PROCRUSTES ANALYSIS AND ITS APPLICATION TO SENSOR INTEGRATION

by

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Abstract

A new technique for the synthesis of data from multiple sensors in a manufacturing process is proposed. This technique, Procrustes analysis, has been used previously in the sensory science field to develop a consensus opinion from data obtained from several judges. Procrustes analysis seeks to transform the data from a sensor/judge so as to better match the data from other sensors. Procrustes analysis works to maximize the agreement for the data from several sensors through translation, rotation, and scaling transformations. The theoretical basis for Procrustes analysis is described, that is expressions are developed for the best fit translation, rotation, and scaling transformations. An algorithm is presented to perform the Procrustes analysis, and is demonstrated through a manufacturing process illustration. It is shown that Procrustes analysis can reduce the disagreement between sensor data and provide a consensus as to the state of the process.

Introduction

The performance of a manufacturing process is heavily dependent on the environment in which it operates and the product it is required to produce. The drive toward increased product performance, i.e., tighter design tolerances, more exotic materials, etc., has placed additional burdens on the manufacturing processes used to produce those products. While expecting more from their manufacturing processes, manufacturers have begun to automate their processes - often removing the human supervisor from the system. We are therefore faced with a situation in which we expect more from the manufacturing processes, but very often we are supplying the processes with less supervision/control. A human controlling a manufacturing process performs a number of functions including:

1. Process observation,
2. Evaluation of process data,
3. Decision about process state,
4. Formulation of corrective action if process state undesirable, and
5. Implementation of corrective action.

For an automated system, hardware and software must replace these functions. When a sensor is introduced into a manufacturing process environment the ultimate goal is to come to some decision about the state of the process. To accomplish this goal, techniques must be available for analyzing sensor data and for making some decision about the process state based on this data. Much research has been undertaken to investigate new sensing techniques [14] and also to more efficiently and intelligently use the information from a sensor such as in [1,2,9,11,12].

To more precisely and accurately identify the state of a manufacturing process one may consider using more than one sensor in a manufacturing process. This seems only reasonable since a human controlling a process uses all his senses (multiple sensors) to observe the process. The use of multiple sensors will increase as we attempt to more closely mimic and ultimately improve upon the human controlled manufacturing process situation. It is clear that the use of multiple sensors provides more information on the evolving state of a manufacturing process than does a single sensor system. It is often unclear, however, how to best use all this information to develop the best possible description of the state of the system.

The use of multiple sensors in a manufacturing process carries with it a number of challenges that must be addressed in order to successfully integrate the sensors. These challenges include:

- Integrating sensor data from very different sources. For example, integrating acoustic emission and accelerometer data.
- Integrating sensor data of varying reliability. For example, ascertaining when the data coming from a sensor is in error, or has become corrupted.
- Synthesizing from the sensor data an estimate of the state of the system. For example, developing from tool temperature data an estimate of the amount of tool flank wear.

A number of approaches have been described for sensor integration including: i) neural networks, ii) the group method of data handling (GMDH), and iii) multiple regression. In [3] Domroese and Chrysochouris compared these approaches and concluded that a neural network was better than the other approaches in empirically characterizing the underlying relationships within a process.

In [4,5], Chrysochouris, et al. describe an approach for sensor synthesis in which the data from sensors may be input to process models from which, for each sensor, state variable estimates may be obtained. This approach is illustrated in Figure 1. The problem then becomes one of integrating or synthesizing these state variable estimates to produce more accurate and precise estimates of the state variables. In these papers these estimates are combined based on knowledge of the probability density functions of the state variables. It is suggested that such knowledge may be obtained through off-line simulations with the process models.

