A New Approach to Estimating the Cutting Process Damping Under Working Conditions

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ABSTRACT. The stability of a machining system against vibration is not only dependent on the structural damping but also dependent on the cutting process damping, which is generated by the chip formation process. In the past, techniques have been developed to estimate the cutting process damping which make use of special experimental set-ups to remove the regenerative effect. In this paper, an approach is proposed to estimate the cutting process damping from data collected under normal working conditions without the need for special action to remove the effect of regeneration. This new approach explicitly recognizes the effect of regeneration in the basic equation of motion which describes the dynamic behavior of the machining system. Vibration data obtained from computer simulation and from physical experiments are analyzed to obtain the cutting process damping through the proposed approach. The results indicate that the proposed approach provides good estimates of the cutting process damping for stable processes. However, when chatter occurs, the proposed approach may poorly estimate the cutting process damping.

INTRODUCTION. Many researchers have proposed that the cutting force can be represented as the product of the instantaneous uncut chip cross-sectional area and a constant, namely the cutting force coefficient. Under this assumption, the cutting force should vary in phase with the variation of chip thickness. However, it has been observed by Doi and Kato [1], that when oscillating a lathe tool normal to the cut surface, the cutting force varies out of phase with the variation of chip thickness. The penetration effect proposed by Tobias and Fishwick [2] explains this phenomenon. Tobias and Fishwick found that owing to the penetration effect, the dynamic cutting force is not only a function of chip thickness variation but also a function of the velocity of the tool as it penetrates the workpiece. The variation in cutting force due to the penetration effect is represented as the product of the velocity of the tool and a constant, namely the penetration coefficient. The penetration coefficient is found to be approximately proportional to the width of cut (or depth of cut) and inversely proportional to cutting speed [2,4-6]. This phenomenon indicates that a damping effect arises in the chip formation process.

The stability of a machining system is affected by the cutting process damping as well as the structural damping [2,5,6,7]. Smith and Tobias [3] found that the cutting process damping effect in the chip formation process generally increases the stability of the machining system against chatter. In order to predict the dynamic response of a machining system more accurately, the cutting process damping should be taken into account.

Variation in cutting forces causes vibration in machining systems. The occurrence of vibration will leave on the workpiece an undulating tooth path. The undulated surface will subsequently affect the chip loading of the next tooth as it passes over this surface. This is called the regenerative effect. The regenerative effect makes it more difficult to measure the cutting process damping. In the past, special experimental set-ups have been used to remove the regenerative effect and, hence, the cutting process damping could be measured easily from data obtained from such experimental systems [7,8]. In this paper, a method is proposed which will enable the measurement of the cutting process damping under working conditions without utilization of any special set-up to eliminate the regenerative effect.

In this paper, the model used to obtain the dynamic response equation for a one-degree of freedom machining system is used to compare the proposed approach for the estimation of the cutting process damping with two other methods [7,8]. These other two approaches are briefly reviewed and the proposed approach for the estimation of the cutting process damping is then developed based on the relation between the cutting process damping ratio and the parameters (system damping and vibration frequency) obtained from a Dynamic Data System (DDS) model [14] derived from vibration data. The proposed approach is studied through computer simulation using both the mathematical formulation for the one degree of freedom machining system and the dynamic face milling force model [12]. Vibration data obtained from a series of physical experiments are also analyzed to calculate the cutting process damping using the proposed approach and are compared to the results obtained through the simulations.

THEORETICAL MODEL FOR THE MACHINING PROCESS. It is assumed that the dynamic response of a machine tool system with one degree of freedom can be well represented by the following equation of motion:

\[ \ddot{x}(t) + 2\zeta \omega_n \dot{x}(t) + \omega_n^2 x(t) = K_c [x(t-\tau) - x(0)] - K_p \dot{x}(0). \]  

(1)

where \( x(t) \) is the relative displacement between the cutter and the workpiece; \( \zeta \) is the structural damping ratio; \( \omega_n \) is the natural frequency of the machining system; \( K_c \) is the cutting force coefficient; \( K_p \) is the penetration coefficient; \( \tau \) is the time delay which is inversely proportional to the cutting speed. If we select \( \zeta_c \) such that \( K_p = 2 \zeta_c \omega_n \), then Eq. (1) can be represented as:

\[ \ddot{x}(t) + 2(\zeta_c + \zeta) \omega_n \dot{x}(t) + \omega_n^2 x(t) = K_c [x(t-\tau) - x(0)]. \]  

(2)

where \( \zeta_c \) is defined as the cutting process damping ratio.

