A DYNAMIC MODEL OF THE CUTTING FORCE SYSTEM IN THE END MILLING PROCESS

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ABSTRACT

A dynamic model for the cutting force system in the end milling process is developed. This model, which describes: a) the dynamic transverse response of the end mill via a distributed parameter model, b) the chip load geometry, and c) the dependence of the cutting forces on the chip load, is used to predict the dynamic cutting force system produced during the end milling process. The cutting force system, may then in turn be used to predict the surface accuracy and surface texture produced by the process. The model incorporates the effects of cutting conditions, end mill geometry and such process noise factors as cutter runout. The model is verified through comparisons of model predicted cutting force signals with measured cutting forces obtained from machining experiments.

1. INTRODUCTION

The end milling process is widely used in industry because of its versatility and efficiency. Applications of the end milling process can be found in many industries ranging from large aerospace manufacturers to small tool and die shops. Reasons for its popularity include the fact that it may be used for the rough and finish machining of such features as slots, pockets, peripheries, and faces of components. Problems which may arise or result from the end milling process include cutter breakage, the generation of a finished part surface which does not satisfy product design specifications, and process instability. Shank or flute breakage typically arises due to excessive levels of cutting force being applied to the end mill. The surface accuracy, or machined surface error [4], and the surface texture [1,2] produced by the process are both highly dependent on the cutting force system. The stability of the end milling process is also dependent on the cutting force system and its interaction with the dynamics of the machining system [9,13,19,20]. The cutting force system is central to each of these problems, and thus it may be viewed as being the primary measure of end milling process performance.

Models of the end milling process have been developed to provide a better understanding of the process, so that the problems described above may be avoided. Such models are capable of predicting the effect of various process conditions on end milling performance measures such as cutting forces and surface accuracy. Such a model was constructed in [3,5] to mechanistically describe an end milling process. This model predicts cutting forces based on a rigid end milling system, and then applies these forces to the end mill and workpiece to cause deflection, i.e., surface error. This "rigid system" assumption for the calculation of cutting forces has produced good results for a class of problems in which the deflections of cutter and workpiece are relatively small. However, for the end milling of thin-walled sections or the machining of deep pockets/cavities with long cutters, these deflections may become large and have a significant effect on the chip load and thus the cutting force and surface accuracy. In addition, since the model does not incorporate the displacement of the cutter relative to the workpiece into the chip load determination and since the dynamic response of system is also not characterized, the model is unable to predict the dynamic behavior of the process.

A subset of the non-rigid end milling system situations has been examined in [12]. Displacements from a cutter deflection model (valid under static loading conditions) were used along with an algorithm for the calculation of chip load based on both cut geometry and system deflections to predict cutting forces and surface error. Measured forces obtained while using an end mill with a large projection length, thus experiencing large amounts of deflection, were accurately predicted by the model. Again, however, the dynamic behavior of the process is not described, since the response of the end mill is characterized with a model valid only under static loading conditions.

The dynamics of the end milling process have been extensively examined in [11,14,15,16,17]. This fundamental research, in particular focuses on the frequencies and corresponding mode shapes of the machine tool system (the machine tool structure, spindle, end mill, fixtures, and workpiece) that may be excited during the cutting process. For the purposes of cutting force system prediction the vibration of the end mill is characterized with a two degree-of-freedom system model such as that depicted in Fig. 1. The assumption that end mill vibration is described by a two degree-of-freedom system implies that any change in the cutter displacement magnitude along the cutter axis may be neglected. Such an assumption is valid for situations in which the axial depth of cut is relatively small. For situations in which the axial depth of cut is large (such as the machining of deep pockets), the variation in displacement magnitude along the cutter axis may be considerable.
A model that describes the vibration of an end mill via a two degree-of-freedom system does not describe this variation. Instead, it describes in an average sense the deflection of the cutter over the axial engagement with the workpiece.

This paper examines the components of a dynamic model for the prediction of the cutting force system in the end milling process. The components of the model, namely a relationship between the end mill/workpiece contact area (chip load) and the cutting forces, an algorithm for deflection-dependent chip load determination, and a distributed parameter model for the dynamic transverse response of an end mill are discussed. The computer-based framework for combining these three components to form a dynamic model of the cutting force system is described. Finally, the model is verified through a series of experiments in which cutting forces are measured and compared with model predictions.

