Solving the Transportation Model

- Last time we looked at the formulation of the Linear Programming problem associated with the Transportation model.
- Suggested that a software tool (e.g., Maple) could be used to solve the LP problem.
Another Transportation Model Solution

- The basic steps of the transportation technique are:
  - Step 1: Determine a starting feasible solution
  - Step 2: Determine an entering variable from among the non-basic variables. If all such variables satisfy the optimality condition, stop, otherwise go to step 3
  - Step 3: Determine a leaving variable (using the feasibility condition) from among the variables of the current basic solution; then find the new basic solution. Return to step 2.
Some definitions

- **Basic variable??**
  - Say we have a transportation problem with two sources and two destinations
  - Four decision variables: $x_{11}, x_{12}, x_{21}, \text{ and } x_{22}$
  - If a decision variable has a non-zero value – it is termed a basic variable – at that point it represents a part of the solution

- **Non-basic variable**
  - A decision variable with a current value of zero
Another Transportation Model Solution

- Consider the transportation table below. Unit transportation costs $c_{ij}$ are shown.

```
<table>
<thead>
<tr>
<th></th>
<th>Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Source 1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$x_{11}$</td>
</tr>
<tr>
<td>Source 2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>$x_{21}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Source 3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$x_{31}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Supply</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
```
Solution – Step 1

- **Northwest-corner method**: Allocate maximum amount allowable by the supply and demand to variable $x_{11}$ (in the northwest corner of the tableau). The satisfied column or row is then crossed out. If column and row are both satisfied – only one (either one) may be crossed out.

- Adjust amounts of supply and demand for all uncrossed-out rows and columns, the maximum feasible amount is allocated to the first uncrossed-out element in the new column or row. Process is completed when exactly one row or one column is left uncrossed-out.
Solution – Step 1

Getting back to the last table:

- $X_{11}=5$, which crosses out column 1. Thus no further allocation can be made in column 1. The amount left in row 1 is 10 units.
- $X_{12}=10$, which crosses out row 1 and leaves 5 units in column 2.
- $X_{22}=5$, which crosses out column 2 and leaves 20 units in row 2.
- $X_{23}=15$, which crosses out column 3 and leaves 5 units in row 2.
- $X_{24}=5$, which crosses out row 2 and leaves 5 units in column 4.
- $X_{34}=5$, which crosses out row 3 or column 4. Since only one row or one column remains uncrossed out, the process ends.
Solution – Step 1

- The resulting starting basic solution is given in the next table, the basic variables are the ones we just listed, and the other $x_{pq}$'s are non-basic. The associated transportation cost is $5 \times 10 + 10 \times 0 + 5 \times 7 + 15 \times 9 + 5 \times 20 + 5 \times 18 = $410
Solution – Step 2

- The entering variable is determined by using the optimality condition of the simplex method.
- In the method of multipliers we associate the multipliers $u_i$ and $v_j$ with row $i$ and column $j$ of the transportation tableau. For each basic variable $x_{ij}$ in the current solution, the multipliers $u_i$ and $v_j$ must satisfy the following equation:

\[ u_i + v_j = c_{ij} \quad \text{for each basic variable } x_{ij} \]
Solution – Step 2

- These equations yield \( m + n - 1 \) equations in \( m + n \) unknowns. The values of the multipliers can be determined from these equations by assuming an arbitrary value for any one of the multipliers and then solving the \( m + n - 1 \) equations in the remaining \( m + n - 1 \) unknown multipliers. Once this is done, the evaluation for each non-basic variable \( x_{pq} \) is given by:

\[
\bar{c}_{pq} = u_p + v_q - c_{pq} \quad \text{for each nonbasic variable } x_{pq}
\]
Solution – Step 2

- So the equations associated with the basic variables in our example are:

\[
\begin{align*}
\text{x}_{11} & : u_1 + v_1 = c_{11} = 10 \\
\text{x}_{12} & : u_1 + v_2 = c_{12} = 0 \\
\text{x}_{22} & : u_2 + v_2 = c_{22} = 7 \\
\text{x}_{23} & : u_2 + v_3 = c_{23} = 9 \\
\text{x}_{24} & : u_2 + v_4 = c_{24} = 20 \\
\text{x}_{34} & : v_3 + v_4 = c_{34} = 18
\end{align*}
\]