The effect that the cutting process damping has on the stability of the machining system can be easily seen from Eq. (2). If the structural damping ratio is much larger than the cutting process damping ratio, the effect of the cutting process damping may be insignificant. However, if the structural damping ratio is relatively small, the cutting process damping will dominate the damping effects of the machining system. Further, it is well known that the cutting process damping dominates the damping effect at low cutting speed, since the cutting process damping ratio is approximately inversely proportional to the cutting speed [2,5,6].
ESTABLISHED METHODS FOR THE ESTIMATION OF CUTTING PROCESS DAMPING.

Thusty's Approach. Thusty [7] developed a novel experimental approach to the estimation of the cutting process damping ratio for a turning process. The special set-up shown in Figure 1 was used to remove the regenerative effect.

![Figure 1. Thusty's Set-up to Eliminate Regeneration [7]](image)

The dynamic response for the machining system above is:

\[ \ddot{x}(t) + 2(\zeta + \zeta_C)k_0 \dot{x}(t) + \omega_0^2 x(t) = -k_c x(t). \]  

For a unit impulse disturbance, the solution of Eq. (3) is given by:

\[ x(t) = e^{-\left(\zeta + \zeta_C\right)k_0 t} \sin \omega_d t, \]  

(4)

where \( \omega_d = \sqrt{\left(1 - (\zeta + \zeta_C)^2\right)\omega_0^2 + k_c}. \)

An estimate for the cutting process damping ratio, \( \zeta_p \), can then be determined by fitting Eq. (4) to the measured vibration/displacement data obtained from the set-up depicted in Figure 1. It should be noted that under the conditions stated above, i.e., no regenerative effect, that the damping in the vibration signal (the system damping) is equal to \( 2(\zeta + \zeta_C)k_0 k_0 \).

**Dynamic Data System (DDS) Approach.** It is possible to characterize the data obtained from a stochastic process with the following differential equation:

\[
(D^m + \alpha_1 D^{m-1} + \ldots + \alpha_{m-1} D + \alpha_m) x(t) = (\beta_0 D^m + \beta_1 D^{m-1} + \ldots + \beta_1 D + \beta_0) z(t),
\]

where it is assumed that:

\[
E[z(t)] = 0, \quad \text{and} \quad E[z(t)z(t-u)] = \delta(u)\sigma^2.
\]

Note that in Eq. (5), \( x(t) \) is the system response, \( z(t) \) is white noise, \( \delta(u) \) is the Dirac Delta Function, \( D \) is the differential operator, \( E \) is the expectation operator, \( \alpha_0, \ldots, \alpha_{m-1} \) and \( \alpha_m \) are the autoregressive parameters, and \( \beta_0, \beta_1, \ldots, \beta_1 \) and \( \beta_0 \) are the moving average parameters. The data obtained by sampling the continuous system at a uniform interval, \( \Delta \), may be modeled using the discrete representation of Eq. (5), viz.,

\[
(1 - \phi_1 B^1 - \phi_2 B^2 - \ldots - \phi_m B^m) x_t = (1 - \theta_1 B^1 - \theta_2 B^2 - \ldots - \theta_m B^m) a_t,
\]

where it is assumed that:

\[
E[a_t] = 0, \quad \text{and} \quad E[a_t a_{t+k}] = \delta_k \sigma^2.
\]

In Eq. (6), \( a_t \) is the discrete white noise disturbance, \( \delta_k \) is the Kronecker Delta Function, \( B \) is the difference operator, \( \phi_1, \phi_2, \ldots, \phi_m \) and \( \phi_0 \) are the autoregressive parameters, and \( \theta_1, \theta_2, \ldots, \theta_m \) and \( \theta_0 \) are the moving average parameters.

