hysteresis and are subject to stochastic errors such as vibration and stick-slip. The individual contribution of each error source may be small, however the combination of these errors can have a large enough effect to significantly deteriorate the accuracy of the measurement. Some attempts have been made to reduce these error sources. Schwider, et al. (1983), Kinnstetter (1988) and Harinaran et al., (1987) developed algorithms to compensate for inaccuracies in the reference phase shifter. These solutions merely use averaging techniques on a greater amount of data and do not adequately eliminate the errors.
This paper will address the problem of inaccurate phase shifts by determining the actual phase shifts and using these actual shifts in the surface reconstruction algorithm. The remainder of this paper contains five sections. The first will describe the laser interferometer setup and provide an overview of interferometry theory. The second section will present the phase shift estimation algorithm. In the third section, a simulation procedure will be used to test the phase shift estimation algorithm. The fourth section will present the results from using the phase shift estimation algorithm on experimentally obtained data. Finally, discussions of the results in the paper will be presented and some conclusions made.

LASER INTERFEROMETRY SYSTEM

Experimental Setup

The hardware elements of the interferometer system include a He-Ne laser, magnetostrictive actuator, optics, and a CCD camera with imaging system. These elements were assembled into a complete system as shown in schematic form in Fig. 1. The system is a modified Twyman-Green interferometer. The light source is a low power He-Ne laser with a wavelength of 637nm. The laser beam is collimated and directed to the beam splitter where it is divided into beams of equal intensity with one being transmitted into one arm of the interferometer and one being reflected into the second arm. One beam reflects off the specimen while the other reflects off the reference mirror. When the two light beams recombine at the beam splitter, the phase difference of the two beams causes interference patterns. The intensity distribution of the interference pattern is captured by a CCD camera and digitized by an image processing unit.

Laser Interferometry Theory

The interference concept is based on the superposition property of coherent light waves. For example, if two waves of initial amplitudes \(a\) and \(b\) with the same wavelength \(\lambda\), leave a source and travel by different paths and then arrive at the same point in space where superposition takes place, the complex amplitude of the resultant wave is equal to the sum of the complex amplitudes of the two initial waves.

Consider two wave trains \(E_1\) and \(E_2\) that are given by,

\[
E_1(x, y) = a e^{2ikx} 
\]

and shown in Fig. 1, where \(a\) and \(b\) denote the wavefront amplitudes, \(k = 2\pi/\lambda\) is the wave number with \(\lambda\) equal to the wavelength of the light, \(l\) is the mean path length from the beam splitter to the reference mirror, and \(w(x,y)\) is the mean path length from the beam splitter to the specimen and represents the surface texture height of the specimens surface. These two waves join at the beam splitter and the wave \(E_p\) that results from the superposition of waves \(E_1\) and \(E_2\) is,

\[
E_p(x, y) = E_1(x, y) + E_2(x, y) 
\]

The energy flux or intensity \(I\) of a wave is proportional to the square of the amplitude of the wave field, so \(E_p E_p^*\), where \(E_p^*\) is the complex conjugate of \(E_p\). From this, the intensity distribution in the interference pattern is

\[
I(x, y, l) = a^2 + b^2 + 2ab\cos2k[w(x, y) - l] 
\]

The intensity distribution can be used to reconstruct the surface topography of the specimen.

Surface Reconstruction

Bruning, et al. (1974) describe a method to obtain a topographic map of the surface using interference patterns which involves moving the reference mirror and obtaining the fringe pattern after each movement. Bruning, et al. (1974) consider an alternative representation for Eq. (4),

\[
I(x, y, l) = a_0 + a_1 \cos(2kl) + b_1 \sin(2kl) 
\]

Equation (5) is a Fourier series with the dc term and first harmonics only, where the coefficients are functions of \(x\) and \(y\). The coefficients at each \(x\) and \(y\) can be found by sampling the fringe pattern after moving the reference mirror in small increments and using the orthogonality properties of the trigonometric functions.

\[
a_0 = \frac{1}{np} \sum_{i=1}^{np} I(x_i, y_i, l) = a^2 + b^2 
\]

\[
a_1 = \frac{2}{np} \sum_{i=1}^{np} I(x_i, y_i, l) \cos 2kl_i = 2ab \cos 2kw(x, y) 
\]

\[
b_1 = \frac{2}{np} \sum_{i=1}^{np} I(x_i, y_i, l) \sin 2kl_i = 2ab \sin 2kw(x, y) 
\]

