Quality Design Using a Computer-Based Dynamic Force Model for the Turning Process

W.J. Endres, M.S.M.E.,
Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign
J.W. Sutherland, Ph.D., P.D.C., Inc. and R.E. DeVoit, Ph.D.,
Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign

SYNOPSIS

A dynamic model of the cutting force system in the turning process has been developed. The model, as implemented as a computer simulation, can be used to evaluate process stability and predict surface error, surface texture, and peak cutting forces. The model incorporates the effects of cutting conditions, tool geometry, and process excitations such as workpiece material inhomogeneity and changes in the chip load due to intermittent cutting. A case study is presented that involves the design of the product and the process by which it is manufactured for a slender cast iron shaft with an axial slot. By using the model as an engineering tool in a Simultaneous Engineering environment, it is shown that a product/process design can be formulated such that product specifications such as surface texture characteristic constraints can be met. In addition, the model is used as an experimental tool to study the robustness of two final process designs to noise variables. The noise variables considered are the depth of cut and the slot width where the noise arises form the dimensional tolerances on the casting diameter and slot width, respectively. In addition to the completion of the design problem, some observations of the process response to variations in product and process design variables are discussed.

NOTATION

- \( A_c(t) \): Time varying chip load.
- \( d_n \): Nominal depth of cut.
- \( \delta \): Time varying depth of cut.
- \( \delta(t) \): Variation of the depth of cut about its nominal value.
- \( d_w \): Diameter of the workpiece.
- \( f_n \): Nominal feedrate.
- \( f(t) \): Time varying feedrate.
- \( \delta f(t) \): Variation of the feedrate about its nominal value.
- \( F_c(t) \): Longitudinal (axial) cutting force.
- \( F_r(t) \): Radial cutting force.
- \( F_t(t) \): Tangential cutting force.
- \( \delta L_c(t) \): Longitudinal tool displacement.
- \( l_w \): Length of the workpiece.
- \( N_s \): Nominal spindle speed.
- \( N_s_a \): Roughness average value for a linear axial trace.
- \( N_s_a \): Roughness average value for an area trace.
- \( r_n \): Tool nose radius.
- \( \delta R(t) \): Radial tool displacement.
- \( \delta R_a(t) \): Radial workpiece displacement.
- \( t \): Time.
- \( \delta t \): Average chip thickness.
- \( \delta T_c(t) \): Tangential tool displacement.
- \( \delta T_w(t) \): Tangential workpiece displacement.
- \( V_c \): Nominal cutting speed.
- \( V(t) \): Time varying cutting speed.
- \( \delta V(t) \): Variation of the cutting speed about its nominal value.
- \( \nu_c(t) \): A vector of the nominal cutting conditions and tool geometry where
  \[ \nu_c(t) = [\alpha_{b,n} \ \delta \gamma_a \ V_c \ f_n \ d_n] \]
- \( \nu(t) \): A vector of the time varying cutting conditions and tool geometry where
  \[ \nu(t) = [\alpha_{b,n} \ \delta \gamma_a \ V(t) \ f(t) \ \delta d(t)] \]
- \( \delta \nu(t) \): A vector of variations of the cutting conditions and tool geometry where
  \[ \delta \nu(t) = [\delta \alpha_{b,n} \ \delta \gamma_a \ \delta V(t) \ \delta f(t) \ \delta d(t)] \]
- \( W_s \): Actual slot width.
- \( W_{a,s} \): Nominal (design) slot width.
- \( \alpha_{b,n} \): Nominal tool back rake angle.
- \( \alpha_{b,n} \): Time varying tool back rake angle.
- \( \delta \alpha_{b,n} \): Variation of the tool back rake angle about its nominal value.
- \( \alpha_{b,n} \): Normal rake angle.
- \( \alpha_{b,n} \): Nominal tool side rake angle.
- \( \delta(t) \): A vector of the tool and workpiece displacements where
  \[ \delta(t) = [\delta R(t) \ \delta T_c(t) \ \delta T_w(t) \ \delta \gamma_a \ \delta \gamma_a(t)] \]
- \( \delta_n \): A normally distributed, zero mean variable.
- \( \gamma_a(t) \): Effective lead angle.
- \( \gamma_a \): Nominal tool lead angle.
- \( \gamma(t) \): Time varying tool lead angle.
- \( \delta \gamma_a \): Variation of the tool lead angle about its nominal value.
- \( \delta \gamma_a(t) \): Angular workpiece displacement.
- \( \tau \): Period of workpiece rotation.

1 INTRODUCTION

As with all conventional machining processes widely used in industry, it is important to understand how the process inputs, such as cutting conditions and tool geometry, affect such measures of process performance as process stability, tool wear, tool breakage, surface error, and surface texture. Knowledge of the peak cutting force, average cutting force, and the variations about this average force can aid an engineer in optimizing the performance of the process well ahead of the cut. It is this that motivates the development of models to describe the input-output relations of a process.

For the determination of the average cutting forces, "static" models that yield the instantaneous average process output can be employed. To determine peak forces and the force variations that strongly affect tool breakage, surface error, and surface texture, it is necessary to make use of a model that describes the dynamics of the process. Note that by "dynamic" it is not simply meant that the outputs, such as cutting forces, are time varying, but rather that the state of the process at some time \( t_0 \) is dependent on the evolution of the state of the process over the time interval \( [0,t_0] \).
Much of the early work done in dynamic modeling of the cutting process was concerned with analyzing the transfer function of the cutting process as a means for studying machine tool chatter. Tlusty and Polacek [1] studied a multi-degree of freedom system and the effects of the time derivatives of process displacements. Much work [2-6] was also done to obtain guidelines for determining an acceptable depth of cut for a given feedrate and spindle speed, often referred to as stability charts. This work yields many quantitative results that are useful in the process design stage, but requires that a large number of experiments be performed for each set of process conditions of interest.

More recent work has been done to develop mechanistic cutting process models that are implemented as computer simulations [7-15]. One difference between these models and the aforementioned transfer function models is that they account for the more complex geometry involved with oblique, single-point, and multi-point tool cutting. Another difference, and a major advantage of this approach, is that the experimental work required to calibrate the model over a wide range of process inputs is greatly reduced. Specifically, this approach attempts to model the process input/output relations mechanistically in order to minimize the variation of the model parameters over a range of cutting conditions, tool geometries, and workpiece materials.

