MODELLING OF SURFACE GENERATION MECHANISMS IN TURNING

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ABSTRACT

A two degree of freedom model of the turning process is develop, that allows for simulation of tool vibration in both the radial and longitudinal directions. The 2-D vibration model is used in conjunction with a 3-D surface generation model to obtain simulated surface maps. A wavelength decomposition of the simulated surfaces is accomplished using a 2-D spectral analysis technique. The wavelength decomposition algorithm provides information on both the wavelengths present in the surfaces and their direction of travel. A relationship between the wavelengths in the simulated surface and the 2-D process model is then developed.

1 INTRODUCTION

Manufacturing a machine component or a finished product is comprised of several stages from making a decision on how to manufacture and assemble the components to actually producing the product. Among these various stages, the machining of components plays an important role in determining the structural and functional stability of the manufactured components. The functional stability of a machined component such as the load bearing capacity, fatigue strength, resistance to wear, resistance to corrosion, etc., depends to a large extent on the surface finish characteristics. The ability to predict the surface texture of a machined part will result in better control of the process and component quality. It is important to understand how the process inputs such as cutting conditions and tool geometry affects the surface texture, based on which the process can be optimized. This aspect of process control provides the motivation to develop models describing input-output models of a process.

Under ideal conditions, i.e., based on purely process geometry, the surface profile of a turned surface can be obtained by following the path of the tool-tip, with respect to the feed intervals. But in reality the actual surface profiles contain components arising from sources other than process geometry. It was recognized that, "To determine the peak force and force variations that strongly affect the tool breakage, surface error and surface texture, it is necessary to make use of a model that describes the dynamics of the process [2]." However, it has been long recognized that the most powerful sources of self excitation, those of mode coupling and of regeneration are not associated with the chip formation mechanics but with structural dynamics of the machine tool and the feedback between subsequent cuts [18], [16] and [17]. Since then there has been a significant amount of work done in the area of dynamics for metal cutting process. The effects of the variation of the uncut chip thickness that are caused by the relative displacement between the cutting tool and workpiece in the presence of vibrations have been studied [2]. Mechanistic models to predict the process dynamics that can be implemented by computer simulations were developed.

The complex nature of the surface generation mechanisms in machining operations, including both the process geometry and the vibrations due to process dynamics, makes predicting surface texture difficult. Therefore, in making attempts at predicting surface texture, simplifying assumptions have been adopted. In turning, tool vibrations in only one-dimension were considered to be significant, but in reality the tool can vibrate in three-dimensional space. This work however will be limited to the study of tool vibration in only the radial and longitudinal directions.

The objective of this research is to study the two dimensional effects of machine tool vibration on
the three dimensional surface texture of a turned part by examining its wavelength structure or spatial frequency content. A dynamic model that accounts for the tool vibrations in the radial and in the longitudinal directions is used to investigate the effect of the process dynamics on turned surfaces. A surface generation model is developed and used to predict changes in the surface profile due to the effect of the tool vibration in the longitudinal and radial direction. The surface texture imparted by the process due to the feed, tool, and process dynamics is studied using two-dimensional spectral analysis. This procedure can be used to predict the surface texture and understand the surface generation mechanisms in turning.

The remainder of the paper is divided into six sections. The first section consists of a review of the relevant literature concerning modelling of the turning process and surface characterization. The second section presents a two-dimensional model for turning dynamics with the third presenting a model of the surface generation mechanisms in turning. The fourth section provides a discussion of two-dimensional spectral analysis techniques applied to surface texture characterization. The fifth section includes a description of the simulation and the results. The sixth section contains the conclusions.

2 LITERATURE REVIEW

Most of the fundamental work in metal cutting process dynamics was aimed at determining the relationships between cutting forces, process geometry and cutting conditions for both orthogonal and oblique cutting. Recent efforts to understand metal cutting process dynamics include that of [3], [4] and [5]. Some other work related to cutting dynamics included relating the cutting process and surface generation models for the turning processes [20] and [2]. This motivated the work related to quantification or characterization of the surface generation process.

Over the years, a large number of parameters have been developed to describe the various aspects of surface texture, many in an ad hoc approach to some functional problem [15]. A summary of the most common parameters is given in [14]. Recently, more sophisticated methods of surface characterization have been developed by the use of wavelength decomposition techniques [7]. Pandit and Shanmugham [9] proposed the use of the DDS methodology [10] to characterize and decompose the wavelength structure in surface profiles. A better understanding of the relationship between the process and surface pattern has been demonstrated by the use of wavelength decomposition techniques [8]. The wavelength structure of the surface bridges the gap between the manufacturing process mechanics and surface pattern. Moon and Sutherland [6] demonstrated the use of wavelength decomposition techniques in investigating the origin of spatial frequencies within a turned surface profile. More recently, the effect of tool wear on a turned surface profile has been studied [12] via spectral analysis.