To illustrate this approach, consider a turning machining process for which it is desired to construct estimates of the state variables: flank wear and surface finish. Let us also assume that the process is instrumented with sensors to measure: acoustic emission, cutting force, and cutting tool temperature. Assuming that models are available to relate the process outputs to the states, then for each sensor, an estimate of both the flank wear and surface finish can be constructed. As pointed out in [5], if the sensors are equally corrupted by noise and if the models adequately reflect the complexity of the process, then the state estimates may simply be averaged to provide more precise estimates of the true process states. In general though, this is not the case, and the state estimates may be deterministically biased from the true states.

The use of process models in the development of synthesized state variable estimates appears to be a significant departure away from other sensor integration techniques that are essentially empirical in nature. It is to be expected that a sensor synthesis system that is mechanistic in nature, as opposed to empirical, would tend to be more...
Process Models

![Block diagram illustrating the use of sensors to construct state estimates](image)

The criterion most often used for Procrustes analysis is to minimize the sum-of-squares between the set of points for a configuration and the corresponding consensus.

![Illustration of Individual Configurations and Consensus](image)

As stated previously, Procrustes analysis seeks to obtain an agreement for several configurations through translation, rotation, and scaling transformations. The rotation of $X_0$, an $(n \times p)$ matrix, may be accomplished by post-multiplying the matrix by a $(p \times p)$ orthogonal rotation matrix, $H$. Uniform scaling may be accomplished by multiplying $X_0$ by the scalar $\rho$. Translation may be accomplished by adding the same $(1 \times p)$ row vector to each row in $X_0$, or equivalently by adding the matrix $T_i$ to $X_0$, where $T_i$ contains an identical row vector. When the translation, rotation, and scaling transformations are applied to $X_0$ they produce a new configuration, $X^*_0 = \rho_1 X_0 H_i + T_i.$

The Procrustes problem is then to select $\rho_i, H_i,$ and $T_i$ so that the residual sum of squares, expressed by Eq. (2), is a minimum.

$$S = \sum_{i<j} \text{Tr} \left[ (\rho_i X_i H_i + T_i) - (\rho_j X_j H_j + T_j) \right]$$

$$+ (\rho_i X_i H_i + T_i)^\top - (\rho_j X_j H_j + T_j)^\top.$$  

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Before we proceed to minimize $S$, some general discussion is in order. It may be observed that $S$ may be minimized by choosing all the scaling coefficients, $p_i$, to be zero. To avoid this trivial solution, Gower [6] suggests the following constraint be imposed:

$$
\sum_{i=1}^{m} p_i^2 tr(X_i^t X) = \sum_{i=1}^{m} tr(X_i^t X) .
$$

(3)

This constraint forces the sum of squares of the data about the origin to be unchanged by the scaling coefficients. Additionally, to allow for different magnitudes of data, the $X_i$ may be initially scaled so that:

$$
\sum_{i=1}^{m} tr(X_i^t X_i) = m .
$$

(4)

Following the completion of the Procrustes analysis, the data may be returned to their original units.

As is evident from Eq. (1), a translation term is included in the transformation. Differentiation of Eq. (2) with respect to this translation term, and equating the result to zero gives the result that all $m$ configurations should be translated to have the same centroid. This centroid may be conveniently chosen to be the origin. To accomplish this translation, the matrix $T_i$, consisting of $m$ identical row vectors, may be formed. A column element in $T_i$ is simply the negative of the column mean of $X_i$.

With respect to the selection of the rotation matrices, $H_i$, it may be noted that since $S$ will be unaffected by orthogonal rotations of the whole system of points (i.e., the points for all sensors and treatments), no unique solution for the $H_i$ can be found. To obtain a solution, one might consider determining the rotation matrices relative to a fixed configuration (e.g., $X_i$). In this paper, however, a solution will be used that produces $m$ rotation matrices. A unique solution may then be determined, as a final step, by referring all final coordinates to the principal axes of the set of centroid points or consensus configuration, $Y$.