2. DYNAMIC MODEL OF THE CUTTING FORCE SYSTEM

For machining processes the relationship between the cutting force system, machine tool system structural response, and chip load is often described with a diagram such as that shown in Fig. 2. As is evident from the figure, such a dynamic model has three elements: 1) A relationship for the cutting force system as a function of the area of contact between the end mill and workpiece (chip load), 2) A model which determines the chip load as a function of process geometry and level of system deflection, and 3) A model for the dynamic response of the machine tool system structure. Also evident from Fig. 2 is the fact that the model contains feedback, that is, the effect of system compliance is an integral part of the system dynamics. A dynamic model of this form has been used to characterize the behavior of an end milling process, and thus additional attention must be given to the three elements of the model described above.

2.1 Cutting Force Model

The development of a computer-based model for the end milling process requires an understanding of the geometry associated with the process. Fig. 3 displays the geometry and sign convention employed for a peripheral milling operation such as will be considered in this paper. The rotation of such an end mill may be simulated by examining the process at discrete angular positions in time, the angular increment being $d\theta$. The $j$th angular position of the end mill, $\theta(j)$, is then given by:

$$\theta(j) = (j-1) \cdot d\theta.$$  \hspace{1cm} (1)

The end mill, having $N_f$ evenly spaced flutes with angular spacing, $\gamma_f$ of $2\pi/N_f$, may be viewed as being composed of $N_a$ axial slices or disks of thickness $dz$. The distance of the $i$th elemental slice from the free end of the mill is therefore:

$$Z(i) = (i-1)dz + dz/2.$$  \hspace{1cm} (2)

Many investigations have been made into the nature of cutting forces in metal cutting operations, building on the pioneering work of those such as Merchant [8]. Sabberwal [10] used these theories to predict the forces in milling operations. Fig. 4 shows the elemental tangential and radial forces acting on one of the flutes for an arbitrary axial element of the end mill. In [3,5,12] these instantaneous cutting forces were assumed to be proportional to the area of contact between the flute and the workpiece. Using this assumption, the cutting forces acting on the $k$th flute on the $i$th axial elemental disk at the $j$th angular position may be found with Eqs. (3) and (4):
2.2 Chip Load Model

The determination of cutting forces requires knowledge of the area of contact between the end mill and the workpiece. For a flank element of thickness dz, this is equivalent to requiring knowledge of the uncut chip thickness, tc. To capture the effect of changing cut geometry due to cutter rotation on the chip thickness, the chip thickness may be expressed as:

\[ t_c(i,j,k) = f_t \sin \beta(i,j,k) \]  \hspace{1cm} (7)

where \( f_t = \text{feed/tool} \), is the feed/tooth.

This equation, developed by Martellotti [6], gives the distance or chip thickness, between successive flute paths under the assumptions that the tooth path is circular, no runout is present, and that the system does not deflect.

Given the existence of cutter runout, it has been shown [5,12] that the chip thickness may be expressed as:

\[ t_c(i,j,k) = \min [m_i \sin \beta(i,j,k) + R(i,k) - R(i,k-m)] \hspace{1cm} m=1,2,3,... \]  \hspace{1cm} (8)

where \( R(i,k) = (R^2 + L^2 + 2R\lambda \cos \lambda \cdot (-\lambda)^{-1})^{1/2} \)

\( \rho \) and \( \lambda \) are parameters which describe the parallel axis offset runout geometry, 
\( \tau \) and \( \phi \) are parameters which describe the cutter tilt geometry,
\( z \) is the distance from the fixed end of the cutter to the axial position of interest, and
\( m \) is an index used to consider preceding flutes.

Eq. (8) characterizes the effect of cutter runout on the chip thickness, and also allows for the possibility that the current flute of interest may not be machining over the surface generated by the immediately preceding flute.