All these previous studies have accounted for process dynamics due to the vibration in one direction only. Studies have revealed that vibration also occurs in the direction tangential to the workpiece surface and it can be quite significant [19] and [1]. Further, surface characterization by wavelength decomposition techniques, used so far, viewed the surface only in two dimensions (height and length), and didn’t consider the fact that the manufactured surfaces are three dimensional in nature. To adequately assess the wavelength structure of a three dimensional surface, it is necessary to use two-dimensional spectral analysis. Two-dimensional spectral analysis determines both the wavelength structure and the direction of travel of each wave [11].

3 2-D VIBRATION MODEL

The machine tool structure represents a three dimensional multi degree-of-freedom structure carrying the tools at one point and the workpiece at the opposite point. Such a structure used to perform a machining process demonstrates an infinite number of modes, each of which are excited to a greater or lesser extent by the material removal process. The complex nature of the dynamic behavior of the machining process can be characterized by the use of a simple transfer function model with the machine tool dynamics represented by a two degree of freedom vibration model. In this paper, the structural dynamics of a turning process is modeled by a two-degree of freedom system, shown in figure 1. The model accounts for the structural vibrations of the machine tool in the radial and longitudinal directions.

As illustrated in figure 1, the nominal depth of cut for the turning process is d0 and f is the feed. The initial diameter of the workpiece is D and the spindle speed of the workpiece is Ns. The structural dynamics of the tool are characterized by a mass mx, and stiffness, kx in the radial direction and a mass my, and stiffness ky in the longitudinal direction. In case of a turning process, as shown in figure 3.1, forces are generated in the radial (X), longitudinal(Y), and tangential (Z) directions. The damping coefficients c,
and $c_y$ describe the tool flank interference force in the radial and longitudinal direction respectively. Based on the system structure and the radial and longitudinal forces $F_x(t)$ and $F_y(t)$ respectively, the equations of motion for the two degree-of-freedom system shown in the figure 1 can be described as:

\[ m_x \ddot{x}(t) + c_x \dot{x}(t) + k_x x(t) = F_x(t) \]  
\[ m_y \ddot{y}(t) + c_y \dot{y}(t) + k_y y(t) = F_y(t) \]

It is known that the radial cutting forces are, in part, a function of the chip load and tool flank interference force [5]. The radial force tends to separate the tool from the workpiece and consequently affect the surface texture. There is radial displacement of the tool $x$, due to the effect of this radial load, as a result of which, the depth of cut actually experienced by the workpiece will be $d_0 - x$. Due to the nature of the process, material left uncut during one workpiece revolution will increase the material to be removed in subsequent revolutions. The dependence of the chip load on both present and past displacements suggests that the radial force may be represented as [21]:

\[ F_x(t) = K_f [d_0 - x(t) - x(t - \tau)] \]

where $x(t)$ is the radial displacement at time $t$, $x(t-\tau)$ is the radial displacement at time $t-\tau$, where $\tau$ is the time required for one revolution of the workpiece and $K_f$ is the coefficient that relates chip load to the force in the radial direction.

**Figure 1: Top view of turning process**

It can be seen from figure 1 that at the same time as the radial displacement there is also a longitudinal displacement which would affect the instantaneous feed of the cutting process. The displacement $y$ caused by the longitudinal force $F_y(t)$ causes a shift in the feed that can be given as $f-y$. The radial displacement and longitudinal displacement causes a coupling effect in the radial and longitudinal forces. Therefore the radial force, in such circumstances, can be represented as:

\[ F_x(t) = K_f [f - y(t) - y(t - \tau)] [d_0 - x(t) + x(t - \tau)] \]

Following a similar procedure, the representation of the longitudinal force can be obtained as:

\[ F_y(t) = K_l [f - y(t) - y(t - \tau)] [d_0 - x(t) + x(t - \tau)] \]

where $y(t)$ is the longitudinal displacement at time $t$, $y(t-\tau)$ is the longitudinal displacement at time $t-\tau$ and $K_l$ is the coefficient that relates chip load to the force in longitudinal direction.