In considering rotation, it may be noted that $H_i$ must be a valid rotation matrix. To insure this, the following constraint must be imposed on the sum of squares function, Eq. (2), that we wish to minimize:

$$
H_i^t H_i = I \quad \text{for } i = 1, 2, \ldots, m
$$

(5)

finding the rotation matrices ($H_i$) and scaling coefficients ($p_i$) that minimize the residual sum of squares. This constraint as well as the constraints defined by Eq. (3) may be imposed on Eq. (2) using the method of Lagrange multipliers to form the objective function to be optimized.

**Best Fit Rotation Matrix**

With the constrained objective function (Lagrangian) in hand, attention may be turned to differentiating the Lagrangian with respect to the rotation matrices and setting the result equal to zero. Simplifying, and expressing the result in matrix form gives:

$$
\rho_i X_i^t (mY - \rho_i X_i H_i) = B_i H_i , \quad i = 1, 2, \ldots, m
$$

(6)

where, $Y = \frac{1}{m} \sum_{i=1}^{m} \rho_i X_i H_i$, and

it may be noted that $Y$ is the consensus configuration, i.e., the matrix containing the centroid points. Additionally, the matrix $B_i$ contains the Lagrange multipliers associated with Eq. (5). Solving for $H_i$ gives:

$$
H_i = m (\rho_i^2 (X_i X_i^t) + B_i)^{-1} \rho_i X_i^t Y .
$$

(7)

Imposing the constraint specified by Eq. (5) produces:

$$
(\rho_i^2 X_i X_i^t + B_i)^{-1} = \frac{1}{m \rho_i^2} (X_i X_i^t)^{-1} .
$$

(8)

By taking the "square root" of both sides of this equation we obtain:

$$
(\rho_i^2 (X_i X_i^t) + B_i)^{-1} = \frac{1}{m \rho_i} (X_i^t Y Y_i X_i^t)^{-1/2} .
$$

(9)

The use of the term "square root" in this situation means that if matrix $C = A A$, then $A$ is the square root of $C$. With Eq. (9) now available, it may be substituted into Eq. (7) giving:

$$
H_i = (X_i^t Y Y_i X_i^t)^{-1/2} X_i^t Y .
$$

(10)

In order to evaluate Eq. (10) the square root of $(X_i^t Y Y_i X_i^t)^{-1}$ must be calculated. To do this, we may express it as $V_i E_i^{(1/2)}$, where $E_i$ is a diagonal matrix containing the eigenvalues and $V_i$ contains the eigenvectors of $(X_i^t Y Y_i X_i^t)^{-1}$. The square root of $(X_i^t Y Y_i X_i^t)^{-1}$ may then be expressed as $(V_i E_i^{(1/2)} V_i^{-1})$ where $E_i^{(1/2)}$ is obtained by taking the square root of each element on the diagonal of $E_i$. Thus, the best fit rotation matrix is given by:

$$
H_i = V_i E_i^{(1/2)} V_i^{-1} X_i^t Y .
$$

(11)

Of course, Eq. (11) does not give a closed form solution for $H_i$, owing to the dependence on $Y$, the centroid or consensus matrix. It does, however, provide us with a basis for an iterative algorithm.

**Best Fit Scaling Coefficient**

Differentiating the Lagrangian with respect to $\rho_i$, setting the result equal to zero, and simplifying gives:

$$
m \rho_i tr(X_i^t Y_i Y_i^t) + \mu \rho_i tr(X_i^t X_i) = 0 ,
$$

(12)

or,

$$
\rho_i = \frac{m \rho_i tr(X_i^t Y_i Y_i^t)}{(m + \mu) \rho_i tr(X_i^t X_i)} .
$$

(13)

where $\mu$ is the Lagrangian multiplier associated with Eq. (13).