Displacements between the tool and workpiece in the X- and Y-directions also impact the chip thickness. The cutting forces produced by the process cause these displacements which in tum influence the cut geometry. This mechanism is generally characterized as being a feedback process, and for an end milling process both primary and regenerative feedback are present. Primary feedback is classified as displacements that immediately impact the chip thickness while regenerative feedback refers to a situation in which current displacements impact the chip thickness encountered at some time in the future. With the incorporation of both primary and regenerative feedback effects, the chip thickness may be expressed as:

\[ t_c(i,j,k) = \min [m_i \sin \beta(i,j,k) + R(i,k) - R(i,k-m)] \]

\[ + (x(i,j) - x(i,n) \sin \beta(i,j,k) + y(i,j) - y(i,n) \cos \beta(i,j,k)) \hspace{1cm} m=1,2,3,... \]  \hspace{1cm} (9)

where \( x(i,j) \) is the relative displacement in the X-direction of the ith axial disk at the jth angular position of the end mill from the workpiece,
\( y(i,j) \) is the displacement in the Y-direction, and
\( n = j \cdot (k-m) \cdot \gamma / \theta \cdot \delta \).

As is apparent from an examination of Eq. (9), the chip thickness for the ith axial slice, jth angular position, and kth flute, is dependent on the cut geometry, including cutter runout, and the displacement history between the end mill and workpiece. A model for the chip thickness similar to that given in Eq. (9) has been used previously [12] to accurately characterize the effects of cutter runout and system flexibilities on the cutting forces produced by the end milling process.
2.3 Model for the Dynamic Response of the End Mill

The components of the machine tool system are all affected to some extent by the cutting forces produced by the end milling process. For the purposes of cutting force system prediction, however, only those components which cause the end mill to displaced relative to the workpiece are of interest. As has been stated previously [4], the end mill itself is often the greatest source of these displacements. A distributed parameter model of the end mill has been developed to characterize its dynamic response to the cutting forces. Unlike a two degree-of-freedom system model, such a model describes the displacement variation along the axis of the cutter. A model for the vibratory response of an end mill may be constructed by considering the end mill to be a beam, cantilevered at one end (the tool holder), and free at the other. The equation of motion for the transverse vibration of a cantilevered beam, such as that shown in Fig. 5, is described by:

\[
\frac{\partial^2}{\partial z^2} \left( E(z) \frac{\partial^2 y(z,t)}{\partial z^2} \right) + f(z,t) = \mu(z) \frac{\partial^2 y(z,t)}{\partial t^2} \tag{10}
\]

where \( y \) is the displacement of the beam in the \( y \)-direction, \( \mu \) is the mass of the beam per unit length, \( E \) is the modulus of elasticity for the beam, \( I \) is the moment of inertia for a cross section of the beam, \( f \) is the forcing function applied to the beam, and \( t \) is time.

Under the assumption that the flexural rigidity, \( EI \), and mass, \( \mu \), are constant, Eq. (10) may be simplified.

Considering the case of free vibration first (\( f(z,t) = 0 \)), Eq. (10) becomes separable in space and time, i.e., \( y(z,t) = Y(z)F(t) \), where \( F(t) \) is harmonic and has a frequency of \( \omega \). The eigenvalue problem may now be expressed as:

\[
EI \frac{d^2Y(z)}{dz^2} = \omega^2 \mu Y(z) \tag{11}
\]

or

\[
\frac{d^2Y(z)}{dz^2} - \eta^4 Y(z) = 0 \tag{12}
\]

where \( \eta^4 = \frac{\omega^2 \mu}{EI} \).

The natural frequencies of the system are then:

\[
\omega_r = (\eta_r L)^2 (EI/\mu L^4)^{1/2} \quad \text{for } r = 1,2,3,\ldots \tag{13}
\]

To complete the formulation of the boundary value problem, the boundary conditions must be specified. The boundary conditions at the fixed or clamped end of the beam (end mill) are:

\[
Y(0) = 0, \quad \frac{dY(z)}{dz} = 0 \text{ at } z=0 \tag{14}
\]

The boundary conditions at the free end of the beam are:

\[
\frac{d^2Y(z)}{dz^2} = 0 \text{ at } z=L, \quad \frac{d^2Y(z)}{dz^2} = 0 \text{ at } z=L \tag{15}
\]

Applying the boundary conditions to the general solution of Eq. (12) produces an equation for the normal modes in terms of the system eigenvalues:

\[
Y_r(z) = C_r [(\sin \eta_r z - \sinh \eta_r z) + D_r (\cos \eta_r z - \cosh \eta_r z)] \tag{16}
\]

where

\[
D_r = \frac{\cos \eta_r L + \cosh \eta_r L}{\sin \eta_r L - \sinh \eta_r L}, \quad C_r = \frac{\cos \eta_r L \cosh \eta_r L + 1}{\eta_r} \quad \text{for } \eta_r \text{ the eigenvalue for the } r \text{th mode, which can be obtained through the numerical solution of the characteristic equation, } \cos \eta_r L \cosh \eta_r L + 1 = 0.
\]

The first eight eigenvalues are summarized in Table 1.