Combining equations (4) and (5) with equations (1) and (2), the system of equations of motion for the system illustrated in figure 1 are:

\[ m_x \ddot{x}(t) + c_x \dot{x}(t) + k_x x(t) = K_f [d_0 - x(t) - x(t - \tau)] \]

\[ m_y \ddot{y}(t) + c_y \dot{y}(t) + k_y y(t) = K_l [f - y(t) - y(t - \tau)] [d_0 - x(t) + x(t - \tau)] \]

These equations represent the process dynamics of the system and can be used to simulate the radial and longitudinal displacements $x$ and $y$ respectively which can be used to estimate the surface texture of such a machining process.

### 4  3-D SURFACE MODEL

A process model for the generation of the surface profile of a turned surface was presented for the presence of one degree-of-freedom dynamics [6]. This model will be extended for the case of the two degree-of-freedom process dynamics discussed above. The topography of a turned surface depends on the time varying position of the tool point during machining [6]. The time varying position of the tool point during machining is dependent on the tool geometry, tool/workpiece vibratory motion, feed and spindle speed. The geometry of the tool tip used to form the surface can be represented by a model of its nose radius. Such a model that has the position and radius of the tool tip as the input can be used to
predict the surface topography. Turned surface profiles often appear as a series of arc segments (scallop) when formed as the tool tip geometry is imparted to the workpiece. Figure 2 illustrates a process model that relates the process dynamics and surface generation mechanism.

Figure 2: Block diagram of process model used for surface generation

Though, typically the surface profiles are collected along the workpiece axis, in this paper we are interested in collecting three dimensional topographic map of the surface. This is important since the displacement in both radial as well as in longitudinal direction are being considered, and these displacements will cause a shift in the actual position of the tool tip, as the instantaneous feed and the depth of cut changes.

5 2-D SPECTRAL ANALYSIS

The surface texture generated by the 3-D surface generation model needs to be characterized so that the resulting texture can be assessed. Characterizing the three-dimensional data can be accomplished using two-dimensional spectral analysis techniques. It has been shown that two-dimensional spectral analysis provides a comprehensive description of both the structure and scales of the pattern in a spatial data set [11]. One-dimensional spectral analysis techniques that have been used for surface characterization don't, fully describe the surface unless the surface is purely two-dimensional or isotropic in nature. Whereas one dimensional spectral analysis can provide information about the wavelength components present in a surface profile, two dimensional Fourier analysis techniques provide information on both the wavelength components and their respective directionality for a three-dimensional surface map [13].

Two dimensional Fourier transform techniques applied to spatial data will determine a Fourier coefficient \( f(\omega_1, \omega_2) \) where \( \omega_1 \) and \( \omega_2 \) are the frequencies in the x and y directions respectively. The Fourier coefficient can be represented as a complex number \( a_{pq} + ib_{pq} \) where \( p \) and \( q \) indicate the indices associated with the x and y directions respectively [11].

Let \( S_{xy} \) denote a two dimensional array of surface heights corrected for their overall mean where, \( (x=1,...,m; y=1,...,n) \) By taking the Fourier transform of the mean-corrected data, a matrix of fourier coefficients can be determined by,

\[
f(\omega_1, \omega_2) = \left( \frac{1}{mn} \right) \sum_{x=1}^{m} \sum_{y=1}^{n} S_{xy} e^{2\pi i \left( \frac{px}{m} + \frac{qy}{n} \right)}
\]

with \( p=0,...,m-1 \) and \( q=0,...,n-1 \) and where,

\[
f(\omega_1, \omega_2) = a_{pq} + ib_{pq}
\]

The surface \( S_{xy} \) is composed of a set of cosine waves,

\[
S_{xy} = \cos \left[ 2\pi \left( \frac{px}{m} + \frac{qy}{n} \right) \right]
\]

where each wave results in a spikes in the fourier spectrum at points \( (p,q) \). The wavelength of each wave \( W(\omega_1, \omega_2) \) is determined by,

\[
W(\omega_1, \omega_2) = \frac{mn}{\sqrt{(m^2 q^2 + n^2 p^2)}}
\]

and the direction of travel of the wave \( \theta(\omega_1, \omega_2) \) is determined by,

\[
\theta(\omega_1, \omega_2) = \tan \left( \frac{pp}{mq} \right)
\]

where \( 0 \leq \theta(\omega_1, \omega_2) < \pi \). The magnitude of each individual fourier coefficient will indicate the importance of the contribution at the respective frequencies. A 2-D power spectrum can be evaluated to provide a measure of the relative significance at each frequency. The power, \( P(\omega_1, \omega_2) \), associated with each wave can be determined by,

\[
P(\omega_1, \omega_2) = \frac{mn}{\sigma^2} \left( a_{pq}^2 + b_{pq}^2 \right)
\]

where,
\[ \sigma^2 = \frac{\sum_{x=1}^{m} \sum_{y=1}^{n} s_{xy}^2}{mn} \]  

Using Eq.’s (11), (12) and (13) the structure of a surface can be represented mathematically in such a way as to provide an understanding of the underlying texture.