Multiplying Eq. (12) by $\rho_i$, summing over $i = 1, 2, \ldots, m$, and recalling Eq. (3), Produces

$$
(m + \mu) = \frac{m \rho_i tr(Y_i Y_i^t)}{\sum_{i=1}^{m} tr(X_i X_i)} .
$$

(14)

Inserting the relationship for $(m + \mu)$ given by Eq. (14) into Eq. (13) gives

$$
\rho_i = \frac{tr(X_i^t Y_i Y_i^t) \sum_{i=1}^{m} tr(X_i X_i)}{m \rho_i tr(Y_i Y_i^t) tr(X_i X_i)} .
$$

(15)
As was the case with the solution for the best fit rotation matrix, the expression for the best fit scaling coefficient, Eq. (15), does not provide a closed form solution for \( p_i \). Once again, however, it does suggest us with a basis for an iterative algorithm to obtain a consensus.

**Procrustes Algorithm**

As has been pointed out, the expressions for the best fit rotation matrix and scaling coefficient do not provide closed form solutions. These expressions, however, may be used an iterative fashion through a Procrustes algorithm such as the one outlined below.

**Step 1:** Translate each \( X_i \) \((i = 1, 2, \ldots, m)\) to the origin.

**Step 2:** To insure that the requirement specified by Eq. (4) is satisfied, scale each \( X_i \) by \( \lambda \), where \( \lambda \) is found from the relationship:

\[
\sum_{i=1}^{m} \lambda^2 \text{tr} \left( X_i X_i^T \right) = m.
\]

The translated, scaled configurations are now stored in \( X_i \), and will be referred to as the "initial" configurations.

**Step 3:** Define the initial consensus, \( Y \), as the mean of the \( X_i \). In other words, \( Y \) is given by the equation:

\[
Y = \frac{1}{m} \sum_{i=1}^{m} X_i.
\]

Initialize the scaling coefficients: \( p_1 = 1 \). Initialize the rotation matrices: \( X_i = I \). Evaluate the residual sum of squares as:

\[
S_R = \sum_{i=1}^{m} \text{tr} \left( (Y - p_i X_i H_i) (Y - p_i X_i H_i)^T \right) \quad (16)
\]

This residual sum of squares characterizes the lack of agreement between the configurations. With the completion of step 3, all initialization procedures required by the algorithm are completed.

**Step 4:** Given the previously determined scaling factor and rotation matrix, \( p_i \) and \( H_i \), and the initial configuration, \( X_i \), the current configuration is \( X_i^* = p_i X_i H_i \). For \( i = 1, 2, \ldots, m \), rotate the current configuration to fit \( Y \). In other words, find the best fit rotation matrix, using Eq. (11), for the current configuration \( X_i^* \), call this matrix \( H_i^* \). The new rotation matrix maps the current configuration into the new configuration. The new configuration is \( X_i^* H_i^* \) or \( p_i X_i H_i H_i^* \). The updated rotation matrix that relates the initial to the new configuration is then \( H_i = H_i H_i^* \).

**Step 5:** Define a new consensus, \( Y \), as the mean of the current \( X_i^* \)'s. Evaluate the residual sum of squares using Eq. (16), call the result \( S_R^* \).

**Step 6:** For \( i = 1, 2, \ldots, m \), find the new best fit scaling coefficient using the following equation that is based on Eq. (15):

\[
\rho_i^* = \left( \frac{\text{tr} \left( p_i X_i H_i Y \right) \sum_{i=1}^{m} \text{tr} \left( X_i X_i^* \right)}{m \text{tr}(Y Y^T) \text{tr}(X_i X_i^*)} \right)^{1/2}.
\]

The new configuration is then \( p_i X_i H_i \). The scaling factor may be updated as \( p_i = \rho_i^* \).

**Step 7:** Define a new consensus, \( Y \), as the mean of the current \( X_i^* \)'s. Evaluate the residual sum of squares using Eq. (16), call this result \( S_R^* \).

**Step 8:** If the reduction in the residual sum of squares for this iteration \( (S_R - S_R^*) \) is less than the convergence criterion, the Procrustes algorithm is completed. Otherwise, set \( S_R = S_R^* \), and move to step 4 to begin another iteration. The selection of a numerical value for the convergence criterion is somewhat subjective. For the example considered in the next section, a criterion of 0.0001 was found to be satisfactory.