The procedure of modeling under the above formulation is essentially a decomposition of the energy of the stochastic system over the smallest number of dynamic modes. The various dynamic modes of the system are given by the eigenvalues of the characteristic equation of Eq. (6),

\[
\lambda^n - \sum_{j=1}^{n} \phi_j \lambda^{n-j} = 0.
\]

(7)

A dynamic mode is normally associated with a single degree of freedom and is given by one of the pairs of complex conjugate roots of Eq. (7). If such roots are denoted by \( \lambda \) and \( \lambda^* \), the natural frequency and the damping ratio of the mode are given by

\[
\omega_m = \frac{1}{2\Delta} \sqrt{\frac{|\ln(\lambda \lambda^*)|^2}{4} + \frac{1}{2\sqrt{\lambda \lambda^*}}}
\]

(8)

\[
\zeta_m = \frac{1}{2\Delta} \sqrt{\frac{|\ln(\lambda \lambda^*)|^2}{4} + \frac{4}{2\sqrt{\lambda \lambda^*}}}
\]

(9)

The DDS approach has been used to measure cutting process damping in the turning operation [8] as well as in milling operations [9,10]. Balakrishnan, et al. [8] used the DDS approach to analyze the signal measured during the cutting process and used a random forcing function to remove the regenerative effect. Therefore, the damping ratio obtained from the DDS model for vibrational data thusly generated is the sum of the cutting process damping ratio and the structural damping ratio. The cutting process damping obtained by this approach for various cutting speeds shows good agreement with those obtained by Thusty's approach [8]. However, it should be pointed out that if displacement data obtained under working conditions (which includes the effect of regeneration) is modeled via the DDS approach, and values for the system frequency (\( \omega_m \)) and the system damping ratio (\( \zeta_m \)) are obtained, the system damping ratio will not necessarily be equal to the sum of the cutting process and structural damping ratios. This has been previously established [8] and will again be confirmed in this paper.

**A Modified DDS Approach for Cutting Process Damping Estimation.**

**Model Development.** The approach used by Thusty and the DDS modeling approach to the estimation of the cutting process damping use either a special workpiece design or a random forcing function to eliminate the regenerative effect. However, since \( \zeta_m \omega_m \pm i \zeta_m \) are a pair of roots of the characteristic equation of Eq. (5) which represents the n-degree-of-freedom extension of the system described by Eq. (2), they are also roots of the characteristic equation of Eq. (5).

Taking the Laplace Transform on both sides of Eq. (2), the following equation can be derived:

\[
s^2 X(s) + 2(\zeta + \zeta_C)k_0 X(s) + \omega_0^2 X(s) = K_c e^{-\alpha s} X(s) - X(s).
\]

Therefore, the characteristic equation for the system represented by Eq. (2) is:

\[
s^2 + 2(\zeta + \zeta_C)k_0 s + \omega_0^2 - K_c e^{-\alpha s} + K_c = 0.
\]
Since \(-\zeta_m\omega_m \pm j\sqrt{1-\zeta_m^2} \omega_m\) are a pair of roots of Eq. (11), the following equation can be obtained by replacing 's' with \(-\zeta_m\omega_m \pm j\sqrt{1-\zeta_m^2} \omega_m\).

\[
(-\zeta_m\omega_m \pm j\sqrt{1-\zeta_m^2} \omega_m)^2 + 2(\zeta_m \omega_m(-\zeta_m\omega_m \pm j\sqrt{1-\zeta_m^2} \omega_m)) + \omega_m^2 \nonumber
\]

\[= K_c(\zeta_m\omega_m \pm j\sqrt{1-\zeta_m^2} \omega_m)^2 - 1].
\]

(12)

Equation (12) is satisfied by equality of its real and imaginary parts which can be written as:

\[\text{RE}: \sigma^2 - \omega^2 - 2(\zeta_m + \zeta_m)\omega_0 \sin(\omega t) + \omega_m^2 = K_c(e^{-\sigma t} \cos(\omega t) - 1) \]  

(13a)

\[\text{IM}: 2\sigma\omega + 2(\zeta_m - \zeta_m)\omega_0 \sin(\omega t) = K_c e^{-\sigma t} \sin(\omega t),
\]

(13b)

where \(\sigma = \zeta_m\omega_m\) and \(\omega = \sqrt{1-\zeta_m^2} \omega_m\).