6 SIMULATION AND RESULTS

The presence of coupled forcing functions in the dynamic model, as shown in equations (6) and (7), makes it a difficult and a tedious process to obtain an analytical solution to the process dynamics models presented here. Also, the basic objective of this paper is not to provide analytical solutions to the of surface generation and tool dynamic models, but to understand the presence of the frequency patterns and the underlying implications of wavelength decompositions. This will provide an understanding of the effect of the tool vibrations in two directions and their effect on the surface generation process. A two-dimensional spectral analysis approach is used to describe the surface generation mechanisms without actually solving the equations analytically.

Computer simulation models were developed and coded in FORTRAN for simulating the process dynamics and the resulting surface texture. The cutting conditions and the values of different parameters used in the simulation procedures are described in table 1. These conditions were chosen to be similar to those actually used in turning tests.

<table>
<thead>
<tr>
<th>Table 1: Conditions of simulated cutting experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workpiece Diameter</td>
</tr>
<tr>
<td>Nose Radius</td>
</tr>
<tr>
<td>Spindle speed</td>
</tr>
<tr>
<td>Feed</td>
</tr>
<tr>
<td>Depth of Cut</td>
</tr>
</tbody>
</table>

Initially a simulation was run assuming that there was no vibration present in the system. The resulting surface texture can be seen in figure 3. As expected the surface is that of the ideal case containing parallel rows separated by the feed distance with each row having a cross-section the shape of the tool nose radius. The two-dimensional Fourier amplitude spectrum of figure 3 is shown in figure 4 with the dominant frequencies tabulated in table 2. Table 2 indicates that in the no vibration case all frequencies present are at 0° or in the feed direction. The large spikes correspond to the feed frequency, with the smaller spikes occurring at a harmonic of twice the feed frequency. These spikes at +/- harmonics of the feed frequency are explained by the symmetry of the Fourier transform along the y-spatial frequency axis. Due to size limitations of the 3-D plotting software only the relevant a portions of the actual spectrum can be plotted.

![Figure 3: Surface map with no vibrations](image)

![Figure 4: Amplitude spectrum of figure 3](image)

<table>
<thead>
<tr>
<th>Table 2: Dominant Wavelengths from Figure 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (1/mm)</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>0, +/- 3.7856</td>
</tr>
<tr>
<td>0, +/- 7.5712</td>
</tr>
</tbody>
</table>

The simulation is then run with radial vibrations only. The surface map generated by this simulation is shown in figure 5. It is apparent that the radial vibrations don’t affect the feed spacing, just the height and size of the scallops made by the nose of the tool. The amplitude spectrum of figure 5 is shown in figure 6 with the dominant frequencies listed in table

118
3. In figure 6 the feed frequency is still the most dominant and the feed harmonic can still be seen, however, the dynamics of the system caused a group of frequencies to occur in the center of the plot. As can be seen in table 3 the group of dominant frequencies in the center of figure 6 all have wavelengths in a small range (0.11mm-0.1321mm) and the waves travel in a small range of directions (81.8°-95°). These frequencies can be explained by taking a closer look at the turning process.

As the tool travels circumferentially around the workpiece the tool is vibrating radially at an infinite number of frequencies [6]. At each revolution some frequencies will out of phase with the previous revolution. This phenomena creates a wave corresponding to each frequency that travels at an angle near 90° depending on the amount of 'out of phase'. In [6] these frequencies in the surface profile appeared as low frequencies due to aliasing and therefore it is difficult to fully understand their cause. The 2-D spectrum however, doesn't alias the frequencies and therefore their cause can be determined.

A simulation is then run with dynamics present only in the longitudinal or feed direction. The surface map generated by this simulation is shown in figure 7. It is evident that the longitudinal vibrations have a significant affect on the feed spacing. The amplitude spectrum of figure 7 is shown in figure 8 with the dominant frequencies listed in table 4. In figure 8 the feed frequency is still the most dominant and the feed harmonic can still be seen. Also, a small mound of frequencies due to cutting dynamics is present similar to figure 6. However, the mound is smaller, and from table 4 it is evident that the longitudinal vibration spreads the variation over a larger number of frequencies and therefore reduces the magnitude of the power related to the cutting dynamics frequencies.