Recent work has produced a mechanistic dynamic model of the turning process [16, 17]. Development of this model employed a similar approach as for previously developed mechanistic dynamic models [13, 15]. This model improves on previous work in that it involves a detailed analytical determination of the chip load under time-varying cutting conditions and tool geometry. In addition, a total of six displacements of the tool and workpiece are considered and the contribution of each is mapped into changes in each of the cutting condition and tool geometry variables. Finally, the structure of the model allows for calculation of intermittent cutting directly, without the need for a change in the model parameters upon entry into and exit from the cut.

With the availability of such a complex simulation model as developed in [16, 17], an engineer can be aided in both product and process design, ultimately producing a superior product at a lower cost. Because the model is capable of predicting the dynamic response of the cutting process, the user is able to evaluate the process performance based on such criteria as process stability, peak cutting forces, and machined surface error and texture in regions of low rigidity and/or intermittent cutting. In addition, because of the mechanistic nature of the model, process simulation over a wide range of cutting conditions and tool geometries is made available to the engineer.

By incorporating a process design tool such as this computer-based simulation into the process design task, the process design engineer can now have access to a complex design tool just as product design engineers have had for many years in the form of Finite Element Analysis software. In the past, the complexity of the cutting process only allowed process design engineers to base decisions primarily on experience and experimental data. This type of design approach can be quite costly in terms of time and money during the ‘on the floor’ experimentation with the machine tool and raw workpieces. With this process design tool, the time and cost involved in the experimental stage of process design can be reduced to a negligible level, as product designers have accomplished by employing Finite Element Analysis in the product design stage. At the same time, this tool can never replace the human factor involved in process design; but in the hands of an experienced process design engineer, it can be a powerful complement to the engineer’s experience and knowledge of the process.

While it was mentioned that the experienced process design engineer can take advantage of his design experience when using the model, the model also provides a tool for the product design engineer from which he can gain some basic knowledge of the process. Though the product design engineer may not have the expertise required to optimize the process, he now has a tool that can be used by both the product and process design engineers simultaneously. With the push for the simultaneous design of the product and process [18], it is very important to have design tools that are mutually useful for both aspects of the overall design process. In the long run, by simultaneously designing the product and process, using computational models such as mechanistic process models [19] and Finite Element models, should ultimately produce a superior product at Job One.

This paper demonstrates how the model can be used in a Simultaneous Engineering framework, allowing a product design engineer, with perhaps only limited knowledge of the cutting process, to improve the product design for manufacturability. It is also shown how the process parameters obtained during the product design stage can be easily varied experimentally, on the computer, in order to study the effects of noise in the actual process parameters. In doing so, the designer can be assured of a process design that is robust to noise variables such as raw material dimensional tolerances, tool wear, material hardness variations, and inaccurate machine settings. By studying the effects of these minor changes using a computer simulation rather than costly shop floor experimentation, the extra time and cost incurred at the start of production can be vastly reduced, and hopefully eliminated.

The paper discusses the method of mechanistic process modeling and its advantages over the traditional transfer function model approach. The development of the model as well as its capabilities [17], focused on a few tools, is also briefly discussed. Finally, the use of the model for quality design is demonstrated in a hypothetical case study where the model, as a computer-based simulation, is implemented in a Simultaneous Engineering environment. The case study focuses on both the product and process design tasks for the turning of a slender cast iron shaft with a slot. In this case, the primary measure of cutting process performance is the surface finish in the vicinity of the slot.

2 MECHANISTIC VERSUS TRANSFER FUNCTION MODELING

The idea of mechanistic modeling is to characterize the process by sets of analytical equations that hold over a wide range of process inputs. The transfer function approach models the dynamic structural response of the process using a transfer function of a given order for which parameters must be experimentally determined for each workpiece material and machine tool. The mechanistic modeling approach employs either discrete (Finite Element Method) or continuous (analytical beam vibration theory) modeling techniques to describe the dynamic structural response. In this case, the dynamic model of the structure yields displacements of the cutting process components based on mechanics of materials. Because the parameters that describe different materials, such as Modulus of Elasticity, Modulus of Rigidity, and density, are well known for all conventional materials, the mechanistic approach does not require experimentation to determine the dynamic structural response model parameters as is necessary for the transfer function approach.

Perhaps more limiting than the need for the determination of a few parameters of the dynamic structural response model is the method used to determine the input to this model, specifically the cutting forces. In other words, transfer function based models generally consider the simple tool geometry of orthogonal or oblique cutting that are related to five or six inputs containing combinations of tool geometry variables. For instance, orthogonal cutting has only one tool geometry variable, namely the rake angle. Even the complex geometry of oblique cutting can be characterized by two variables, namely the rake and inclination angles. With fewer geometry variables to consider, based on the cutting process inputs, it is relatively simple to determine the chip load and hence the cutting forces in each direction.

For the more complex geometries that exist in turning or end milling, the inputs to the dynamic structural response model vary extensively over the many combinations of tool
geometry variables as well as for workpiece materials. In turning, there are four tool geometry variables, namely the side rake angle, back rake angle, lead angle, and nose radius. Variation of different variables in this case strongly affects the input to the dynamic structural response model. The transfer function approach requires extensive experimentation at each combination of parameters that are of interest. The mechanistic approach allows modeling of the cutting forces based on the cutting conditions and tool geometries over a large range of these parameters based on a minimal set of experiments for a given material.

In addition to the accommodation of the tool variables for complex tool geometries, the mechanistic method allows variation of cutting conditions to any combination that lies within the range of model calibration. The transfer function approach requires experimental data for individual combinations of cutting conditions which is quite cumbersome in the process design stage when an engineer would like to have the freedom to vary conditions in a relatively unrestricted fashion. The mechanistic modeling approach provides this freedom at the expense of a minimal number of experiments over a large range of cutting conditions of parameters. Another disadvantage of the transfer function approach is that it requires similar, yet more abundant, tests to be performed on the workpiece in question. On the other hand, the mechanistic approach is not limited to a particular workpiece but rather a class of workpieces. Therefore, the mechanistic model has a major advantage over the transfer function model in that it can be implemented during product design. The transfer function model would possibly require additional experimental work for each change in the product design. This is perhaps the most limiting factor for the transfer function approach as applied in a Simultaneous Engineering framework.