If values are known for all the system parameters then we can, of course, solve the above for \(\sigma\) and \(\omega\), and thus obtain values for \(\zeta_m\) and \(\omega_m\). If, however, the cutting process damping is not known but displacement data is available, and modeled via DDS to obtain estimates of \(\zeta_m\) and \(\omega_m\), then an estimate of the cutting process damping ratio can be obtained from Eq. (13), viz.,

\[(\zeta_m + \zeta_m) = \frac{(\sigma^2 - \omega^2 + \omega_m^2) e^{-\sigma t} \sin(\omega t) + 2\sigma \omega_0 e^{-\sigma t} \cos(\omega t) - 1}{2\omega_0 e^{-\sigma t} \sin(\omega t) + 2\omega_0 e^{-\sigma t} \cos(\omega t) - 2\omega_0}.
\]

(14)

Therefore, the cutting process damping ratio can be estimated by using the roots obtained from a DDS model for vibration data, based on the characteristic equation of Eq. (2), without the need to physically remove the effect of regeneration. Hereafter, this approach will be known as the Modified DDS Approach.

The key to the Modified DDS Approach to the estimation of the cutting process damping is not so much the use of DDS to obtain estimates of the system parameters but rather the explicit recognition of the regeneration mechanism in the basic equation of motion, viz., Eq. (2).

In principle, any reasonable estimates of the system parameters \(\omega_0\), \(\zeta_m\), \(\omega_m\), and \(\zeta_m\) could be used in Eq. (14) to obtain an estimate of the cutting process damping ratio \(\zeta_m\). However, under actual working/production conditions the DDS modeling methodology provides a reasonable means to determine the system parameters from measured displacement data.

Verification of the Proposed Approach. In order to examine the Modified DDS approach to the estimation of the cutting process damping ratio, a computer program was developed to generate vibration data for the system described by Eq. (2). Initially, a special case, namely, the system described by Eq. (3) was considered. The purpose of this was to demonstrate that the DDS modeling approach can, in general, produce accurate estimates of the system parameters.

Two values of the system damping were obtained in the following ways. First, the system damping and vibration frequency can be observed directly in Eq. (4), which is the solution for the system described by Eq. (3). Second, the data output by the computer program for system simulation (via Eq. (2)) can be analyzed by the DDS modeling method. The results for determining the system damping and vibration frequency are summarized in Table 1. As shown in the table, when compared to the exact solution, the DDS approach appears to very accurately estimate the system damping.

In a second set of experiments, simulation was used to generate the vibration data using the same values of \((\zeta_m + \zeta_m), \omega_0, \zeta_m, \text{and } K_c\) as given in Table 1 but with the addition of various time delays. The purpose of these simulation experiments was to compare estimates for the system damping and frequency obtained from the DDS model with the calculated values from Eq. (13), and in particular, to investigate whether the roots of the characteristic equation for a system described by Eq. (2) could be obtained by using the DDS approach.

DDS was used to estimate the system damping and vibration frequency from the simulated data. The system damping and vibration frequency were also calculated directly from Eq. (13). Finally, using the Modified DDS Approach the parameter estimates from the DDS models were used in Eq. (14) to calculate the sum of the structural and the cutting process damping. The results are summarized in Table 2.

Examining Table 2, the following observations can be made:

1. Except for one case, there is good agreement between the roots of the characteristic equation of the machining system obtained from the DDS approach and those obtained from Eq. (13) directly. For this one case the system damping ratio is negative. Although the system damping ratio is negative, damping obtained from the DDS approach is still positive. It should be noted that, for a real system, when system damping is negative the amplitude of tool oscillation becomes sufficiently large and the tool will leave the workpiece for part of the oscillatory cycle. Under such a condition, the nonlinearity of the system may increase the chance for error in utilizing the method for estimating system parameters.

2. The values for the system damping ratio given in the table change as the time delay changes. The system damping is smaller than the sum of structural damping and cutting process damping \((\zeta_m + \zeta_m) = 0.025\) in Table 1) in all cases examined. It is seen that the regenerative effect generally decreases the stability of the machining system. It is also observed that the vibration frequency changes as the time delay changes.

3. The results clearly show that the damping ratio obtained from the DDS approach for a system without special provision to remove the regenerative effect is the system damping ratio instead of the sum of cutting process and the structural damping ratios.