The natural modes given by Eq. (16) may be normalized by requiring that:

\[
\int_0^L \mu Y_r^2(z) dz = 1. \tag{17}
\]

<table>
<thead>
<tr>
<th>Eigenvalue Number</th>
<th>( \eta_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.8751041</td>
</tr>
<tr>
<td>2</td>
<td>4.6940911</td>
</tr>
<tr>
<td>3</td>
<td>7.8547574</td>
</tr>
<tr>
<td>4</td>
<td>10.9955407</td>
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<tr>
<td>5</td>
<td>14.1371683</td>
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<tr>
<td>6</td>
<td>17.2787595</td>
</tr>
<tr>
<td>7</td>
<td>20.4203522</td>
</tr>
<tr>
<td>8</td>
<td>23.5619449</td>
</tr>
</tbody>
</table>

The normalization procedure specified by Eq. (17) allows the constant \( C_r \) to be solved for:

\[
C_r = \frac{m([D_r^2 \sin \eta_r L + (D_r^2 + 1) \sin \eta_r L \cosh \eta_r L] + 1 \cdot \frac{\sin \eta_r L \cosh \eta_r L}{\eta_r} \cdot \frac{\cos \eta_r L \cosh \eta_r L + 1}{\eta_r})}{D_r^2 \sin \eta_r L \cosh \eta_r L + (D_r^2 + 1) \sin \eta_r L \cosh \eta_r L} \tag{18}
\]

The response of a cantilever beam to excitation may be viewed as the superposition of the system eigenfunctions, given by Eq. (16), multiplied by the corresponding time dependent modal displacements. The utilization of the modal coordinate system concept allows for the treatment of the different modes.
independently. Based on this modal analysis, the system response may be expressed as:

\[ y(z,t) = \sum_{r=1}^{\infty} Y_r(z) q_r(t) \quad (19) \]

where \( q_r(t) \) is the displacement of the \( r \)th mode at time, \( t \), and is governed by the ordinary differential equation:

\[ \frac{d^2 q_r(t)}{dt^2} + 2\zeta \omega_n \frac{dq_r(t)}{dt} + \omega_n^2 q_r(t) = Q_r(t) \quad (20) \]

\( Q_r(t) \) is the force applied to the \( r \)th mode at time, \( t \).

In general, the modal force must be obtained through the following relationship:

\[ Q_r(t) = \int_{0}^{L} f(z,t) Y_r(z) \, dz \quad (21) \]

However, since the cutting force model views \( f(z,t) \) as a number of point loads applied at various locations along the axis of the end mill, some simplification is possible. The forcing function in the Y-direction may be expressed as:

\[ f(z,t) = dF_y(y, i, j) \delta(z-z_i) u(t) \quad (22) \]

where \( dF_y(y, i, j) \) is the force applied to the \( i \)th elemental slice of the end mill in the Y-direction at the \( j \)th angular position (corresponding to time, \( t \)), \( \delta \) is the Dirac delta function, \( u(t) \) is a unit step function, \( z_i = L - Z(i) \) is the distance of the \( i \)th elemental slice from the fixed end of the end mill, and \( Z(i) \) being the distance from the free end as defined previously.

Substituting this relationship for the force into Eq. (21) gives an equation for the modal force:

\[ Q_r(t) = \sum_{i=1}^{N_z} dF_y(y, i, j) Y_r(z_i) u(t) \quad (23) \]

In summary, the dynamic response of the end mill in the Y-direction is given by Eq. (19), with the modal displacements governed by Eq. (20), the mode shape specified by Eq. (16), and the modal force given by Eq. (23). A similar set of equations have also been developed to describe the dynamic displacement of the cutter/beam in the X-direction.