Once the Procrustes algorithm is completed, further analysis of the data may be undertaken. Analysis of variance and other statistical tests of significance may be used to assess differences between treatments. Principal component analysis may be used to further decompose the structure in both the individual configurations and the consensus.

**Sensor Integration in a Machining Process via Procrustes Analysis**

To illustrate how Procrustes analysis may be used to synthesize state estimates for a manufacturing process, let us consider the following scenario. It is desired to synthesize state estimates for flank wear and surface finish for a turning machining process that has been instrumented with sensors to measure acoustic emission, cutting force, and cutting tool temperature. It is assumed that process models are available to construct, for each sensor, estimates of both the flank wear and surface finish. Each time the process is examined, therefore, the output from a sensor is used to develop an estimate of both state variables. The problem faced is then how to integrate the three estimates of each state variable. Procrustes analysis, in the form of the algorithm described in the preceding section, may be used to perform this synthesis.

Continuing with the illustration, let us assume that at some point in time in the examination of the ongoing turning process seven sets of state variable estimates have been obtained. These estimates are summarized in Table 1. The Procrustes algorithm begins by translating these sets of data to the origin. This may be accomplished by subtracting the following row vectors from each row of the raw data shown in Table 1: Sensor 1 - (68.2857, 72.2857), Sensor 2 - (73.5714, 81.8571), Sensor 3 - (68.2857, 72.5714). The scale factor, \( \lambda \), introduced to scale the total sum of squares of the data to be equal to the number of sensors (equal to three in this case) is then found to be 0.00786625. After the initial translation and scaling transformations, the initial configurations are obtained; these are shown in Table 2.

The initial configurations may be plotted as shown in Fig. 3. From the figure and Table 1 it is clear that while there is some agreement between the state estimates, there is also conflict. The lack of agreement between the sensors may be characterized by calculating the residual sum of squares of the configurations about their consensus mean. This residual sum of squares is 0.205547. As a measure of the agreement between configurations this sum of squares actually is relatively small since the calculations have been made after translation (removing positional bias between the configurations) and scaling (greatly reducing the magnitude of the data in this case). To get a feeling for the magnitude of the residual sum of squares, it may be compared to the total sum of squares of the data about the origin (equal to \( m = 3 \)). Using this comparison, it may be observed that the lack of agreement between the configurations represents approximately 7% of the total variation.

Transactions of NAMRI/SME 350  Volume X, 1992
Table 1: Raw State Variable Estimates

<table>
<thead>
<tr>
<th>Process Examination</th>
<th>Sensor 1 Acoustic Emission</th>
<th>Sensor 2 Cutting Force</th>
<th>Sensor 3 Tool Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>A1</td>
<td>A2</td>
<td>A1</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>81</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>70</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>72</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>66</td>
<td>70</td>
<td>73</td>
</tr>
<tr>
<td>5</td>
<td>89</td>
<td>68</td>
<td>103</td>
</tr>
<tr>
<td>6</td>
<td>115</td>
<td>71</td>
<td>132</td>
</tr>
<tr>
<td>7</td>
<td>134</td>
<td>74</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 2: Initial Configurations: \( X_i (i=1, 2, 3) \)

\[
X_1 = \begin{bmatrix}
-0.40343 & 0.06855 \\
-0.34050 & -0.01798 \\
-0.28543 & -0.00225 \\
-0.01798 & -0.001798 \\
0.16294 & -0.003371 \\
0.36747 & -0.01011 \\
0.51693 & 0.01349
\end{bmatrix}
\]

\[
X_2 = \begin{bmatrix}
-0.55513 & -0.21127 \\
-0.38994 & -0.21913 \\
-0.34274 & -0.17193 \\
-0.00450 & -0.01461 \\
0.23149 & 0.06405 \\
0.45961 & 0.22138 \\
0.60121 & 0.33151
\end{bmatrix}
\]

\[
X_3 = \begin{bmatrix}
-0.40343 & -0.00450 \\
-0.30117 & -0.09889 \\
-0.25397 & -0.06743 \\
-0.01011 & -0.02809 \\
0.16294 & -0.00450 \\
0.34387 & 0.06630 \\
0.46186 & 0.13710
\end{bmatrix}
\]