- By letting \( u_1 = 0 \), the values of the multipliers are successively determined as \( v_1 = 10, \ v_2 = 0, \ u_2 = 7, \ v_3 = 2, \ v_4 = 13, \) and \( u_3 = 5 \).
Solution – Step 2

- The evaluations of the non-basic variables are thus given as:

  \[ x_{13}: \quad \tilde{c}_{13} = u_1 + v_3 - c_{13} = 0 + 2 - 20 = -18 \]
  \[ x_{14}: \quad \tilde{c}_{14} = u_1 + v_4 - c_{14} = 0 + 13 - 11 = 2 \]
  \[ x_{21}: \quad \tilde{c}_{21} = u_2 + v_1 - c_{21} = 7 + 10 - 12 = 5 \]
  \[ x_{31}: \quad \tilde{c}_{31} = u_3 + v_1 - c_{31} = 5 + 10 - 0 = 15 \]
  \[ x_{32}: \quad \tilde{c}_{32} = u_3 + v_2 - c_{32} = 5 + 0 - 14 = -9 \]
  \[ x_{33}: \quad \tilde{c}_{33} = u_3 + v_3 - c_{33} = 5 + 2 - 16 = -9 \]

- Since \( x_{31} \) has the most positive \( \tilde{c}_{pq} \), it is selected as the entering variable.
Solution – Step 3

- This step is equivalent to applying the feasibility condition in the simplex method. However, since all the constraint coefficients in the original transportation model are either zero or one, the ratios of the feasibility condition will always have their denominator equal to 1. Thus the values of the basic variables will give the associated ratios directly.
Solution – Step 3

- For the purpose of determining the minimum ratio, we construct a closed loop for the current entering variable. The loop starts and ends at the designated non-basic variable. It consists of successive horizontal and vertical (connected) segments whose end points must be the basic variables, except for the end points that are associated with the entering variable. This means that every corner element of the loop must be a cell containing a basic variable.
Solution – Step 3

This loop may be defined in terms of the basic variables as:

\[ x_{31} \rightarrow x_{11} \rightarrow x_{12} \rightarrow x_{22} \rightarrow x_{24} \rightarrow x_{34} \rightarrow x_{31}. \]
Solution – Step 3

- The new solution is:

- Its new cost is $0 \times 10 + 15 \times 0 + 0 \times 7 + 15 \times 9 + 10 \times 20 + 5 \times 0 = $335$. This cost differs from the one associated with the starting solution by $410-335=75$, which is equal to the number of units assigned to $x_{31}$ ($=5$) multiplied by $c_{31} (=15)$.
Solution – Step 3

- What are the basic variables?
- The previously basic variables $x_{11}$ and $x_{22}$ are now zero – do they become non-basic? For now, these zero basic variables are treated as any other positive basic variables.
- The new basic solution obtained is now checked for optimality by computing the new multipliers as shown next:
The values of $c_{pq}$ are given by the numbers in the southwest corner of each nonbasic cell. The nonbasic variable $x_{21}$ with the largest positive $c_{pq}$ thus enters the solution. The closed loop associated with $x_{21}$ shows that either $x_{11}$ or $x_{22}$ can be the leaving variable. We arbitrarily select $x_{11}$ to leave the solution.
Solution – Step 3

- Next table shows the new basic solution. The new values of $u_j$, $v_j$, and $c_{pq}$ are computed anew. This table gives the entering and leaving variables as $x_{14}$ and $x_{24}$, respectively.
Solution – Step 3

With this change, we obtain the new solution:

\[
\begin{array}{cccc}
\nu_1 = 5 & \nu_2 = 0 & \nu_3 = 2 & \nu_4 = 11 \\
\hline
u_1 = 0 & 10 & 5 & 0 & 20 & 11 \\
u_2 = 7 & 5 & 10 & -18 & 20 & 10 \\
u_3 = -5 & 12 & 0 & 14 & 16 & 18 \\
\end{array}
\]
Solution – Step 3

- Since all the $c_{pq}$ are nonpositive, the optimum solution has been attained.
- The optimal solution is summarized as follows.
  - Ship 5 units from (source) 1 to (destination) 2 at $5 \times 0 = $0,
    10 units from 1 to 4 at $10 \times 11 = $110,
    10 units from 2 to 2 at $10 \times 7 = $70,
    15 units from 2 to 3 at $15 \times 9 = $135,
    and 5 units from 3 to 1 at $5 \times 0 = $0.