Therefore,

\[
2kw(x, y) = \arctan \left( \frac{b_1}{a_1} \right) 
\]

\[
l = l_i = \frac{i \lambda}{2n} 
\]

The integer \(p\) denotes the number of periods or fringes over which the interference pattern is sampled and \(n\) is the quantization of each period. Therefore, for each position \((x, y)\) in the interference pattern, the phase or wavefront that corresponds to the surface height \(w(x, y)\) can be found within some multiple of \(2\pi\).

The method developed by Bruning, et al. (1974) is the basis for all commercial interferometry measurement systems and is

Transactions of NAMRI/SME 346 Volume XXIII, 1995
frequently called Phase Shift Interferometry (PSI). The major drawback to this method is that it requires a large (about 100) number of samples to obtain the surface topography \( w(x,y) \). Brophy (1990), Schwider, et al. (1983) and others have expanded on the PSI method to develop simple fast algorithms similar in form to Eq. (9), to determine \( w(x,y) \) with as few as three fringe patterns. A popular example of such an algorithm involving four fringe patterns with \( \pi/2 \) phase shift between fringe patterns is shown in Eq. 10,

\[
     w(x, y) = \frac{\lambda}{4\pi} \tan\left( \frac{I_4 - I_2}{I_1 - I_3} \right)
\]

where, \( I_i \) is the intensity in the \( i \)th fringe pattern at pixel position \((x,y)\). This and similar algorithms require a specified amount of phase shift between fringe patterns. Any deviation from the desired phase shift will have a deleterious effect on the measurement accuracy. Schwider, et al. (1983) showed that inaccuracies in the phase shift introduced a frequency of twice the fringe spacing frequency onto the reconstructed surface. This will be shown in a later section.

**PHASE SHIFT ERROR COMPENSATION**

**Phase Shift Estimation**

The measurement error due to inaccurate phase shifts can be greatly reduced by first determining what the actual phase shifts were and then by using those actual phase shifts in the surface reconstruction algorithm. In this paper, a system of equations has been developed and solved for these shifts. From Eq. (4), with \( A_j = a_j^2 + b_j^2 \) and equivalent to the mean of the cosine wave and with \( B_j = 2ab \) and equivalent to the amplitude of the cosine wave, the light intensity values at a pixel for each fringe pattern can be represented as:

\[
     I_{ij} = A_j + B_j \cos(\omega_j + \phi_j)
\]

where \( i = 1,2,3,4 \) is the fringe number, \( j \) is the pixel number, \( \omega \) is a representation of the surface height, and \( \phi \) is the phase shift between fringe patterns with \( \phi_0 = 0 \).

Using a four fringe reconstruction algorithm at three pixels gives a system of twelve equations and twelve unknowns as follows:

\[
     I_{11} = A_1 + B_1 \cos(\omega_1 + \phi_0)
\]

\[
     I_{21} = A_1 + B_1 \cos(\omega_1 + \phi_2)
\]

\[
     I_{31} = A_1 + B_1 \cos(\omega_1 + \phi_3)
\]

\[
     I_{41} = A_1 + B_1 \cos(\omega_1 + \phi_4)
\]

\[
     I_{12} = A_2 + B_2 \cos(\omega_2)
\]

\[
     I_{22} = A_2 + B_2 \cos(\omega_2 + \phi_2)
\]

\[
     I_{32} = A_2 + B_2 \cos(\omega_2 + \phi_3)
\]

\[
     I_{42} = A_2 + B_2 \cos(\omega_2 + \phi_4)
\]

\[
     I_{13} = A_3 + B_3 \cos(\omega_3)
\]

\[
     I_{23} = A_3 + B_3 \cos(\omega_3 + \phi_2)
\]

\[
     I_{33} = A_3 + B_3 \cos(\omega_3 + \phi_3)
\]

\[
     I_{43} = A_3 + B_3 \cos(\omega_3 + \phi_4)
\]

A modified Gauss-Seidel numerical technique was chosen to solve the system of non-linear equations for the twelve unknowns. Gauss-Seidel is an iterative scheme where each equation is solved for one variable (James, et al. 1985). Then using initial guesses, each equation is solved for a variable that then replaces the initial guess. The iteration continues until the change in the variables drops below a specified value. Due to the non-linearity of Eqs. (12-23) a poor initial guess could cause a large error in the variable estimate using Gauss-Seidel, therefore a modified Gauss-Seidel technique is used.