It may be noted at this point that a decision concerning the state of the process most typically would be made for an example such as this in the (flank wear, surface finish) coordinate space. Such a decision might be: flank wear too large, surface finish too large, and process satisfactory at present. These decisions could also be made in the coordinate space displayed in Fig. 3, using the transformed values for the flank wear and surface finish. Decision criteria such as those shown in Fig. 4 might be employed and applied to a consensus in such a situation to evaluate the state of a process.

Table 4 along with the associated new configurations. The residual sum of squares associated with the deviation of these new configurations from their consensus was found to be 0.0234216. With the completion of this scaling step, the first Procrustes iteration is completed. The configurations at the end of the first Procrustes iteration are shown in Fig. 5. By comparing Figs. 3 and 5, and examining the associated residual sum of squares (0.02655 and 0.0234) it is clear that a much greater agreement between the three configurations has been achieved through the rotation and scaling steps.

Figure 3: Initial Configurations prior to Procrustes Analysis

![Figure 3](image)

Figure 4: Example of Process State Decisions in Transformed Coordinate Space

![Figure 4](image)

The Procrustes algorithm is said to have converged when the reduction in the residual sum of squares from one iteration to the next is less than some specified convergence criterion. The convergence criterion selected for this example was 0.0001. A total of six iterations were required to achieve convergence. The effect of the rotation and scaling steps on the residual sum of squares for each iteration are summarized in Table 5. As is evident, at convergence the residual sum of squares is 0.0029252, or the disagreement between the configurations accounts for only about 0.1% of the total variation in the data.
Table 3: Rotation Matrices and Resulting Configurations for First Procrustes Iteration

\[
H_1 = \begin{bmatrix}
0.96463 & 0.26361 \\
-0.26361 & 0.96463
\end{bmatrix}, \quad H_2 = \begin{bmatrix}
0.97624 & -0.21672 \\
0.21672 & 0.97624
\end{bmatrix}, \quad H_3 = \begin{bmatrix}
0.99940 & 0.03459 \\
-0.03459 & 0.99940
\end{bmatrix}
\]

\[
X_1 = \begin{bmatrix}
-0.40723 & -0.04022 \\
-0.32371 & -0.10710 \\
-0.27474 & -0.07741 \\
-0.01260 & -0.02208 \\
0.16607 & 0.01043 \\
0.35714 & 0.08711 \\
0.49509 & 0.14928
\end{bmatrix}, \quad X_2 = \begin{bmatrix}
-0.58772 & -0.08594 \\
-0.42816 & -0.12942 \\
-0.37186 & -0.09357 \\
-0.00755 & -0.01329 \\
0.23987 & 0.01236 \\
0.49667 & 0.11651 \\
0.65876 & 0.19334
\end{bmatrix}, \quad X_3 = \begin{bmatrix}
-0.40303 & -0.01845 \\
-0.29756 & -0.10925 \\
-0.25148 & -0.07617 \\
-0.00914 & -0.02843 \\
0.16300 & 0.00114 \\
0.34137 & 0.07816 \\
0.45684 & 0.15299
\end{bmatrix}
\]

Table 4: Scaling Factors and Resulting Configurations for First Procrustes Iteration

