**13.0 BICYCLE EXAMPLE**

(Updated Spring 2005)

Interested in time it takes to pedal up a hill. 7 variables of interest:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Set</td>
<td>Up</td>
<td>Down</td>
</tr>
<tr>
<td>2-Dynamo</td>
<td>Off</td>
<td>On</td>
</tr>
<tr>
<td>3-Handlebars</td>
<td>Up</td>
<td>Down</td>
</tr>
<tr>
<td>4-Gear</td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>5-Raincoat</td>
<td>On</td>
<td>Off</td>
</tr>
<tr>
<td>6-Breakfast</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>7-Tires</td>
<td>Hard</td>
<td>Soft</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>12</th>
<th>13</th>
<th>23</th>
<th>123</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Introduce variables 4,5,6,&7 as follows:

4=12 I=124
5=13 => I=135 Generators
6=23 I=236
7=123 I=1237

Selected generator member of family: I= ± 124 = ± 135 = ± 236 = ± 1237

Defining relationship:
I=124=135=236=1237
2@ a time =2345=1346=347=1256=257=167
3@ a time =456=1457=2467=3567
Run the experiment.

<table>
<thead>
<tr>
<th>Test</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>69</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>52</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>83</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>71</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>88</td>
</tr>
</tbody>
</table>

Use Base design calc. matrix to calc l_i’s

\[
l_1 = \frac{-69 + 52 - 60 + 83 - 71 + 50 - 59 + 88}{4} = 3.5
\]
\[
l_{123} = \frac{-69 + 52 + 60 - 83 + 71 - 50 - 59 + 88}{4} = 2.5
\]
\[
l_1 = 66.5 \text{ est } 1
\]
\[
l_1 = 3.5 \text{ est } 1 + 24 + 35 + 67
\]
\[
l_2 = 12.0 \text{ est } 2 + 14 + 36 + 57
\]
\[
l_3 = 1.0 \text{ est } 3 + 15 + 26 + 47
\]
\[
l_{12} = 22.5 \text{ est } 12 + 4 + 37 + 56
\]
\[
l_{13} = 0.5 \text{ est } 13 + 5 + 27 + 46
\]
\[
l_{23} = 1.0 \text{ est } 23 + 6 + 17 + 45
\]
\[
l_{123} = 2.5 \text{ est } 34 + 25 + 16 + 7
\]

- Could plot l’s on NPP to find important ones
- In this case, we know from past experiments that \( \sigma_y = 3 \)

\[
\sigma_e^2 = \sigma_{effect}^2 = \frac{4\sigma_y^2}{N}
\]

therefore

\[
\sigma_e = \left[ \frac{4 \cdot 3^2}{8} \right]^{\frac{1}{2}} = \sqrt{4.5} \approx 2.1
\]

It can be easily found out that \( l_2 \) and \( l_{12} \) are statistically significant with 95% confidence.
Simple conclusion: $\mu_2$ & $\mu_4$ are important
Could also be: $\mu_2$ & $\mu_{12}$ important or $\mu_2$ & $\mu_{14}$ important or...

To unconfound, sort out what is/is not important, let’s run another 8 tests. Overlaying our knowledge on the experiment results suggests that probably 4 & 14 is important, 2nd experiment will unconfound 4 and its interactions.

<table>
<thead>
<tr>
<th>Test</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>63</td>
</tr>
<tr>
<td>10</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>82</td>
</tr>
<tr>
<td>11</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>73</td>
</tr>
<tr>
<td>12</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>53</td>
</tr>
<tr>
<td>13</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>64</td>
</tr>
<tr>
<td>14</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>84</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>72</td>
</tr>
<tr>
<td>16</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>45</td>
</tr>
</tbody>
</table>

Signs of column 4 in this recipe matrix are opposite/flipped/ “folded” from those of 1st recipe matrix. All the other signs are the same. Let’s examine the recipe matrix to see what the generators are for this 2nd $2^{7-4}$ design. 4= -12, 5=13,6=23,7=123. So the defining relation for the 2nd design is:

\[
I=-124=135=236=1237 \\
=-2345=-1346=-347=1256=257=167 \\
=-456=-1457=-2467=3567=-1234567
\]