4. When the phase angle is close to 0° (e.g. time delays equal to 0.010 or 0.012 second), system damping ratio is relatively high. When the phase angle is close to 90° (e.g. time delays equal to 0.011 second), system damping ratio is relatively low. This result is completely consistent with results previously obtained in [11].

5. It is seen that the estimate of the damping ratio \((\zeta_m + \zeta_m)\) obtained from the modified DDS approach accurately estimates the true damping ratio \((\zeta_m + \zeta_m) = 0.025\) in both cases where the process is stable. When the system is unstable the estimate of the system damping obtained from the DDS approach is in error, and thus the estimate of the damping ratio \((\zeta_m + \zeta_m)\) is also in error. If another method was available for accurately estimating the system parameters from the data when the system is unstable, the system parameters could be used in the proposed approach to accurately produce an estimate of the damping ratio.

APPLICATION OF THE MODIFIED DDS APPROACH UNDER WORKING CONDITIONS

Experimental Set-Up and Test Conditions. The experimental set-up used to conduct machining experiments and measure the displacement data is shown in Figure 2 [13]. The workpiece used in the experiments is very flexible, with one principal direction and very rigid in the other two principal directions and can therefore be approximated by a one degree of freedom system. This workpiece is shown in Figure 3. The stiffness of the workpiece is very low so that the dynamics of the much more rigid components of the cutting system, such as the spindle, machine tool and fixture, can be neglected.

Finite element analysis was used to study the dynamic properties of the workpiece including the natural frequencies, corresponding stiffnesses and mode shapes [13]. Table 3 shows the first three natural frequencies, corresponding stiffnesses and principal directions of vibration. Upon examining the stiffnesses in Table 3, it can be seen that the workpiece is much more flexible for the first mode shape than it is for the other two mode shapes. It is therefore to be assumed that motion only occurs in the x direction as defined by mode shape 1.
Table 1. Simulated Conditions and Results (without regenerative effects)

<table>
<thead>
<tr>
<th>Simulated Conditions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency $\omega_n$</td>
</tr>
<tr>
<td>Damping Ratio $(\zeta + \zeta_c)$</td>
</tr>
<tr>
<td>$\Delta$</td>
</tr>
<tr>
<td>$K_c$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Approach</th>
<th>Damping Coefficient</th>
<th>Vibration Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Exact Solution</td>
<td>$2(\zeta + \zeta_c)\omega_n$</td>
<td>$\sqrt{(1 - (\zeta + \zeta_c)^2)\omega_n^2 + K_c}/2\pi$</td>
</tr>
<tr>
<td></td>
<td>157.07</td>
<td>514.97</td>
</tr>
<tr>
<td>2. DDS Model</td>
<td>$2\zeta_m\omega_m$</td>
<td>$\sqrt{1 - \zeta_m^2}\omega_m/2\pi$</td>
</tr>
<tr>
<td></td>
<td>156.60</td>
<td>514.56</td>
</tr>
</tbody>
</table>

Table 2. Simulated Conditions and Results (with regenerative effects)

<table>
<thead>
<tr>
<th>Time</th>
<th>System Damping Ratio</th>
<th>Vibration Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay (sec)</td>
<td>$\zeta_m$</td>
<td>$\sqrt{1 - \zeta_m^2}\omega_m/2\pi$</td>
</tr>
<tr>
<td>Eq. (13)</td>
<td>DDS</td>
<td>(Eq. (13))</td>
</tr>
<tr>
<td>0.010</td>
<td>0.0110</td>
<td>0.0109</td>
</tr>
<tr>
<td>0.011</td>
<td>-0.0015</td>
<td>0.0018</td>
</tr>
<tr>
<td>0.012</td>
<td>0.0093</td>
<td>0.0105</td>
</tr>
</tbody>
</table>

*Obtained via the Modified DDS Approach using Eq. (14)

Figure 2. Setup for Machining Experiments

Figure 3. Experimental Workpiece and Proximity Sensor
Table 3. Dynamic Characteristics of the Workpiece