3 MODEL DEVELOPMENT AND CAPABILITIES

The dynamic model used for the computer simulations is a dynamic mechanistic model of the turning process that is capable of predicting cutting forces and surface topography [17]. The model consists of the following five components which are shown in Fig 1 in block diagram form:

- A nonlinear Chip Geometry Model (CGM) which accommodates time varying inputs:
  - Model input (the vector of time varying cutting conditions and tool geometry).
  - Model outputs (the chip load, the average chip thickness, and the effective lead angle).
- An empirical Cutting Force Model (CFM):
  - Model inputs (the outputs of the CGM and the vector of time varying cutting conditions and tool geometry).
  - Model outputs (the radial, longitudinal, and tangential cutting forces).

- A multi-degree of freedom Machining System Structural Response Model (MSSRM):
  - Model inputs (the outputs of the CFM).
  - Model output (the vector of tool and workpiece displacements).

- A nonlinear Displacement Feedback Model (DFM):
  - Model input (the output of the MSSRM).
  - Model output (the vector of variations of the cutting conditions and tool geometry).

- A Surface Topography Model (STM):
  - Model input (the vector of time varying cutting conditions and tool geometry).
  - Model outputs (the surface error/texture data and characterization parameters).

3.1 The Chip Geometry Model (CGM)

The first component of the dynamic process model is the Chip Geometry Model, which must be able to accommodate the time varying cutting conditions and tool geometry. The model yields three time varying outputs, chip load, average chip thickness, and effective lead angle, which are essential in determining the cutting forces in the radial-tangential-longitudinal (R, T, L) coordinate frame. An important characteristic of the Chip Geometry Model is that it accounts for the nose radius rather than simply modeling the chip load as the product of the feed and depth of cut \((f(t) \cdot d(t))\). Accounting for the nose radius in this manner is particularly important at high feedrates, large nose radii, and small depths of cut, where the error in the \(f(t) \cdot d(t)\) method can be large. Even more important is when there is a change in the cutting conditions and tool geometry over a revolution, where the nose radius effect is exaggerated.

The time varying chip load is found by analytically integrating between the two cutting edge profiles. The analytical approach was chosen over numeric integration to minimize the number of computations performed at each time step. The analytical solution permits the determination of the chip load at each time step by simply evaluating a set of equations and limits of integration. The chip load integration is performed by dividing the chip into four elements as shown in Fig 2 for the non-displaced system. Division of the chip into these elements facilitates a piecewise integration for the chip load of the displaced system using either of the Cartesian or polar coordinate systems shown in Fig 2.

The other two outputs, the average chip thickness and the effective lead angle, are strongly affected by the nose radius and vary significantly for the dynamic case as compared to the static case. The average chip thickness is used to determine the coefficients which relate \(F_r(t)\) and \(F_t(t)\) to the chip load, while the effective lead angle is used to distribute \(F_t(t)\) into its components in the radial and longitudinal directions.

3.2 The Cutting Force Model (CFM)

The Cutting Force Model describes the material's resistance to cutting. The model is similar in form to the 'static' force model of [12] and relates the tangential and friction cutting forces to the chip load by

\[
F_T(t) = K_T(t) A_d(t) \tag{1}
\]

and

\[
F_F(t) = K_F(t) A_d(t), \tag{2}
\]

where the coefficients \(K_T(t)\) and \(K_F(t)\) are empirical in nature and are calculated using the following relations:

\[
b_a(K_T(t) - a \cdot p(t)) \tag{3}
\]

and

\[
b_b(K_F(t) - b \cdot p(t)), \tag{4}
\]

where \(a = [a_0 \ a_1 \ a_2 \ a_3]\), \(b = [b_0 \ b_1 \ b_2 \ b_3]\), and

\[
p(t) = \begin{bmatrix} 1 & b \cdot c(t) & ln(V(t)) & \alpha(t) \end{bmatrix}^T.
\]

The components of \(a\) and \(b\) are empirically determined constants and \(\alpha(t)\) is a function of \(a_i(t)\) and \(\alpha_i\).

The Cutting Force Model is calibrated for a given material by performing cutting tests on a 'rigid' workpiece and recording the average values of \(F_T(t)\) and \(F_F(t)\) over a range of cutting conditions and tool geometries.

Finally, to account for the inhomogeneous material hardness that introduces noise into the cutting forces described by \(K_T(t)\) and \(K_F(t)\), the values found using Eqs. (3) and (4) are perturbed by a normally distributed, zero mean, percent of \(K_T(t)\) and \(K_F(t)\), \(\delta_{\text{normal}}\), yielding

\[
K_T(t) = (1 + \delta_{\text{normal}}) K_T(t) \tag{5}
\]

and

\[
K_F(t) = (1 + \delta_{\text{normal}}) K_F(t). \tag{6}
\]

The tangential and friction cutting forces are now found by replacing \(K_T(t)\) and \(K_F(t)\) in Eqs. (1) and (2) with \(K_T(t)\) and...
The model developed for the cutting process dynamics includes the tool and workpiece dynamics and assumes that the remainder of the machining system is rigid. Figure 3 shows the distinction between the cutting process and the other dynamic elements of the machining system. For the machining of a workpiece with a large length-to-diameter ratio, the assumption of a rigid machine tool structure, as compared to the flexibility of the cutting process components, is reasonable.

In modeling the dynamic response of the tool holder and workpiece, distributed parameter techniques are used [20]. Modal analysis and beam vibration theory are used to obtain the vibration of the tool and workpiece beams as functions of time and position. In developing the dynamic models, it was necessary to determine the degree of freedom in which the displacements that significantly affect the chip geometry and cutting forces occur. The following list describes all possible displacements that could affect the response of the process through such variables as the cutting conditions and tool geometry.