2.4 Numerical Method for Dynamic Response Determination.

In the preceding sections models have been presented for: a) the cutting force components in the X- and Y-directions, b) the chip load, and c) the dynamic response of the end mill. The equation of motion given in Eq. (20), and the corresponding equation of motion in the X-direction, when integrated give the X and Y positions of the end mill in the modal coordinate system. In order to accomplish this integration a fourth order predictor-corrector scheme is used. This technique, as the name implies, consists of two steps, a prediction of the displacement and velocity one step ahead into the future, and secondly, after having moved to that future point in time, adjusting the displacement and velocity based on additional information. The predicted modal displacement and velocity of the end mill one step ahead into the future are given by Eqs. (24) and (25):

\[ \frac{d^2 q(t + \Delta t)}{dt^2} = \frac{dq(t + 3\Delta t)}{dt} - \frac{4}{3} \left( \frac{d^2 q(t + \Delta t)}{dt^2} + \frac{2}{3} \frac{d^2 q(t)}{dt^2} \right) \quad (24) \]

\[ q(t + \Delta t) = \frac{4}{3} \left( \frac{d^2 q(t + \Delta t)}{dt^2} + \frac{2}{3} \frac{d^2 q(t + \Delta t)}{dt^2} \right) \quad (25) \]

After having moved one time step or angular position into the future, the cutting forces and associated modal forces may be calculated based on the predicted position of the end mill. The modal acceleration may then be calculated:

\[ \frac{d^2 q(t)}{dt^2} = Q(t) - 2\zeta \omega_n \frac{dq(t)}{dt} - \omega_n^2 q(t) \quad (26) \]

The modal displacement and velocity of the end mill may then be corrected based on knowledge of the current estimates of the acceleration, velocity, and displacement:

\[ q(t) = \frac{1}{8} [q(t) - q(t - \Delta t) - q(t + \Delta t) + \Delta(t) \left( \frac{d^2 q(t)}{dt^2} + \frac{d^2 q(t - \Delta t)}{dt^2} - \frac{d^2 q(t + \Delta t)}{dt^2} \right)] \quad (27) \]

\[ \frac{dq(t)}{dt} = \frac{1}{8} \left( \frac{d^2 q(t)}{dt^2} + \frac{d^2 q(t - \Delta t)}{dt^2} - \frac{d^2 q(t + \Delta t)}{dt^2} \right) \quad (28) \]

Several iterations of the corrector step may be necessary for the displacements and velocities to converge. In summary, Eqs. (24)-(28) may be used in concert with the models described previously to determine the modal displacements associated with response of the end mill in the Y-direction (a similar set of equations are available for the X-direction).

3. EXPERIMENTAL WORK

In [12] a series of down milling experiments were conducted. The results of these experiments may be used to verify the ability of the dynamic cutting force prediction model to accurately predict the cutting forces produced by an end milling process. Two sets of tests were performed on a Kearney and Trecker horizontal milling machine using 19 mm (0.75 inch) diameter, 30 degree helix angle, four fluted, high speed steel end mills. The workpiece material was a large, rigidly held block of 390 aluminum alloy. The first set of tests were performed with an end mill having a projection length of 73 mm (2.875 inch). These experiments were conducted in order to estimate empirical functions for \( K_t \) and \( K_r \). The functions obtained were [12]:

\[ K_t = 387 \, t_c \quad (29) \]

\[ K_r = 0.0018 \, t_c \quad (30) \]

where \( t_c \) is the average chip thickness value over one cutter revolution, and for a rigid system with no cutter runout is given, as reported in [5], by:

\[ t_c = \frac{f_t \, d_r}{\text{RAD} \cos^{-1}(1 - \frac{d_r}{\text{RAD}})} \quad (31) \]

The average chip thickness is, in general, dependent on the amount of cutter runout and the level of system deflection. However, the average chip thickness given by Eq. (31) is a reasonable estimate of the actual average chip thickness.

The second set of tests were conducted with an end mill having a projection length of 133 mm (5.25 inch). The conditions
for the eight tests performed with this relatively large end mill length are summarized in Table 2, in addition, the average chip thickness for each set of conditions is also displayed. All the tests described in Table 2 were performed without coolant at a constant cutting speed of 31.7 m/min (104 ft/min - 530 rpm). Instantaneous cutting forces were measured with a Kistler dynamometer (Model 9255) and the force signals were digitized (sampling rate of 1000 Hz) and stored using a microprocessor-controlled data acquisition system.