In this modified technique, instead of only one equation being solved for each variable, each equation that contains the variable is solved for that variable. Then using initial guesses a value for that variable is obtained from each equation. A variable average from all equations solved for a variable is then taken to reduce the effects of a bad initial guess estimate. For example, consider the solution for \( \omega_1 \) using initial conditions \( A_{10}, B_{10}, \phi_{20}, \phi_{30} \) and \( \phi_{40} \) which will be defined later, as shown in Eq. (24).

\[
     \omega_1 = \left( \frac{\cos \left( \frac{I_{11} - A_{10}}{B_{10}} \right)}{\cos \left( \frac{I_{21} - A_{10}}{B_{10}} \right)} - \phi_{20} \right) + \left( \frac{\cos \left( \frac{I_{31} - A_{10}}{B_{10}} \right)}{\cos \left( \frac{I_{41} - A_{10}}{B_{10}} \right)} - \phi_{40} \right) / 4
\]

Each equation that contains \( \omega_1 \), Eqs. (12-15), is solved for \( \omega_1 \). The value for \( \omega_1 \) is then taken as the average of the values obtained from Eqs. (12-15). This value for \( \omega_1 \) is then used in solving for the remainder of the values.

Each variable is solved for in this manner replacing the initial guesses with the value obtained. The solution order is first \( \omega_1 \) followed by \( \phi_2, \phi_3 \) and \( \phi_4 \). After a new value for each variable has been obtained, Eqs. (12-23) are solved for \( I_{ij} \). The solution is complete when \( |I_{ij(solved)} - I_{ij(actual)}| < 0.5 \) for all equations. An error threshold of 0.5 is chosen because \( I_{actual} \) is an integer number while \( I_{solved} \) is real. The solution yields an estimate for the actual phase shifts \( \phi_2, \phi_3 \) and \( \phi_4 \).

The initial guesses play an important role in this numerical calculation. If the initial guesses are not suitably close to the actual values, the numerical solution will not converge. The initial guess for the phase shifts \( \phi_0 \) are taken as the nominal values \((0,\pi/2,3\pi/2)\). Since the constant \( A_j \) is the mean value of the cosine function:

\[
     A_{j0} = \left( \sum_{i=1}^{4} I_{ij} \right) / 4
\]

provides a good estimate since the cosine wave is being sampled four times at approximately \( \pi/2 \) intervals. Since \( B_j \) is the amplitude of the cosine wave:

\[
     B_{j0} = \max|I_{ij}-A_{j0}|
\]

provides an adequate estimate of the amplitude of the cosine wave.

**Surface Reconstruction Algorithm**

In this section, a surface reconstruction algorithm that can use the estimated phase shifts from the previous section will be developed. Since traditional surface reconstruction algorithms were derived for some constant phase shift, the actual phase shifts cannot be used until an appropriate algorithm is developed. The
conventional algorithms such as shown in Eq. (10), all take the form:

\[ w(x, y) = \frac{\lambda}{4\pi} \left( \sum_{i=1}^{n} \frac{c_i I_i}{d_i I_i} \right) \]  

(27)

where \( n \) is the number of fringe patterns in the algorithm, \( \lambda \) is the wavelength of light and \( w(x, y) \) is the reconstructed surface height. A PSI algorithm can be constructed for almost any set of phase shifts by using a diagram as shown in Fig. 2. The diagram is constructed by vectors representing the frame intensities \( I_i \) out to the radius of the circle. Each successive vector is separated angularly from the previous one by an angle equal to the constant phase shift. A vector lying only in the horizontal direction corresponds to an intensity only dependent on \( \cos(\phi) \), while a vector lying only in the vertical direction corresponds to an intensity dependent only on \( \sin(\phi) \). The reconstruction algorithms take the form of \( \text{atan}(\sin(\phi)/\cos(\phi)) \). Therefore to determine the numerator of Eq. (27) a vector in the vertical direction can be constructed and the denominator of Eq. (27) can be determined by constructing a vector in the horizontal direction.