(a) Radial compression of the tool due to $F_{r}(t)$.
(b) Bending of the tool due to $F_{b}(t)$.
(c) Bending of the tool due to $F_{t}(t)$.
(d) Radial compression of the workpiece due to $F_{w}(t)$.
(e) Longitudinal compression of the workpiece due to $F_{l}(t)$.
(f) Bending of the workpiece due to $F_{w}(t)$.
(g) Bending of the workpiece due to $F_{t}(t)$.
(h) Bending of the workpiece due to $F_{b}(t)$.
(i) Torsion of the workpiece due to $F_{t}(t)$.

Displacements (a) and (f) are considered to be significant since they directly affect the depth of cut. Displacement (c) affects the tool lead angle, feedrate, and perhaps the depth of cut, and is therefore considered significant. Displacements (b) and (g) are considered to be significant since they decrease the depth of cut and alter the tool back rake angle. In addition, their time derivatives affect the cutting speed. The time derivative of displacement (i) directly affects the cutting speed and is therefore considered significant. Displacement (h) is considered to be negligible since the moment arm is the workpiece radius, which in general, is small for workpieces with a large length-to-diameter ratio. Finally, displacements (d) and (e) result from elastic deformations along the cutting edge which are considered to be small since the tool is cutting through the material. Therefore, the six displacements considered are $\delta r(t)$, $\delta t(t)$, $\delta l(t)$, $\delta b(t)$, $\delta w(t)$, and $\delta u(t)$.

3.4 The Displacement Feedback Model (DFM)

The net effect of the six displacements on each of the cutting conditions and tool geometry variables can be obtained by superposition. The vector of cutting conditions and tool geometry is dependent on the vector of their nominal values and the cutting process displacements by the relation

$$v(t) = v_0 + \delta v(t),$$  
where $\delta v(t)$ is found using a nonlinear operator $F(t): \mathbb{R}^6 \rightarrow \mathbb{R}^3$, viz.,

$$\delta v(t) = F[v(t)].$$

The operator $F$ is found by computing each component of $\delta v(t)$ as a summation of the contributions of each component of $v(t)$. Each of these contributions is determined based on geometrical relations of the displaced system.

3.5 The Surface Topography Model (STM)

The Surface Topography Model simply determines the height of the generated surface based on tool geometry parameters such as the nose radius, and in cases of very large displacements over a revolution the end cutting edge angle or the lead angle. In addition, the model accounts for the process displacements as incorporated into the time varying cutting conditions and tool geometry via the Displacement Feedback Model. The outputs of the model are in the form of a three-dimensional surface plot as well as numerical surface characterization parameters such as the roughness average, peak-to-valley, and valley line values. The surface plot is constructed such that a zero profile height corresponds to the valley of a feed groove for the non-displaced system. By generating the surface prediction in this manner, it is possible to detect regions of undercutting and observe steady-state displacements.

The numerical surface characterization parameters, roughness average ($R_a$), peak-to-valley (PTV), and center line (CL) values, are determined for traces of linear distances equal to five cycles of the dominant frequency in both the axial and circumferential directions. The axial measurements ($R_a$, PTV, CL) are made for a constant value of the workpiece angle while the circumferential measurements ($R_a$, PTV, CL) are made along the valley of a feed groove to eliminate the effects of the feedrate. In addition to these parameters, another parameter is calculated over an area of the workpiece. The area for which it is calculated is that for which the three-dimensional surface plot is constructed. This parameter, $R_{area}$, attempts to characterize the overall surface texture in the sense of volume of material above the center line as compared to the volume of material below the center line. It is actually the equivalent of the traditional $R_a$ value that is calculated for an area trace rather than a linear trace.

3.6 Model verification

The three capabilities of the model that required verification were:

- the ability to predict the static cutting forces over the range of the Cutting Force Model using the Chip Geometry Model,
- the ability to accurately predict the steady-state cutting forces over the entire length of the workpiece, and
- the ability to predict the dynamic response of the cutting process, specifically the peak forces and oscillations during intermittent cutting.

The material used for all verification tests was 304 stainless steel. The Cutting Force Model was calibrated for KC950 grade (TiCAl2O14TIN coated carbide) inserts using a 3x4 factorial design scheme. Verification cutting tests were performed on a large (rigid) workpiece and compared to model predictions under static conditions. The model predicted $F_r$ and $F_p$ quite well with percent errors falling in the range of -3.87 to 1.09 percent and -5.25 to 5.57 percent, respectively.

For verification of the response during intermittent turning, it was necessary to use a lower grade insert (K68) due to breakage problems with the KC950 grade inserts. With the gain in tool toughness obtained through the use of the K68 instead of the KC950 grade insert came significantly higher tool wear. Taking this into account, the model again accurately predicted the steady-state cutting forces at varying axial positions along the workpiece.

Finally, the dynamic response resulting from intermittent cutting, as shown in Fig 4, reveals that the model predicted response is quite similar to that of the measured cutting force. The damping ratio (the sum of the structural and cutting process) was set at 0.40 to match the measured data as closely as possible.

4 CASE STUDY

This case study demonstrates how this model can be used for the simultaneous design of a product and the machining process used to manufacture it. The dynamic nature of the model
allows various characteristics of the process and the final product to be observed. Such characteristics include undercutting, steady state error (error of form), and surface texture as measured in both the axial and circumferential directions. In fact, it can be seen that simply considering the axial direction for surface texture can be quite misleading. Also, the regenerative effect can be observed and conditions can be adjusted to minimize its effect on the surface texture. Finally, the model is used as a computer-based experimental tool to study the effects of noise in input variables on the final process design. By using the model to run a set of experiments, the robustness of the final process design with respect to variations in the noisy variables can be studied in a time and cost efficient manner.

4.1 Product specifications and approach to the design problem

The final product, upon which this case study is based, is a cast iron slotted shaft of length 508.00±0.010 mm and diameter 50.80±0.05 mm. The shaft has a slot of length 150.00 mm as shown in Fig 5 where W s is the nominal (design) width of the slot. The width of the slot is a design variable and must satisfy the relation 17.005W s ≤19.00 mm. The surface texture, as characterized by the roughness average values for both the linear and area samples, must satisfy the following constraints:

- Maximum \( R a ≤16.0 \) μm,
- Mean \( R s ≤8.0 \) μm,
- Maximum \( R a ≤16.0 \) μm,
- Mean \( R s ≤8.0 \) μm, and
- \( R s, a r e a ≤12.0 \) μm.