The estimated runout parameters for these eight tests were \( \rho = 0.048 \) mm (0.0019 inch), \( \lambda = 10 \) degrees, \( \tau = 0.040 \) degrees, and \( \phi = 0 \).

4. MODEL VALIDATION AND COMPARISONS

The conditions associated with the experiments in Table 2 were used as inputs to the dynamic end milling model to obtain predicted instantaneous cutting forces. For the simulations the 390 Aluminum workpiece was assumed to be rigid, and the only structural dynamics considered were those associated with the end mill. Damping arises in a machining process due to both structural (hysteretic) and cutting process sources. Tusty [18] suggests that the process damping results from interference of the tool flank with the machined work surface, and for the relatively low cutting speeds considered in this paper, the process damping dominates any structural effects. Structural damping may be introduced into the system through the damping ratio in Eq. (26) and process damping may be incorporated through the forcing function.

Fig. 6 shows the predicted cutting force in the Y-direction for the first ten cutter revolutions of test 5. Evident from this figure is the transient nature associated with the start-up of the simulation as the forces increase from zero to take on their "steady-state" oscillatory nature. This start-up phenomenon is due to the fact that the simulation assumes that the starting position of the end mill is in a "dwell" (a location where the end mill is allowed to rotate with the table feed disengaged). For a "dwell" the displacement of the end mill relative to the workpiece may be assumed to be zero, thus providing initial conditions for the simulation.

<table>
<thead>
<tr>
<th>Test</th>
<th>Radial Depth of Cut (mm)</th>
<th>Feed, mm/rev (inch/rev)</th>
<th>Axial Depth of Cut (mm)</th>
<th>Spindle Speed, rpm</th>
<th>End Mill Length, (mm)</th>
<th>Average Chip Thickness, (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.35 (0.250)</td>
<td>0.1575 (0.0062)</td>
<td>25.4 (1.000)</td>
<td>530.0</td>
<td>133 (5.25)</td>
<td>0.0852 (0.00336)</td>
</tr>
<tr>
<td>2</td>
<td>6.35 (0.250)</td>
<td>0.0635 (0.0025)</td>
<td>25.4 (1.000)</td>
<td>530.0</td>
<td>133 (5.25)</td>
<td>0.0843 (0.00335)</td>
</tr>
<tr>
<td>3</td>
<td>1.27 (0.050)</td>
<td>0.0635 (0.0025)</td>
<td>25.4 (1.000)</td>
<td>530.0</td>
<td>133 (5.25)</td>
<td>0.0843 (0.00335)</td>
</tr>
<tr>
<td>4</td>
<td>1.27 (0.050)</td>
<td>0.1575 (0.0062)</td>
<td>25.4 (1.000)</td>
<td>530.0</td>
<td>133 (5.25)</td>
<td>0.0843 (0.00335)</td>
</tr>
<tr>
<td>5</td>
<td>1.27 (0.050)</td>
<td>0.0635 (0.0025)</td>
<td>12.7 (0.500)</td>
<td>530.0</td>
<td>133 (5.25)</td>
<td>0.0843 (0.00335)</td>
</tr>
<tr>
<td>6</td>
<td>1.27 (0.050)</td>
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<tr>
<td>7</td>
<td>6.35 (0.250)</td>
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<td>8</td>
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<td>0.1575 (0.0062)</td>
<td>12.7 (0.500)</td>
<td>530.0</td>
<td>133 (5.25)</td>
<td>0.0843 (0.00335)</td>
</tr>
</tbody>
</table>

Table 2 Levels of the Independent Variables for Tests 1-8, from reference [10]

A comparison of four revolutions of the measured and model predicted cutting forces in the Y-direction for tests 1 and 5 are shown in Figs 7 and 8. The instantaneous cutting forces predicted by the dynamic model are shown for their 7th, 8th, 9th, and 10th revolutions, so that the effect due to the simulation start-up is diminished. Also shown in these figures are the instantaneous cutting forces predicted by the rigid system model described in [3,4,5]. An examination of Figs 7 and 8 shows that the dynamic model accurately predicts both the shape and the relative magnitude of the measured cutting force signals. The quality of the predictions made by the dynamic model is emphasized by their superiority with respect to the predictions made by the rigid system model. Table 3 summarizes the measured and model predicted average and peak cutting forces in the Y-direction for all eight tests listed in Table 2. Once again, the cutting forces predicted based on the rigid system model are included for comparison purposes. An examination of