<table>
<thead>
<tr>
<th>Natural Frequency</th>
<th>Stiffness</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>463 Hz</td>
<td>10,607 lbf/in</td>
<td>x</td>
</tr>
<tr>
<td>1,263 Hz</td>
<td>90,782 lbf/in</td>
<td>y</td>
</tr>
<tr>
<td>1,597 Hz</td>
<td>214,866 lbf/in</td>
<td>xy</td>
</tr>
</tbody>
</table>

An impulse response test was conducted on the workpiece shown in Figure 3 and the x-direction vibration signal/displacement was measured by the proximity probe, also shown in the figure. Figure 4 shows the graph of the x-direction vibration/displacement signal versus time for this test. DDS modeling was used to estimate the structural damping ratio and the natural frequency of the workpiece from the data. The data was sampled at 1,250 Hz. The complex conjugate roots of the characteristic equation were $\lambda_1 = 0.6650 \pm 0.7189 j$, thus producing (via Equations (8,9)) estimates of the natural frequency, $\omega_n$, and the structural damping, $\zeta$. The natural frequency was found to be 461 Hz. This frequency is nearly identical to the frequency for the first mode shown in Table 3, determined from the finite element analysis. The damping ratio found was 0.090. The stiffness of the workpiece and the cutting force coefficient were determined experimentally [13].

Table 4 contains a list of cutting conditions used for the machining tests. Because the workpiece stiffness is quite low, the depth of cut may not have to be very large to produce unstable cutting conditions. Therefore, a relatively small depth of cut was used for the tests. The standard deviation of the vibration/displacement signal for each machining condition is also summarized in Table 4.

Data Analysis and Discussion: The vibration data obtained from the tests were analyzed by the Modified DDS Approach to estimate the cutting process damping ratio. Figure 5 shows some of the sampled vibration data.

For the various test conditions, the autoregressive roots of the DDS models were used via Eqs. 8 and 9 to determine the system damping ratio, $\zeta_m$, and the vibration frequency, $\omega_n$. For the Modified DDS Approach, the estimates of $\zeta_n$ and $\omega_n$ obtained from DDS were used in Eq. (14) to determine the cutting process damping ratio, $\zeta_c$. The results are summarized in Table 5.

Table 4. Cutting Conditions and Results for Physical Experiments

<table>
<thead>
<tr>
<th>Cutter Geometry</th>
<th>Constant Cutting Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Teeth</td>
<td>8</td>
</tr>
<tr>
<td>Feedrate</td>
<td>10 ipm</td>
</tr>
<tr>
<td>Cutter Radius</td>
<td>2 inch</td>
</tr>
<tr>
<td>Width of Cut</td>
<td>2 inch</td>
</tr>
<tr>
<td>Axial Rake Angle</td>
<td>$7^\circ$</td>
</tr>
<tr>
<td>Depth of Cut</td>
<td>0.01 inch</td>
</tr>
<tr>
<td>Radial Rake Angle</td>
<td>$7^\circ$</td>
</tr>
<tr>
<td>Lead Angle</td>
<td>$15^\circ$</td>
</tr>
<tr>
<td>Nose Radius</td>
<td>3/64 inch</td>
</tr>
<tr>
<td>Spindle Speed (rpm):</td>
<td>690 / 720 / 750 / 780 / 810</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1100 4180 2200 880 2420</td>
</tr>
<tr>
<td>of Vibration (µin)</td>
<td></td>
</tr>
</tbody>
</table>

In Table 5 it is seen that the larger the system damping ratio, $\zeta_m$, the smaller the vibration magnitude (as previously seen in Table 4) and that the frequency measured at most conditions differs from the natural frequency of 461 Hz. This is because the frequency measured through the DDS approach by analyzing the data obtained during the cut is the vibration frequency instead of natural frequency of the machining system. Table 5 also shows that the cutting process damping ratio at the speed of 720 rpm is much different from others.

Examining Table 4 and Figure 5b, it is seen that the vibration magnitude at this speed is much larger than the others. Based on observations during the test and these results it appears that this condition produces an unstable process. As expected, the DDS modeling approach fails to properly estimate the system damping, and thus, the Modified DDS approach does not obtain the right value for the cutting process damping under this unstable situation.