The simulation traces made to obtain \( R a \) values are made at four axial intervals and five workpiece angles, evenly distributed between the end and start of the interruption yielding a total of twenty values. The simulation traces made to obtain \( R s \) values are made at four axial positions and five workpiece angles, evenly distributed between the end and start of the interruption, again yielding twenty values. Figure 6 shows the locations at which the traces are taken for the twenty revolutions of the simulations.

Due to production requirements, each workpiece should be machined in no more than 3.25 min. For good surface finish, a negative geometry tool (\( r = 1.191 \) mm, \( \gamma = 15 \) deg, and \( \alpha_s = \alpha_r = -5 \) deg) is to be used at a nominal cutting speed of at least 110 m/min. At this cutting speed and the final workpiece diameter, the nominal spindle speed must be at least 690 r/min. Therefore, to meet the production time requirement, one machining pass must be used at a nominal feedrate of at least 0.03 mm/rev for the minimum nominal spindle speed of 690 r/min.

To ensure removal of material over the entire surface of the workpiece under varying raw casting diameters and off-axis chucking, the nominal diameter of the casting is specified to be 53.00 mm. A tolerance on the casting diameter is specified to be ±0.40 mm. For this nominal diameter and tolerance, the depth of cut required to remove all material in one machining pass is \( d = 1.100\pm0.200 \) mm. In addition, a tolerance of ±0.20 mm is specified for the slot width.

4.2 The product/process design stage

The product/process design problem. The only product design variable is the slot width while there are two process design variables, specifically spindle speed and feedrate. Because of the large length-to-diameter ratio, minimization of workpiece dynamics in the area of the interruption are immediately suspect as the primary design challenge. Based on knowledge of the first modal eigenfunction of a fixed-simply supported beam, which has its maximum value at approximately \( x/L = 0.6 \), the maximum workpiece flexibility would be expected to occur 304.80 mm from the chuck. Unfortunately, the slot is present at this position no matter which end of the workpiece is held by the chuck. Therefore, the starting positions for all simulations were 304.80 mm from the chuck, as this was considered the worst case scenario.

Due to the high flexibility, a large steady-state workpiece displacement is expected. In the actual machining situation, control of the tool position to maintain a constant depth of cut based on measurements of the machined workpiece diameter could be used to compensate for this steady state error. Therefore, a simulation was performed at 740 r/min and the nominal depth of cut. The steady state depth of cut for this case was about 0.150 mm less than the nominal. The remainder of the simulations were run for a nominal depth of cut equal to the sum of the actual nominal value (1.100 mm) and 0.150 mm (i.e. \( d = 1.250 \) mm). This was done to ensure that the steady-state depth of cut in the simulations would be large enough to result in a final diameter within the tolerance of ±0.05 mm.

Because the cutting process is very complex, it is difficult to scientifically determine what a good process design should try to accomplish. For instance, to minimize the regenerative effect, a process designer may consider choosing a spindle speed that results in an integer number of cycles of the dominant vibration over one revolution. But, due to the complexity of the process, the frequency while cutting deviates from the natural frequency in practice. First, the vibration frequency is lowered from the natural frequency due to process damping. Second, the vibration frequency while cutting is increased from the natural frequency due to the forcing function's dependence on the vibration through \( K_s(t) \) and \( K_p(t) \). In addition, because \( K_s(t) \) and \( K_p(t) \) vary with changes in the cutting speed, this effect is made even more complex.

Initial design support simulations. To gain an initial understanding of the problem at hand, ten simulations were performed for the average slot width (18.00 mm), nominal depth of cut (1.250 mm), and ten spindle speeds, incremented by 10 r/min and starting from the minimum value of 690 r/min. The simulation results show varying response to the intermittent cutting excitation. Figure 7 shows four plots of the predicted machined surface. The out of phase nature as the axial dimension is traversed is slightly different for each causing the surface undulations to die out more quickly for \( N_s = 730 \) r/min and \( N_s = 780 \) r/min. Figure 8 shows the \( R_a \) and \( R_s \) values for the twenty traces obtained at each spindle speed. In addition, each plot includes the \( R_s, a r e a \) value and the mean of the \( R_a \) and \( R_s \) values for each spindle speed. As can be seen, there is a significant amount of scatter for the twenty points at each spindle speed, with the maximum for each speed being exceptionally high. This extraordinary high maximum value results from the large surface texture changes as the tool begins to feed itself into the slot in the initial cutting. This area is one for which it is particularly difficult to meet surface characterization parameters. Often times, a process design can only hope to minimize the effects of this region on the remainder of the surface through minimizing the regenerative effect, perhaps with the spindle speed as mentioned previously.

Three other valuable observations can be made from Fig 8. First, the maximum \( R_s \) values are significantly lower than the maximum \( R_a \) values. This would possibly be expected due to the high flexibility of the system as this causes significant dynamic response as a function of workpiece angle. The second observation is that, while the maximum \( R_a \) values seem to vary slowly (smoothly) as a function of spindle speed, the maximum \( R_s \) values vary sharply suggesting that the \( R_a \) parameter is relatively sensitive to changes in spindle speed. In addition, the \( R_s \) values tend to have multiple occurrences near the maximum for the higher spindle speeds while the maximum values at the low spindle speeds are far separated from the rest of the points. The third observation is simply that the surface characterization parameters meet all five of the specifications for only three of the ten spindle speeds (730, 740, and 750 r/min).

Returning to the three-dimensional surface plots of Fig 7, it can be seen that the surface height at the point of re-entry to the cut (the edge of the interruption) varies from one spindle speed
to the next. This is due to the variation in the number of cycles of free vibration that have occurred while in the interruption. As can be seen in Fig 9 where \( W_{0,n} \) is equivalent to 39 deg of workpiece rotation, the depth of cut response varies significantly depending on its magnitude upon re-entry to the cut. To minimize the step-up response, and to obtain a relatively smooth transition from non-cutting to cutting, it seems as though it would be advantageous to adjust the width of the slot such that there is an integer number of cycles of free vibration during the interruption.