<table>
<thead>
<tr>
<th>ρ_1 = 1.0663</th>
<th>ρ_2 = 0.9116</th>
<th>ρ_3 = 1.0953</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.43422 &amp; -0.04289</td>
<td>-0.53578 &amp; -0.07834</td>
<td>-0.44143 &amp; 0.02021</td>
</tr>
<tr>
<td>-0.34517 &amp; -0.11420</td>
<td>-0.39032 &amp; -0.11798</td>
<td>-0.32591 &amp; -0.11966</td>
</tr>
<tr>
<td>-0.29296 &amp; -0.08254</td>
<td>-0.33899 &amp; -0.08530</td>
<td>-0.27544 &amp; -0.08343</td>
</tr>
<tr>
<td>-0.01344 &amp; -0.02355</td>
<td>-0.00689 &amp; -0.01211</td>
<td>-0.01001 &amp; -0.03114</td>
</tr>
<tr>
<td>0.17708 &amp; 0.01113</td>
<td>0.21867 &amp; 0.01127</td>
<td>0.17853 &amp; 0.00125</td>
</tr>
<tr>
<td>0.38081 &amp; 0.09289</td>
<td>0.45277 &amp; 0.10621</td>
<td>0.37389 &amp; 0.08560</td>
</tr>
<tr>
<td>0.52790 &amp; 0.15917</td>
<td>0.60054 &amp; 0.17625</td>
<td>0.50036 &amp; 0.16757</td>
</tr>
</tbody>
</table>

The best fit rotation matrices and scaling coefficients for the last iteration as well as the final configurations are shown in Table 6. The final configurations are also displayed graphically in Fig. 6. An examination of the figure shows that the configurations are now in good agreement. With these three configurations and their consensus available, attention may be turned to the next stage of the problem; making a decision about the process state based on the sensor data. This will be much easier now that the sensor data is in agreement.

Once the Procrustes analysis has been completed, the relationship between the process states from the models/sensors is understood. As subsequent information from the sensors arises, the transformations developed using Procrustes analysis may be applied to the data to map it into the transformed space. This new data may then be evaluated under the assumption that suitable decision-making criteria have been formulated in the transformed space.

In addition to using Procrustes analysis to develop a consensus for data collected from several sensors, the analysis also provides other information on the sensors and the process models used to develop state variable estimates. Examining Table 6 again, consider the scaling factors given by ρ_i. It is seen that the factor for the second sensor is less than one, while the factors for the first and third sensor are greater than one. This suggests that the second sensor and its associated process models overestimate the larger flank wear and surface finish values. Such information would of course be useful in further refining the process models.

![Figure 5: Configurations Following the Completion of the First Procrustes Iteration](image)

![Figure 6: Final Configurations from the Procrustes Analysis](image)

Summary and Conclusions

Examining the rotation matrices of Table 6 also reveals some interesting information concerning the sensors and their associated process models. The rotation matrix for sensor 1 indicates that its data and coordinate axes were rotated by about 15.3° to match the...
Table 5: Residual Sum of Squares After Completing the Indicated Step  
(Initial Residual Sum of Squares = 0.205547)

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Rotation Step</th>
<th>Scaling Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.086414</td>
<td>0.023422</td>
</tr>
<tr>
<td>2</td>
<td>0.0079497</td>
<td>0.0041545</td>
</tr>
<tr>
<td>3</td>
<td>0.0041545</td>
<td>0.0032165</td>
</tr>
<tr>
<td>4</td>
<td>0.0032165</td>
<td>0.00029834</td>
</tr>
<tr>
<td>5</td>
<td>0.00029834</td>
<td>0.0029252</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Rotation Matrices, Scaling Factors, and Configurations at Convergence

\[ \rho_1 = 1.1195 \quad \rho_2 = 0.8222 \quad \rho_3 = 1.1803 \]

\[
H_1 = \begin{bmatrix} 0.96459 & 0.26377 \\ -0.26377 & 0.96459 \end{bmatrix} \quad H_2 = \begin{bmatrix} 0.97620 & -0.21686 \\ 0.21686 & 0.97620 \end{bmatrix} \quad H_3 = \begin{bmatrix} 0.99940 & 0.03461 \\ -0.03461 & 0.99940 \end{bmatrix}
\]