![Figure 5: Surface map with radial vibrations](image)

![Figure 6: Amplitude spectrum of figure 5](image)

Table 3: Dominant Wavelengths from Figure 6

<table>
<thead>
<tr>
<th>Frequency (1/mm)</th>
<th>Direction ω (ω₁, ω₂)</th>
<th>Wavelength (mm) W(ω₁, ω₂)</th>
<th>Power (%) P(ω₁, ω₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,+/-.3.7856</td>
<td>0°</td>
<td>0.2642</td>
<td>2.268</td>
</tr>
<tr>
<td>8.7068,.7571</td>
<td>94.97°</td>
<td>0.1144</td>
<td>0.515</td>
</tr>
<tr>
<td>+/-7.5712,0</td>
<td>90°</td>
<td>0.1321</td>
<td>0.4607</td>
</tr>
<tr>
<td>9.0859,.3786</td>
<td>92.39°</td>
<td>0.11</td>
<td>0.4424</td>
</tr>
<tr>
<td>7.9497,1.1357</td>
<td>81.869°</td>
<td>0.11245</td>
<td>0.4356</td>
</tr>
</tbody>
</table>

![Figure 7: Surface map with longitudinal vibrations](image)

![Figure 8: Amplitude spectrum of figure 7](image)
Table 4: Dominant Wavelengths from Figure 8

<table>
<thead>
<tr>
<th>Frequency (1/mm) (\omega_1,\omega_2)</th>
<th>Direction (\theta(\omega_1,\omega_2))</th>
<th>Wavelength (mm) (W(\omega_1,\omega_2))</th>
<th>Power (%) (P(\omega_1,\omega_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, +/-3.7856</td>
<td>0°</td>
<td>0.2642</td>
<td>2.53</td>
</tr>
<tr>
<td>0, +/-7.5712</td>
<td>0°</td>
<td>0.1321</td>
<td>0.340</td>
</tr>
<tr>
<td>9.464, 1.893</td>
<td>78.7°</td>
<td>0.1036</td>
<td>0.254</td>
</tr>
<tr>
<td>7.950, 2.650</td>
<td>71.5°</td>
<td>0.1193</td>
<td>0.253</td>
</tr>
<tr>
<td>9.085, 1.136</td>
<td>82.9°</td>
<td>0.1092</td>
<td>0.231</td>
</tr>
</tbody>
</table>

The final simulation is run including the cutting dynamics in both the longitudinal and radial directions. The surface map generated by this simulation is shown in figure 9. It is evident that the inclusion of both radial and longitudinal vibrations creates the roughest surface as would be expected, because both the tool height and feed spacing is changing. The amplitude spectrum of figure 9 is shown in figure 10 with the dominant frequencies listed in table 5. The same mound of frequencies in the center of the plot is present. It is difficult to determine which frequencies are due to the radial vibration and which are due to the longitudinal vibration because independently they each generate similar frequencies.

Figure 9: Surface map with both vibrations

Figure 10: Amplitude spectrum of figure 9

Table 5: Dominant Wavelengths from Figure 10

<table>
<thead>
<tr>
<th>Frequency (1/mm) (\omega_1,\omega_2)</th>
<th>Direction (\theta(\omega_1,\omega_2))</th>
<th>Wavelength (mm) (W(\omega_1,\omega_2))</th>
<th>Power (%) (P(\omega_1,\omega_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, +/-3.7856</td>
<td>0°</td>
<td>0.2642</td>
<td>1.32</td>
</tr>
<tr>
<td>7.950, 0</td>
<td>90°</td>
<td>0.1258</td>
<td>0.495</td>
</tr>
<tr>
<td>7.571, -0.7570</td>
<td>95.7°</td>
<td>0.1314</td>
<td>0.449</td>
</tr>
<tr>
<td>8.328, -0.3792</td>
<td>92.6°</td>
<td>0.1199</td>
<td>0.370</td>
</tr>
<tr>
<td>7.950, -0.7570</td>
<td>84.6°</td>
<td>0.1252</td>
<td>0.334</td>
</tr>
</tbody>
</table>

7 CONCLUSIONS

- A simulation model of a turning process was developed that includes cutting dynamics and stochastic components with both longitudinal and radial vibrations.
- The model provides a surface topography map that can be studied to better understand the effect that process dynamics has on a turned surface.
- A 2-D spectral analysis technique was employed to evaluate the surface maps and gain a better understanding of how the process dynamics effect the surface texture of turned parts. This technique showed a distinct advantage over the 1-D spectral analysis used in [6].
- Finally, this model may be used to assist in the design of machine tools and manufacturing processes.

8 REFERENCES


9 BIOGRAPHICAL SKETCHES

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