The damping ratio for free vibration of metals is very small (approximately 0.005) causing the damped free vibration frequency to be approximately the natural frequency. Because the natural frequency is well known, the slot width corresponding to n cycles of free vibration is easily determined, viz.

\[
W_s (n) = 2d_w \sin \left( \frac{0.36 \pi N_f}{60 \omega_n} \right) 
\]

where \( d_w \) is the uncut workpiece diameter, \( N_f \) is the spindle speed measured in r/min, and \( \omega_n \) is the natural frequency of the dominant mode measured in Hz.

Second iteration of design support simulations. For the nominal raw casting, the natural frequency of the dominant vibration (38 kHz) is 106.39 Hz. Therefore, for the minimum spindle speed of 690 r/min, the slot width must be 17.77 mm for one cycle of free vibration to occur during the interruption. The maximum spindle speed that satisfies both Eq. (9) and the maximum allowable slot width is 740 r/min. So, as a second iteration in the product/process design stage, simulations were conducted for the same spindle speeds as before only now with slot widths that result in one cycle of free vibration (\( W_{0,n} = W_s (0) \)). For spindle speeds greater than 740 r/min, the maximum slot width of 19.00 mm is used which results in \( W_{0,n} > W_s (0) \). The resulting \( R_{sa} \) and \( R_{sa} \) values, along with their mean values and the \( R_{area} \) value, are shown in Fig 10. Figure 10 also includes the results of the first iteration for comparison purposes.

As can be seen from the data obtained from iteration Two, maintaining one cycle of free vibration during the interruption actually results in an inferior surface for most spindle speeds. At this point, the following observations can be made from carefully examining the data shown in Fig 10:

- The maximum \( R_{sa} \) values are decreased in Iteration Two for the spindle speeds of 690 r/min, 770 r/min, and 780 r/min. In addition, the maximum \( R_{sa} \) values show the same trend for these speeds where the changes are much smaller. Simulations were also performed at 790 r/min and 800 r/min to check for the presence of a trend starting at 780 r/min. As can be seen from Fig 10, the surface characterization parameters increase beyond 780 r/min.

- The maximum \( R_{sa} \) values are increased in Iteration Two for the spindle speeds of 710 to 760 r/min. Again, the maximum \( R_{sa} \) values show the same trend for these speeds where the changes are much smaller.

- Though the maximum \( R_{sa} \) values are significantly higher in Iteration Two at the spindle speeds of 710 to 760 r/min, the mean value of the \( R_{sa} \) value is lower. This could be an indication that the displacement oscillations die out more quickly for \( W_{0,n} = W_s (0) \) resulting in a greater percentage of small values on the opposite side of the workpiece that balance the exceptionally high maximum \( R_{sa} \) values.

The three observations above tend to imply that maintaining \( W_{0,n} = W_s (0) \) may not be the way to obtain the best surface for intermittent cutting as based on the criteria used here. On the other hand, some values of \( W_{0,n} > W_s (0) \) degrade the surface finish while others improve it. Table 1 shows the slot widths used in Iterations One and Two as a percentage of \( W_s (0) \).

<table>
<thead>
<tr>
<th>Spindle Speed (r/min)</th>
<th>( W_s ) for Iteration 1 (% of ( W_s (0) ))</th>
<th>( W_s ) for Iteration 2 (% of ( W_s (0) ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>690</td>
<td>101.8</td>
<td>100.0</td>
</tr>
<tr>
<td>700</td>
<td>100.3</td>
<td>100.0</td>
</tr>
<tr>
<td>710</td>
<td>99.0</td>
<td>100.0</td>
</tr>
<tr>
<td>720</td>
<td>97.5</td>
<td>100.0</td>
</tr>
<tr>
<td>730</td>
<td>96.2</td>
<td>100.0</td>
</tr>
<tr>
<td>740</td>
<td>94.9</td>
<td>100.0</td>
</tr>
<tr>
<td>750</td>
<td>93.6</td>
<td>98.7</td>
</tr>
<tr>
<td>760</td>
<td>92.4</td>
<td>97.4</td>
</tr>
<tr>
<td>770</td>
<td>91.2</td>
<td>96.1</td>
</tr>
<tr>
<td>780</td>
<td>90.0</td>
<td>94.9</td>
</tr>
</tbody>
</table>

Comparing the lower maximum \( R_{sa} \) values in Fig 10 to these percentages, it seems as though less than one cycle of free vibration is desirable, but not too much less as percentages below 93.6 percent cause higher maximum \( R_{sa} \) values. From the data obtained in the first two iterations, it becomes somewhat obvious that spindle speeds of 730, 740, 750, and 780 r/min with slot widths of \( W_s (0) \) where 0.936\%50.962 produce the lowest maximum \( R_{sa} \) and \( R_{sa} \) values.

Third iteration of design support simulations. As a third iteration in the product/process design stage, it was attempted to find the best combination of \( N_{sa} \) and \( W_{0,n} \) from the levels mentioned above. At this point, to meet the surface texture specifications it was necessary to lower the maximum \( R_{sa} \) and \( R_{sa} \) values by one to three microns from their levels in Iteration Two. The \( R_{area} \) parameter, on the other hand, need not be reduced. It should be noted though that the \( R_{area} \) value is much less sensitive to changes in the spindle speed and its reduction, if necessary, would be considered as much of a challenge as for the \( R_{sa} \) and \( R_{sa} \) values.

The simulations for the third iteration were performed for the four spindle speeds mentioned above and three values of \( n \) (0.936, 0.949, and 0.962). This amounts to twelve simulations, of which four had already been performed in Iterations One and Two (\( N_{sa} = 730 \) r/min, \( n = 0.962 \); \( N_{sa} = 740 \) r/min, \( n = 0.949 \); \( N_{sa} = 750 \) r/min, \( n = 0.936 \); \( N_{sa} = 780 \) r/min, \( n = 0.949 \)). Figure 11 shows the maximum and mean \( R_{sa} \) and \( R_{sa} \) values, as well as the \( R_{area} \) value. From the plot, it can be seen that all combinations of spindle speeds and slot widths satisfy all the constraints on \( R_{sa} \), \( R_{sa} \), and \( R_{area} \) as stated in the product specification.