Table 3 Measured, Rigid Model Predicted, and Dynamic Model Predicted Cutting Forces in the Y-direction for Tests 1-8
(The percent difference between the model predicted and measured value is indicated in parentheses.)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1140.9</td>
<td>1147.1 (0.55)</td>
<td>1126.9 (1.23)</td>
<td>1418.9</td>
<td>2703.5 (90.53)</td>
<td>1350.5 (4.82)</td>
</tr>
<tr>
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<td>902.9</td>
<td>2036.7 (125.57)</td>
<td>895.5 (0.83)</td>
</tr>
<tr>
<td>3</td>
<td>151.7</td>
<td>109.9 (27.57)</td>
<td>144.2 (4.90)</td>
<td>306.9</td>
<td>449.7 (46.52)</td>
<td>329.2 (7.62)</td>
</tr>
<tr>
<td>4</td>
<td>218.8</td>
<td>205.9 (5.89)</td>
<td>215.6 (1.48)</td>
<td>422.6</td>
<td>766.4 (81.37)</td>
<td>417.4 (1.22)</td>
</tr>
<tr>
<td>5</td>
<td>112.1</td>
<td>103.2 (7.94)</td>
<td>110.6 (1.35)</td>
<td>391.4</td>
<td>647.2 (65.34)</td>
<td>346.1 (11.57)</td>
</tr>
<tr>
<td>6</td>
<td>60.5</td>
<td>53.4 (11.76)</td>
<td>72.9 (20.51)</td>
<td>266.9</td>
<td>431.5 (61.67)</td>
<td>272.7 (2.17)</td>
</tr>
<tr>
<td>7</td>
<td>358.1</td>
<td>286.0 (20.12)</td>
<td>372.9 (4.15)</td>
<td>733.9</td>
<td>1486.1 (102.48)</td>
<td>813.6 (10.85)</td>
</tr>
<tr>
<td>8</td>
<td>705.5</td>
<td>565.8 (19.80)</td>
<td>611.2 (13.36)</td>
<td>1201.0</td>
<td>2001.6 (66.67)</td>
<td>1232.1 (2.60)</td>
</tr>
</tbody>
</table>
Table 3 reveals that the peak cutting forces predicted based on the dynamic model are substantially closer to the measured peak forces than are the predicted peak forces made by the rigid system model.

To quantitatively examine the cutting forces shown in Figs. 7 and 8 their frequency content was determined. The autospectra for the measured, rigid system model predicted, and dynamic model predicted cutting forces in the Y-direction for tests 1 and 5 are shown in Figs. 9 and 10. Especially evident from these figures are the spindle rotation and flute passing frequencies (8.8 Hz and 35.3 respectively), which are indicated on the autospectra associated with the measured force signals. For a machining situation with no runout, the frequency content is distributed at the flute passing frequency and its harmonics, while the introduction of runout shifts some of the frequency content to the spindle rotation frequency. From the figures it is apparent that the autospectra associated with the dynamic model predictions seem to describe the frequency content associated with the measured force signal autospectra quite well. The autospectra associated with the rigid system model predicted cutting forces differs from both the dynamic model and measured cutting force signal autospectra because the flexible nature/dynamic response of the end mill tempers the effect of cutter runout.