To determine the values for \( c_i \) and \( d_i \), a system of linear equations can be solved:

\[ \sum_{i=1}^{n} c_i = 0 \]  

(28)

\[ \sum_{i=1}^{n} c_i \cos \Phi_i = 0 \]  

(29)

\[ \sum_{i=1}^{n} c_i \sin \Phi_i = 1 \]  

(30)

\[ \sum_{i=1}^{n} d_i = 0 \]  

(31)

\[ \sum_{i=1}^{n} d_i \cos \Phi_i = 1 \]  

(32)

\[ \sum_{i=1}^{n} d_i \sin \Phi_i = 0 \]  

(33)

Eqs. (28) and (31) are to eliminate the bias intensity, Eqs. (29) and (30) create a vertical vector for the numerator and Eqs. (32) and (33) create a horizontal vector for the denominator. Using these equations for the case of four fringe patterns provides six equations and eight unknowns. This problem can still be solved by choosing values for two of the unknowns. An obvious choice is to set \( c_i = 0 \) since \( I_1 \) is in the horizontal direction and the numerator is a vector in the vertical direction, and \( d_2 = 0 \) since \( I_2 \) is mostly in the vertical direction and the denominator is a vector in the horizontal direction. The set of linear equations (28-33) can then be easily solved for \( c_i \) and \( d_i \), providing an algorithm (Eq. 27) that can be used to accurately reconstruct the surface.

**SIMULATION**

A simulation example will be presented to test the new phase shift estimation and surface reconstruction algorithms and compare the new algorithms with the conventional algorithm. In the simulation, the surface being measured by the laser interferometry system was assumed to be perfectly flat, with the reference mirror tilted at an angle to give a value for \( \Phi \) at each pixel. For each pixel the values for the mean \( A \) and amplitude \( B \) were assumed to be normal random variables, \( A \sim N(100,5) \) and \( B \sim N(80,5) \). The phase shifts were set at \( \Phi = 95^\circ, 170^\circ \) and \( \Phi = 290^\circ \) corresponding to displacement errors of 5.65mm, 11.3mm and 22.6mm respectively. The resulting fringe patterns are shown in Fig. 3. For ease in understanding, two-dimensional fringe patterns (one line of pixels perpendicular to the fringes) will be presented here, however the results can easily be extended to the three-dimensional case.

To demonstrate the measurement error that occurs when phase shift error is present, the reconstruction algorithm shown in Eq. (10) is used to generate the surface shown in Fig. 4. It is evident that the phase shift errors have imparted significant errors into the measurement, in fact causing a frequency of twice the fringe spacing frequency with a peak to valley value of 16nm (Schwider, et al. 1983).

To reduce the measurement error, the phase shift estimation will be used and a new surface reconstruction equation will be determined. Since the phase estimation method requires four intensity values at three different pixel positions, a pixel was chosen at each end of the fringe pattern (10th and 390th pixel) and in the middle (200th pixel). Using Eqs. (12-26) and the modified Gauss- Seidel numerical technique, a solution was obtained in 5 iterations. Table 1 shows the intensity data for each pixel. The intensity values have no absolute units, but are digitized in a range (0-255) of intensity levels. Table 2 shows the actual variable values, initial variable guesses and estimated variable values for each pixel.

Using the estimated values for \( \Phi_2, \Phi_3 \) and \( \Phi_4 \) and solving Eqs. (28-30) with \( c_i = 0 \) and \( d_2 = 0 \) the resulting reconstruction algorithm is:

\[ w(x, y) = \frac{4\pi}{\lambda} \text{atan} \left( \frac{0.633I_2 - 0.2026I_3 - 0.4304I_4}{0.5617I_1 - 0.4754I_3 - 0.0863I_4} \right) \]  

(34)

Using Eq. (34) to reconstruct the simulated surface using the fringe patterns shown in Fig. 3 yielded the surface shown in Fig. 5. The presence of a frequency at twice the fringe spacing frequency with a peak to valley value less than 1nm is still evident due to the difference between the actual phase shift and the estimated phase shifts. However, in comparing Fig. 5 with Fig. 4 it is obvious that the phase shift estimation and new reconstruction algorithm have significantly reduced the magnitude of measurement error.
TABLE 1: INTENSITY VALUES FOR THE SIMULATION