It appears choosing the middle value of the range of the three spindle speeds of 730, 740, and 750 r/min would yield a process design that should be quite robust to variations in spindle speed. A slightly different approach is taken in that the assumption is made that the spindle speed (and feedrate as will be mentioned later) can be set accurately and are maintained while cutting is in progress. Under this assumption, it seems as though it would be desirable to satisfy the constraints by the largest margin possible. Therefore, \( N_{sa} = 780 \) r/min was chosen as a candidate for the final process design spindle speed setting. But, since the \( R_{area} \) values for \( N_{sa} = 780 \) r/min are close to the maximum allowable value, \( N_{sa} = 750 \) r/min was also chosen as a candidate for the final process design spindle speed setting. Because the other two spindle speeds (\( N_{sa} = 730 \) r/min and \( N_{sa} = 740 \) r/min) show no better response than \( N_{sa} = 750 \) r/min, they were no longer considered as candidates for the spindle speed of the final process design.

By having two candidates at this point, the robustness of the process design can be compared for two cases, both of which satisfy the product specification constraints for the nominal design parameters.

To provide for good robustness to noise in the slot width, the slot widths corresponding to \( n = 0.949 \) are chosen for the final product design. Therefore, the two spindle speed/nominal slot width combinations were chosen to be:

1. \( N_{sa} = 750 \) r/min with \( W_{0,n} = W_s (0.949) = 18.31 \) mm, and
2) \( N_p = 780 \) r/min with \( W_{p,n} = W_{p,0} = 19.00 \) mm.

4.3 Process robustness analysis stage

The final step in the product/process design stage is to determine the sensitivity of the process response to noise in the process inputs. There are four process inputs that could be noisy. The first two are the depth of cut and slot width. The noise in these variables results from the tolerances placed on the raw casting. For instance, the depth of cut, as mentioned previously, will lie in the range of 12.000 ± 0.200 mm where \( d_{cut} = 1.250 \) mm. The noise in the slot width also results from the casting tolerance as mentioned before yielding the actual slot width of \( W_{p,n} = W_{p,0} = 0.20 \) mm where \( W_{p,0} \) is the design value. The other two noise variables are the spindle speed and feedrate. For brevity in the amount of simulations required for the robustness evaluation, it is assumed that the machine settings (spindle speed and feedrate) can be set very accurately and do not drift from these settings. In other words, only two noise variables instead of four will be considered reducing the number of simulations from \( 2^{4-1} = 2^{4} = 16 \) for each of the two spindle speeds to \( 2^{2-1} = 2^{2} = 4 \) for each of the two nominal slot width combinations. A diagram of the experimental design for the robustness analysis is shown in Fig. 12.

For the two noise variables mentioned, a sensitivity analysis was performed for each of the two spindle speeds. Table 2 shows the resulting maximum and mean values for each parameter setting. In addition, the range and standard deviation of each of these parameters is given. It is obvious that the model response, as measured by the five parameters considered, is very sensitive to changes in depth of cut. On the other hand, both process designs are quite robust to variations in the slot width. This suggests that perhaps an attempt should be made to use two machining passes. On the other hand, it is likely that the cost incurred by doubling the number of machining passes could outweigh the added cost resulting from imposing a slightly tighter tolerance on the raw casting diameter. A final observation is that as the depth of cut decreases, the surface characteristics between the nominal and actual parameters drop. Therefore, if at all possible, the nominal depth of cut should be specified to be as low as possible for the given casting tolerance and expected off-center chucking.

<table>
<thead>
<tr>
<th>Slot Width Level</th>
<th>Depth of Cut Level</th>
<th>( R_{2,area} ) Max (( \mu )m)</th>
<th>( R_{2,area} ) Mean (( \mu )m)</th>
<th>( R_{2,area} ) mean (( \mu )m)</th>
<th>( R_{2,area} ) Max (( \mu )m)</th>
<th>( R_{2,area} ) Mean (( \mu )m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>12.009</td>
<td>12.432</td>
<td>4.155</td>
<td>14.931</td>
<td>5.915</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>9.188</td>
<td>8.873</td>
<td>3.007</td>
<td>11.265</td>
<td>4.043</td>
<td></td>
</tr>
<tr>
<td>+</td>
<td>11.227</td>
<td>12.333</td>
<td>4.066</td>
<td>15.154</td>
<td>5.607</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>2.903</td>
<td>4.060</td>
<td>1.169</td>
<td>5.640</td>
<td>2.081</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.629</td>
<td>2.218</td>
<td>0.634</td>
<td>2.781</td>
<td>1.063</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slot Width Level</th>
<th>Depth of Cut Level</th>
<th>( R_{2,area} ) Max (( \mu )m)</th>
<th>( R_{2,area} ) Mean (( \mu )m)</th>
<th>( R_{2,area} ) mean (( \mu )m)</th>
<th>( R_{2,area} ) Max (( \mu )m)</th>
<th>( R_{2,area} ) Mean (( \mu )m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>11.585</td>
<td>12.920</td>
<td>4.242</td>
<td>15.224</td>
<td>6.042</td>
<td></td>
</tr>
<tr>
<td>+</td>
<td>11.227</td>
<td>13.176</td>
<td>4.315</td>
<td>16.018</td>
<td>5.772</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>2.509</td>
<td>3.485</td>
<td>0.656</td>
<td>5.332</td>
<td>1.364</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.413</td>
<td>1.805</td>
<td>0.442</td>
<td>2.816</td>
<td>0.782</td>
<td></td>
</tr>
</tbody>
</table>

If strict attention is paid to the constraints on \( R_{2,area} \) and \( R_{2,area,n} \). \( N_p = 780 \) and \( N_p = 780 \) r/min don't satisfy these constraints, respectively. On the other hand, in the presence of such high variability, the minute overshoot of these two parameters (\( R_{2,area} = 12.000 \) \( \mu \)m and maximum \( R_{2,area,n} = 16.018 \) \( \mu \)m) could be safely ignored. Therefore, it was assumed that either case satisfies the specified constraints on the surface characterization parameters. Finally, based on the desire to minimize the variability over the range of noise variables based on the range and standard deviation of the surface characterization parameters of Table 2, \( N_p = 780 \) r/min was chosen as the spindle speed setting determined from conducting the product and process design tasks simultaneously.

