Graphs & Networks

- Subgraphs
- Graph coloring
- Activity networks
- Route problems
- Network flow
- Visibility graph problems
- Covering problems
Project Management

- Service organizations have to take on large, complex projects at some point
  - An airline opening new routes or pulling a jumbo jet out of service for major maintenance – large expenses if tasks are delayed
  - Department store – installing new inventory control system – lost sales and large ordering costs if timetables unmet
  - A hospital modernizing operating rooms
Project Management

- **Elements:**
  - Planning
  - Scheduling
  - Control

- Aid to planning – work breakdown structure (WBS), which is a hierarchical tree – tasks required to achieve the objective

- Divides work into manageable components such as tasks, subtasks, and work elements
WBS

1.0 Move the hospital (Project)
   1.1 Move patients (Task)
     1.1.1 Arrange for ambulance (Subtask)
       1.1.1.1 Prepare patients for move
       1.1.1.2 Box patients personnel effects
   1.2 Move furniture
     1.2.1. Contract with moving company
       •
       •
       •

Fitzsimmons 2006
Project Scheduling

- Project Scheduling: determine the project activities in the time sequence in which they have to be performed
- Materials and people needed at each stage of production are computed, and the time each activity will take is also determined
  - Gantt charts
Gantt Chart

- Reflects time estimates and can be easily understood
- Horizontal bars for each activity
- Letters on left of each bar – activities that precede it
Project Scheduling

- Gantt charts not easily updated
- Don’t adequately illustrate interrelationships between activities and resources
- CPM (Critical Path Method) & PERT (Program Evaluation Review Technique) are two widely used network techniques – they have the ability to consider precedence relationships and interdependency of activities
Activity Network

- **Arcs** – activities
- **Nodes** – events
- **Event** – requires completion all events
Critical Path Method

- CPM is an approach to determine the start and finish dates for individual activities in a project
- A result of this method is the identification of a critical path, or chain of crucial activities throughout project
## Critical Path Method

<table>
<thead>
<tr>
<th>Item</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity duration</td>
<td>$t$</td>
<td>The expected duration of an activity</td>
</tr>
<tr>
<td>Early start</td>
<td>$ES$</td>
<td>The earliest time an activity can begin if all previous activities are begun at their earliest times</td>
</tr>
<tr>
<td>Early finish</td>
<td>$EF$</td>
<td>The earliest time an activity can be completed if it is started at its early start time</td>
</tr>
<tr>
<td>Late start</td>
<td>$LS$</td>
<td>The latest time an activity can begin without delaying the completion of the project</td>
</tr>
<tr>
<td>Late finish</td>
<td>$LF$</td>
<td>The latest time an activity can be completed if it is started at its latest start time</td>
</tr>
<tr>
<td>Total slack</td>
<td>$TS$</td>
<td>The amount of time an activity can be delayed without delaying the completion of the project</td>
</tr>
</tbody>
</table>
Critical Path Method

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>ES = EF \text{predecessor (max)}</td>
<td>Earliest Start</td>
</tr>
<tr>
<td>EF = ES + t</td>
<td>Earliest Finish</td>
</tr>
<tr>
<td>LF = LS \text{successor (min)}</td>
<td>Latest Finish</td>
</tr>
<tr>
<td>LS = LF - t</td>
<td>Latest Start</td>
</tr>
<tr>
<td>TS = LF - EF</td>
<td>Time Slack</td>
</tr>
<tr>
<td>TS = LS - ES</td>
<td>Free Slack</td>
</tr>
</tbody>
</table>

or

<table>
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<tbody>
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<td>TS = LS - ES</td>
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<td>TS = LF - EF</td>
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</table>
More on CPM

- Associated with each activity is a cross to be filled in with the activity schedule times, which are calculated as follows:

\[ TS = LS - ES \]

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More CPM
More on Activity Networks

- In many cases \( t \) values are assumed to be deterministic (i.e., constant)
- In other cases, expected duration is mean of known probability distributions
- PERT Approach – Beta Dist.
  - 3 time estimates
  - Most likely time \((M)\), optimistic time \((O)\), and pessimistic time \((P)\)
  - For each activity: Mean \(=\frac{(O + 4M + P)}{6}\)
  - For each activity: Std Dev \(=\frac{(P – O)}{6}\)
  - Add variances along path
  - End up with Mean and S.D. of Completion Time