  The total transportation cost of the schedule is $315.

- Initial solution was $410
Service Supply Relationships

- We’ve been talking about transportation of supplies and the importance of the delivery system.
- This is a key aspect in terms of the supplier-customer interaction.
- Now we are going to focus on the supply chain and how its elements interact with one another.
Service Supply Relationships

- Supply chain management is a total systems approach to delivering manufactured products to the end customer.
- The implications for management arise from the fact that service supply relationships are inherently bidirectional and rarely involve more than two levels of interaction.
Supply Chain Management

- Shortened product life cycles and increasing globalization of markets require a total systems view of the entire supply chain.

- Competitive pressures – firms think globally when identifying suppliers and locating manufacturing operations
  - e.g., a PC may have subassemblies manufactured in the US with parts supplied from Asia, and final assembly-to-order near customers in Europe
Supply Chain Management

- The challenge of supply chain management is to balance the requirements of reliable and prompt customer delivery with manufacturing and inventory costs.
- The supply chain is modeled as a network that captures the relationship between asset costs (e.g., inventory and capital equipment) and the time domain characteristics of customer service (e.g., responsiveness and reliability in customer delivery).
Network Model

- The physical goods supply chain can be viewed as a network of value-adding material-processing stages each defined with supply input, material transformation, and demand output.
Customer-Supply Duality

- Nature of services create a customer-supplier duality – result in service supply relationship rather than supply chain associated with manufactured goods (physical object passed among entities).

- Services can be considered as acting on:
  - Minds (e.g., education, entertainment, religion)
  - Bodies (e.g., transportation, lodging, health care)
  - Belongings (e.g., auto repair, dry cleaning, banking)
  - Information (e.g., tax preparation, insurance, legal defense)
Customer-Supply Duality

- All services act on something provided by the customer. The implication is that customers are also acting as suppliers in the service exchange – this is the customer-supplier duality.
## Single-Level Bidirectional Service Supply Relationship

<table>
<thead>
<tr>
<th>Service Category</th>
<th>Customer -Supplier</th>
<th>Input Output</th>
<th>Service Provider</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minds</td>
<td>Student</td>
<td>&gt;Mind Knowledge&gt;</td>
<td>Professor</td>
</tr>
<tr>
<td>Bodies</td>
<td>Patient</td>
<td>&gt;Tooth Filling&gt;</td>
<td>Dentist</td>
</tr>
<tr>
<td>Belongings</td>
<td>Investor</td>
<td>&gt;Money Interest&gt;</td>
<td>Bank</td>
</tr>
<tr>
<td>Information</td>
<td>Client</td>
<td>&gt;Documents 1040&gt;</td>
<td>Tax Preparer</td>
</tr>
</tbody>
</table>
# Two-Level Bidirectional Service Supply Relationship

<table>
<thead>
<tr>
<th>Service Category</th>
<th>Customer -Supplier</th>
<th>&gt;Input Output&gt;</th>
<th>Service Provider</th>
<th>&gt;Input Output&gt;</th>
<th>Provider’s Supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minds</td>
<td>Patient</td>
<td>&gt;Disturbed Treated&gt;</td>
<td>Therapist</td>
<td>&gt;Prescription Drugs&gt;</td>
<td>Pharmacy</td>
</tr>
<tr>
<td>Bodies</td>
<td>Patient</td>
<td>&gt;Blood Diagnosis&gt;</td>
<td>Physician</td>
<td>&gt;Sample Test Result&gt;</td>
<td>Lab</td>
</tr>
<tr>
<td>Belongings</td>
<td>Driver</td>
<td>&gt;Car Repaired&gt;</td>
<td>Garage</td>
<td>&gt;Engine Rebuilt&gt;</td>
<td>Machine Shop</td>
</tr>
<tr>
<td>Information</td>
<td>Home Buyer</td>
<td>&gt;Property Loan&gt;</td>
<td>Mortgage Company</td>
<td>&gt;Location Clear Title&gt;</td>
<td>Title Search</td>
</tr>
</tbody>
</table>