<table>
<thead>
<tr>
<th>Pixel</th>
<th>10</th>
<th>200</th>
<th>390</th>
</tr>
</thead>
<tbody>
<tr>
<td>I₁</td>
<td>176</td>
<td>175</td>
<td>183</td>
</tr>
<tr>
<td>I₂</td>
<td>70</td>
<td>90</td>
<td>123</td>
</tr>
<tr>
<td>I₃</td>
<td>22</td>
<td>21</td>
<td>33</td>
</tr>
<tr>
<td>I₄</td>
<td>150</td>
<td>124</td>
<td>108</td>
</tr>
</tbody>
</table>

TABLE 2: VARIABLE VALUES FOR SIMULATION

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Initial Guess</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>100.6</td>
<td>104.5</td>
<td>100.4</td>
</tr>
<tr>
<td>B₁</td>
<td>79.3</td>
<td>82.5</td>
<td>79.8</td>
</tr>
<tr>
<td>φ₀</td>
<td>18.0°</td>
<td>-----</td>
<td>18.3°</td>
</tr>
<tr>
<td>A₂</td>
<td>97.6</td>
<td>102.5</td>
<td>97.9</td>
</tr>
<tr>
<td>B₂</td>
<td>77.5</td>
<td>81.5</td>
<td>76.8</td>
</tr>
<tr>
<td>φ₁</td>
<td>0.2°</td>
<td>-----</td>
<td>-0.1°</td>
</tr>
<tr>
<td>A₃</td>
<td>105.1</td>
<td>111.75</td>
<td>104.8</td>
</tr>
<tr>
<td>B₃</td>
<td>81.5</td>
<td>78.75</td>
<td>81.9</td>
</tr>
<tr>
<td>φ₂</td>
<td>-17.8°</td>
<td>-----</td>
<td>-17.2°</td>
</tr>
<tr>
<td>φ₃</td>
<td>95</td>
<td>90</td>
<td>94.5</td>
</tr>
<tr>
<td>φ₄</td>
<td>170</td>
<td>180</td>
<td>170.2</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>270</td>
<td>290.4</td>
</tr>
</tbody>
</table>

\[
\phi(x, y) = \frac{4\pi}{\lambda} \tan \left( \frac{0.5788I_2 - 0.1697I_3 - 0.4091I_4}{0.4288I_1 - 0.5455I_3 + 0.1166I_4} \right) \tag{35}
\]

Using Eq. (35) to reconstruct the experimentally measured surface using the fringe patterns shown in Fig. 6 yielded the surface shown in Fig. 8. The presence of a frequency at twice the fringe spacing frequency can still be distinguished due to the difference between the actual phase shift and the estimated phase shifts. However, in comparing Fig. 8 with Fig. 7 it is obvious that the phase shift estimation and new reconstruction algorithm have significantly reduced the magnitude of measurement error.

SUMMARY AND CONCLUSIONS

This paper has addressed the problem of phase shift errors on the surface measurement in interferometry systems. An algorithm was developed to estimate the actual phase shifts and from these actual phase shifts generate a new surface reconstruction algorithm. The phase shift estimation algorithms utilizes a modified Gauss-Seidel numerical technique to solve twelve nonlinear equations in twelve unknowns. The twelve equations were developed using interferometry theory based on the wave theory of light. To generate the twelve equations, the intensity values of four fringes at three different pixel values are required. Solving these equations yields an estimate of the actual phase shift in the interferometer. These estimated phase shifts can then be used to generate a new/reduced error surface reconstruction equation.

To verify that a reduction in measurement error would result from using these algorithms, and therefore validate the algorithms, a simulation experiment was designed. In the simulation experiment a set of four fringe patterns was generated with known phase shifts and assuming that the measured surface was perfectly
flat. It was shown that the error reduction algorithms significantly reduced the measurement error.

Finally, an actual experiment was run to determine if the error reduction algorithms could be used successfully on real experimental fringe patterns. As in the simulation case, the algorithms also yielded a reduction in measurement error. This demonstrated that the error reduction algorithms can be effectively incorporated into interferometry systems to reduce the measurement error due to inaccurate phase shifts.

ACKNOWLEDGMENT

Partial support by the US DOEd award #P200A10123 is gratefully acknowledged.

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