Fig. 7 Predicted Cutting Force in the Y-Direction for Test 1
(a) Measured Force, (b) Rigid Model Predicted Force,
(c) Dynamic Model Predicted Force

Fig. 8 Predicted Cutting Force in the Y-Direction for Test 5
(a) Measured Force, (b) Rigid Model Predicted Force,
(c) Dynamic Model Predicted Force
For all the test conditions given in Table 2 the spindle rotation and flute passing frequencies are well beneath the lowest natural frequency associated with the 133 mm (5.25 inch) long end mill (630 Hz). The effect of higher spindle speeds on the dynamic cutting force prediction model was investigated through additional simulations. The predicted cutting forces in the Y-direction for the conditions of test 1, based on spindle speeds of 530, 2100, and 3500 rpm respectively, are shown in Fig. 11. The three force signals within the figure have approximately the same cutting force magnitude and general shape, although it does appear that at the larger spindle speeds additional higher frequencies are present within the force signal. The frequency content of the dynamic model predictions was also determined, and is illustrated by the autospectra of Fig. 12. It should be noted that the frequency axes of Fig. 12 have been normalized by dividing by the spindle rotation frequency. As was suggested by the cutting force signals of Fig. 11, Fig. 12 demonstrates that the frequency content of the force signal at 2100 and 3500 rpm is somewhat different from that at 530 rpm. The larger spindle speeds appear to shift some of the power associated with the spindle rotation and flute passing frequencies to higher frequencies. One explanation for this phenomenon is that at the larger spindle speeds the frequencies associated with the end mill begin to become excited by the higher order harmonics in the cutting force signal. This excitation impacts the chip load and therefore the cutting forces, and thus it is reflected in the autospectra of the cutting force signal.

Fig. 9 Autospectra for Signals in the Y-Direction for Test 1 (a) Measured Force Signal, (b) Rigid Model Predicted Force Signal (c) Dynamic Model Predicted Force Signal

Fig. 10 Autospectra for Signals in the Y-Direction for Test 5 (a) Measured Force Signal, (b) Rigid Model Predicted Force Signal, (c) Dynamic Model Predicted Force Signal
5. SUMMARY

A dynamic model of the cutting force system in the end milling process has been developed. This model has the following unique features:

- A model for the instantaneous chip load which includes the effects of past and present displacements of the end mill relative to the workpiece, the effects of the cutter helix angle, and the effects of cutter runout.
- A model for the dynamic response of the end mill. This model differs from those developed previously in that it describes the dynamic response via a distributed parameter representation of the end mill. Such a formulation describes the end mill displacement as a function of the distance along the axis of the cutter, thus allowing for the prediction of the cutting force system when the axial depth of cut is large.

The model has been verified through machining experiments conducted over a wide range of machining conditions at a relatively low spindle speed. Cutting forces predicted by the dynamic model have been compared with both measured and rigid system model predicted force signals, and the following conclusions may be made:

- The dynamic model accurately predicts both the shape and magnitude of the measured cutting force signal.

![Graphs and charts](a) ![Graphs and charts](b) ![Graphs and charts](c)

Fig. 11 Dynamic Model Predicted Cutting Force in the Y-Direction for the Conditions of Test 1 at Spindle Speeds (a) 330 rpm, (b) 2100 rpm, (c) 3500 rpm.

Fig. 12 Autospectra for Dynamic Model Predicted Cutting Forces in the Y-Direction for the Conditions of Test 1 at Speeds of (a) 330 rpm, (b) 2100 rpm, (c) 3500 rpm.
• The predictions made by the dynamic end milling model are far superior to those made by the rigid system model.
• Comparisons of the autospectra associated with the predicted and measured cutting force signatures indicate that the dynamic model accurately describes the frequency content in the measured data.
• Model simulations suggest that increasing the spindle speed shifts the spectral content of the force signal away from the spindle rotation and flute passing frequencies.

The development of a dynamic model for the prediction of cutting forces in the end milling process permits the examination of additional measures of process performance including the stability of the process as a function of spindle speed, feed, etc. and the machined surface texture produced by the process. In addition, the importance of high speed/high productivity milling [11] necessitates that increased attention be given toward the application of this and other models to high spindle speed machining.

The model for forced cutter response described in this paper represents a substantial improvement over other models that have been presented to describe end mill dynamics in that it describes the changing displacement of the cutter relative to the workpiece along the axis of the mill. It is recognized, however, that such a model may be improved through the incorporation of:
• Shear deformation and rotatory inertia effects
• The dynamics of the spindle, tool holder, workpiece, and remaining elements of the machining system structure in addition to the end mill
• Stiffness directionality for two-fluted end mills
• Spindle/cutter wind-up and torsional vibration effects

In addition, process damping effects and model verification through cutting experiments performed at higher cutting speeds must also be investigated.

6. REFERENCES