5 SUMMARY

A dynamic model of the cutting force system in the turning process has been developed. Some of the features of the model include:

- A nonlinear Chip Geometry Model (CGM) that analytically calculates the exact chip load and is able to accommodate time varying inputs of cutting conditions and tool geometry. This model calculates the instantaneous chip load, average chip thickness, and effective lead angle.

- An empirical Cutting Force Model (CFM) that can also accommodate time varying inputs of average chip thickness, cutting conditions, and tool geometry. In addition, the Cutting Force Model incorporates the random excitation that is caused by the material inhomogeneity.

- A multi-degree of freedom System Structural Response Model (MSRSM) that includes the tool and workpiece dynamics in a total of six degrees of freedom.

- A nonlinear Displacement Feedback Model (DFM) that maps the six displacements produced by the Machining System Structural Response Model into deviations about their nominal values for five cutting condition and tool geometry variables.

- A Surface Topography Model (STM) that generates surface error/texture data in the form of three-dimensional surface plots and axial, circumferential, and area roughness average values based on the tool geometry and process displacements.

The models capabilities as a design tool were demonstrated by employing it as a design tool in a product/process design case study. The case study involved the machining of a slender cast iron shaft with a slot. The final product specifications included constraints on surface characterization parameters such as roughness average values as well as a dimensional tolerance on the finished diameter. The challenge of the product/process design problem was to meet a production time requirement in a single machining pass while satisfying the constraints on surface finish and finished diameter.

Use of the model allowed simulation of the machining process throughout the entire product design stage. The model was used in a Simultaneous Engineering environment as an analytical engineering tool providing the opportunity to obtain a quality design of both the product and the process used to manufacture it. The single product design variable, specifically the nominal (design) slot width, was varied along with one of the two cutting condition variables, specifically the spindle speed. In doing so, the three iterations of process simulations resulted in two candidate spindle speed/nominal slot width combinations that satisfied the final product specifications.

The final task involved a robustness analysis of the two cases. The two noise variables chosen were the depth of cut (cast diameter) and slot width since the workpiece raw casting dimensions are specified with tolerances on each of these dimensions. Again, using the model as a post-design experimental tool, much time and cost were saved over the traditional 'on the floor' experimental methods with the machine tool and a raw workpiece. The results of this analysis showed that either of the two cases produced acceptable results, but that one of the two showed less variability over the variation in noise vari-
ables.

Finally, some key characteristics of the process response to the variation in product and process design variables that were observed during the design and robustness analysis stages are:

- The circumferential roughness average values tend to be larger than their axial counterparts indicating the importance of multi-directional surface characterizations in the presence of dynamic cutting.

- The roughness average values, as measured in the axial direction, have a much slower (smoother) response to changes in the spindle speed as compared to those measured in the circumferential direction.

- Designing the slot width such that about 93.6 to 96.2 percent of a full cycle of free vibration occurs during the interruption yields the best process response as measured by roughness average values.

- A final product/process design tends to be rather robust to slight variations (noise) in the slot width while the opposite is true for slight variations in the depth of cut. In other words, the dimensional tolerance on the raw workpiece diameter should be made as small as possible to reduce the variability in the process response under variations in the depth of cut as caused by this tolerance.

- Generally, better surface characteristics are obtained for smaller depths of cut.

The model enables an engineer to observe phenomena such as those mentioned above. As complex as the actual process is, and as complex as this simulation model is, the general metal cutting process has many characteristics that are not fully understood. These areas are still open research issues and efforts made to better understand them would be beneficial enhancements to a model such as this. In particular, two specific phenomena, for which it would be beneficial to better understand, are (1) the origin and magnitude of cutting process damping, and (2) a more accurate mechanistic prediction of the effective lead angle over wide ranges of feedrates.

ACKNOWLEDGEMENTS

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REFERENCES


Fig 1  Block diagram of the dynamic model.

Fig 2  The static chip load that has been divided into four elements for the purpose of integration for the chip load elemental areas.

Fig 3  The components of a machining system showing the distinction between the cutting process and the machine tool structure.
Fig 4  A comparison of the predicted and actual tangential cutting force during intermittent cutting. The process parameters are:

- Distance from chuck = 304.80 mm,
- \(d_w=48.30\) mm, \(l_w=508.00\) mm,
- \(d_r=1.270\) mm, \(f_s=0.305\) mm/rev, and
- \(V=122\) m/min.

(a) Actual.
(b) Predicted.
Fig 5 A schematic of the workpiece considered for the case study.

Fig 6 The locations of traces taken for $R_{a,s}$ and $R_{a,c}$ values from the computer simulations.
Fig 7 A comparison of four three-dimensional surface plots from iteration One.
(a) $N_i=690$ t/min.
(b) $N_i=720$ t/min.
(c) $N_i=750$ t/min.
(d) $N_i=780$ t/min.
Fig 8 Surface characterization parameters obtained for simulations during Iteration One where solid marks denote the mean values and crisscrossed marks denote the $R_{\text{mean}}$ values.
(a) $R_{A_{\text{m}}}$
(b) $R_{A_{\text{c}}}$

Fig 9 Depth of cut and chip load at the point of re-entry to the cut.
(a) $N_r=690$ r/min.
(b) $N_r=780$ r/min.
Fig 10 Surface characterization parameters obtained for simulations during Iterations One and Two where solid marks denote the mean values and crisscrossed marks denote the $R_{a,mean}$ values.
(a) $R_a$
(b) $R_{a,e}$

Fig 11 Surface characterization parameters obtained for the twelve simulations (three slot widths at each of four spindle speeds) during Iteration Three.

Fig 12 A schematic of the experimental design used to study the robustness of two spindle speed/nominal slot width combinations in the presence of noise in the depth of cut and slot width variables.
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