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**Michigan Tech**

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Value Sources in Service Supply Relationships

- **Bi-directional Optimization** –
  - Doing what is best from customer’s perspective while doing the best for the service enterprise.

- **Managing Productive Capacity**
  - Transfer: make knowledge available (e.g., web based FAQ database)
  - Replacement: substitute technology for server (e.g., digital blood pressure device)
  - Embellishment: enable self-service by teaching (e.g., change surgical dressing).

- **Management of Perishability**
  - Minimize negative impact of idle time on productive capacity of distributed service workforce.
Outsourcing Services

- **Benefits**
  - Allows the firm to focus on its core competence
  - Service is cheaper to outsource than perform in-house
  - Provides access to latest technology
  - Leverage benefits of supplier economy of scale

- **Risks**
  - Loss of direct control of quality
  - Jeopardizes employee loyalty
  - Exposure to data security and customer privacy
  - Dependence on one supplier compromises future negotiation leverage
  - Additional coordination expense and delays
  - Atrophy of in-house capability to perform service
## Taxonomy for Outsourcing Business Services

<table>
<thead>
<tr>
<th>Importance of Service</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Property</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Facility Support:</td>
<td>-Laundry</td>
<td>-Repairs</td>
</tr>
<tr>
<td>-Janitorial</td>
<td>-Maintenance</td>
<td></td>
</tr>
<tr>
<td>-Waste disposal</td>
<td>-Product testing</td>
<td></td>
</tr>
<tr>
<td><strong>Equipment Support:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Repairs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Maintenance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Product testing</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>People</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employee Support:</td>
<td>-Food service</td>
<td>-Training</td>
</tr>
<tr>
<td>-Plant security</td>
<td>-Education</td>
<td></td>
</tr>
<tr>
<td>-Temporary personnel</td>
<td>-Medical care</td>
<td></td>
</tr>
<tr>
<td><strong>Service Process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Facilitator:</td>
<td>-Bookkeeping</td>
<td>-Adverting</td>
</tr>
<tr>
<td>-Travel booking</td>
<td>-Public relations</td>
<td></td>
</tr>
<tr>
<td>-Packaged software</td>
<td>-Legal</td>
<td></td>
</tr>
<tr>
<td><strong>Professional</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Advertising</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Public relations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Legal</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Outsourcing Considerations

Focus on Property

Facility Support Service

- Low cost
- Identify responsible party to evaluate performance
- Precise specifications can be written

Equipment Support Service

- Experience and reputation of vendor
- Availability of vendor for emergency response
- Designate person to make service call and to check that service is satisfactory
Outsourcing Considerations

Focus on People

Employee Support Service

- Contact vendor clients for references
- Specifications prepared with end user input
- Evaluate performance on a periodic basis

Employee Development Service

- Experience with particular industry important
- Involve high levels of management in vendor identification and selection
- Contact vendor clients for references
- Use employees to evaluate vendor performance
Outsourcing Considerations

Focus on Process

Facilitator Service

• Knowledge of alternate vendors important
• Involve end user in vendor identification
• References or third party evaluations useful
• Have user write detailed specifications

Professional Service

• Involve high level management in vendor identification and selection
• Reputation and experience very important
• Performance evaluation by top management
Outsourcing Services

- Is outsourcing the best option out there in terms of improving efficiency? How about the environmental impacts?
- Shift impacts from industrialized world to developing world?
- Outsourcing concept does stimulate our thinking about “who is doing what” – how we are satisfying customer needs
- Perhaps needs traditionally met via goods and products can be met through services – to be discussed in next class...