\[
X_1 = \begin{bmatrix} -0.45590 & -0.04511 \\ -0.36239 & -0.11996 \\ -0.30757 & -0.08671 \end{bmatrix} \quad X_2 = \begin{bmatrix} -0.48322 & -0.07059 \\ -0.35204 & -0.10635 \\ -0.30575 & -0.07689 \end{bmatrix} \quad X_3 = \begin{bmatrix} -0.47571 & -0.02178 \\ -0.35122 & -0.12896 \\ -0.29683 & -0.08991 \end{bmatrix}
\]

Consensus. The rotation matrix for sensor 2 indicates that its data and coordinate axes should be rotated by about 12.5° to match the consensus. The fact that these rotations differ by almost 30° may suggest the presence of a process model inadequacy or a problem with one of the sensors. It may be observed that the rotation matrix for sensor 3 corresponds to a rotation of only 2°. One might be led to believe that this sensor is the best since it is closest in agreement to the consensus. This is not necessarily the case, a small rotation means only that this data lies in between, in a rotational sense, the other sets of data.

Previous work has suggested that sensor data may be input to manufacturing process models to develop estimates for the state variables of the process. A new technique for the synthesis of the state variable estimates from multiple sensors has been proposed. This technique, Procrustes analysis, seeks to translate, rotate, and scale the data from a sensor so as to better match the data from other sensors. In short, Procrustes analysis is a technique for resolving conflict/disagreement between sensor data.

The theoretical basis for Procrustes analysis has been described. It has been seen that the technique searches for transformations that minimize the residual sum of squares between the configurations. The method of Lagrange multipliers was used to incorporate into the function to be minimized constraints insuring non-zero scaling coefficients and orthogonal rotation matrices. It was seen that the expressions for the best fit rotation and scaling transformations were not closed form solutions, but rather, were dependent on the consensus. To obtain a consensus despite the lack of closed form solutions, an iterative algorithm was presented to perform the Procrustes analysis.

To illustrate the workings of the Procrustes algorithm, an example was considered. This example focused on a turning process being monitored by three sensors: acoustic emission, cutting force, and tool temperature. Each sensor was assumed to have process models associated with it that could develop estimates of the flank wear and surface finish based on the data from the sensor. Seven examinations/treatments of the turning process were presented, and then the Procrustes algorithm was applied to these data/configurations. The disagreement between the 3 sensor data sets was reduced from approximately 7% of the total variation in the data to approximately 0.1%.

In addition to resolving conflict between sensor data and providing a consensus as to the state of the system, Procrustes analysis also has some other features. For the example considered, examination of the scaling factors revealed that one of the sensors appeared to consistently overestimate the larger values for flank wear and surface finish. Such a systematic tendency may be an indication of a problem with a model and/or sensor. Structural differences between the various configurations were also identified from the rotation matrices for the configurations. Once again these differences are an indication of a problem in the system. As is evident, one of the major benefits of Procrustes analysis is the identification of subtle relationships and structure between and within different sets of data.

A problem that is often mentioned with respect to the synthesis of data from multiple sensors is that of the varying reliability of the sensors. Another problem often described is that different sensors are not corrupted equally by noise. For either problem it would be desirable to weight the information from some sensors more than the information from others. Although this remains as future work, it is believed that such a weighting scheme could be readily incorporated into the Procrustes framework.

In this paper Procrustes analysis has been applied to the problem of synthesizing state variable estimates from estimates provided by several sensors/models. It has been seen that Procrustes analysis is a technique that is well suited to situations in which multivariate data from several sources are to be compared, contrasted, and/or reduced. There are many other engineering applications where Procrustes analysis may be usefully employed. It is the hope of these authors that this paper makes a contribution to not only the sensor integration problem but also to some of these other applications as well.
References