The dynamic face milling force model [12] was also used in an iterative fashion to estimate the cutting process damping ratio. This was done by adjusting the cutting process damping ratio used in the model so that the standard deviation of the vibration magnitude obtained from the model predictions was equal to that obtained from the experiments. Table 6 shows the estimated cutting process damping ratio and the associated standard deviation of vibration for the model predictions for each of the cutting conditions, based on the estimated value of the cutting process damping ratio.

Table 5. Cutting Conditions, Autoregressive Roots, and Estimates of the System Frequency and Damping and the Cutting Process Damping

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>DOC (inch)</th>
<th>ARMA Model</th>
<th>Roots $\omega_n/2\pi$ (Hz)</th>
<th>$\zeta_m$</th>
<th>$\zeta_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>690</td>
<td>.01</td>
<td>(10,9)</td>
<td>.63674j, 67373</td>
<td>463.1</td>
<td>0.0326</td>
</tr>
<tr>
<td>720</td>
<td>.01</td>
<td>(14,13)</td>
<td>.71642j, 67828</td>
<td>474.2</td>
<td>0.0057</td>
</tr>
<tr>
<td>750</td>
<td>.01</td>
<td>(18,17)</td>
<td>.71932j, 66134</td>
<td>477.1</td>
<td>0.0097</td>
</tr>
<tr>
<td>780</td>
<td>.01</td>
<td>(12,11)</td>
<td>.64233j, 6091</td>
<td>474.0</td>
<td>0.0511</td>
</tr>
<tr>
<td>810</td>
<td>.01</td>
<td>(18,17)</td>
<td>.79232j, 56424</td>
<td>501.9</td>
<td>0.0110</td>
</tr>
</tbody>
</table>

* Estimates based on the DDS Modeling Approach
** Estimates based on the Modified DDS Approach

Figure 4. x-Direction Vibration vs. Time from an Impulse Response Test
Figure 5a. Typical Sampled x-direction Vibration Signal for 690 rpm

Figure 5b. Typical Sampled x-direction Vibration Signal for 720 rpm

Figure 5c. Typical Sampled x-direction Vibration Signal for 750 rpm

Table 6. Comparison of Estimates of the Cutting Process Damping Ratio Via the Modified DDS Approach and from the Dynamic Force Model Simulation

<table>
<thead>
<tr>
<th>Cutting Speed (rpm)</th>
<th>Cutting Process Damping Ratio</th>
<th>Standard Deviation of Vibration (.001 in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \zeta_0 ) **</td>
<td>( \zeta_+ )</td>
</tr>
<tr>
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</tr>
<tr>
<td>810</td>
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</table>

** From actual data using the Modified DDS Approach
+ From the Dynamic Force Model simulation data via vibration data "matching" with the actual data.

These results are very interesting and appear to demonstrate at least two things simultaneously: (i) that the Modified DDS Approach is accurately estimating the cutting process damping ratio, and, (ii) that the dynamic face milling force model does a good job of predicting the vibration magnitude for a given set of cutting conditions. This also again confirms that the estimated damping from the DDS modeling approach alone is not the sum of the cutting process and structural damping.

CONCLUSIONS.

1. The explicit recognition of the effect of regeneration in the basic equation of motion makes it possible to use data collected under working/production conditions to estimate the cutting process damping ratio without the need to take special provision to eliminate the regenerative effect. Such was accomplished by using estimates of the system parameters
obtained from a DDS model for the displacement data. This modeling strategy has been referred to here as the Modified DDS approach.

2. The results of this paper, both in terms of simulated data and data obtained from machining tests clearly demonstrate that if the regenerative effect is not eliminated, the damping ratio obtained through DDS modeling of the data is the system damping ratio rather than the sum of the cutting process damping ratio and the structural damping ratio. This result has, of course, been previously demonstrated.

3. The Modified DDS Approach can estimate the cutting process damping ratio under working conditions without any special provisions to remove the regenerative effect. However, when chatter occurs, the DDS modeling strategy poorly estimates the level of system damping thus leading to a poor estimate of the cutting process damping.

4. The excellent agreement between the value of the cutting process damping ratio obtained via the Modified DDS Approach and the value based on the dynamic face milling force model not only reinforces the credibility of the new method but also reinforces the predictive ability of the dynamic face milling model.

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REFERENCES.