Therefore,
\[
l_1' = 67.0 \text{ is the estimate of the effect of I} \\
l_1' = -2.0 \text{ is the estimate of the effect of 1-24+35+67} \\
l_2' = -12.5 \text{ is the estimate of the effect of 2-14+36+57} \\
l_3' = -1.5 \text{ is the estimate of the effect of 3+15+26-47} \\
l_{12}' = -21.5 \text{ is the estimate of the effect of 12-4+37+56} \\
l_{13}' = -1.5 \text{ is the estimate of the effect of 13+5+27-46}
\]
\[ l_{23}' = -3.0 \text{ is the estimate of the effect of } 23+6+17-45 \]
\[ l_{123}' = -2.0 \text{ is the estimate of the effect of } -34+25+16+7 \]

\[
\frac{(l_1 + l_1')}{2} = 66.75 \text{ est } I \quad \frac{(l_1 - l_1')}{2} = -0.5 \text{ est Higher Order Terms} \\
\frac{(l_2 + l_2')}{2} = 0.75 \text{ est } 1+35+67 \quad \frac{(l_1 - l_1')}{2} = 2.75 \text{ est } 24 \\
\frac{(l_2 + l_2')}{2} = -0.25 \text{ est } 2+36+57 \quad \frac{(l_2 - l_2')}{2} = 12.25 \text{ est } 14 \\
\frac{(l_3 + l_3')}{2} = -0.25 \text{ est } 3+15+26 \quad \frac{(l_3 - l_3')}{2} = 1.25 \text{ est } 47 \\
\frac{(l_{12} + l_{12}')}{2} = 0.5 \text{ est } 12+37+56 \quad \frac{(l_{12} - l_{12}')}{2} = 22.0 \text{ est } 4 \\
\frac{(l_{13} + l_{13}')}{2} = -0.5 \text{ est } 13+5+27 \quad \frac{(l_{13} - l_{13}')}{2} = 1.0 \text{ est } 46 \\
\frac{(l_{23} + l_{23}')}{2} = -1.0 \text{ est } 23+6+17 \quad \frac{(l_{23} - l_{23}')}{2} = 2.0 \text{ est } 45 \\
\frac{(l_{123} + l_{123}')}{2} = 0.25 \text{ est } 25+16+7 \quad \frac{(l_{123} - l_{123}')}{2} = 2.25 \text{ est } 34 \\
\]

It can be seen that all interactions and main effects of 4 are unconfounded.

If \( \sigma_y = 3, \sigma_{eff} = \left( \frac{4\sigma^2}{N} \right)^{\frac{1}{2}} = \left( \frac{4 \cdot 3^2}{16} \right)^{\frac{1}{2}} = 1.5 \). So, average (I), 14, and 4 are important and estimated to be 66.75, 12.25, and 22.0 respectively.

Model for response is:

\[ \hat{y} = 66.75 + \frac{22.0}{2}x_4 + \frac{12.25}{2}x_1x_4 \]

**Summary:**

- First we conducted the “principal fraction” from the family of generators

  \[ I = \pm 124 = \pm 135 = \pm 236 = \pm 1237, \text{ i.e., we used the generators with all “+” signs:} \]
  \[ 4=12,5=13,6=23,7=123 \]

- Second we conducted another 2\(^{7-4}\) design with generators 4=–12,5=13,6=23,7=123. We noted that the recipe matrix for this 2nd design was identical to that of our first design except that column 4 was folded. We saw that the results of this fold-over design when combined with those of the 1st design unconfounded the effects of 4 and all of its 2-factor interactions.

Another type of follow-up design - to clear up confounding left by a 1st design is the Mirror Image Design which flips all signs in the recipe matrix
of the 1st design.

<table>
<thead>
<tr>
<th>Test</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>14</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>15</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>16</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The mirror image design when combined with the 1st design will unconfound all the main effects. Let’s examine the recipe matrix to see what the generators happen to be:

\[ 4 = -12, 5 = 13, 6 = 23, 7 = 123. \]

So defining relation is:

\[ I = -124 = -135 = -236 = 1237 \]
\[ = 2345 = 1346 = 1256 = -257 = -167 \]
\[ = -456 = 1457 = 2467 = 356 \]
\[ = -1234567 \]

Therefore, assuming 3-factor interactions and higher are negligible,

<table>
<thead>
<tr>
<th>1st Design</th>
<th>2nd Design: Mirror Image Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_I ) est I</td>
<td>( l_I' ) est I</td>
</tr>
<tr>
<td>( l_1 ) est 1+24+35+67</td>
<td>( l_1' ) est 1-24-35-67</td>
</tr>
<tr>
<td>( l_2 ) est 2+14+36+57</td>
<td>( l_2' ) est 2-14-36-57</td>
</tr>
<tr>
<td>( l_3 ) est 3+15+26+47</td>
<td>( l_3' ) est 3-15-26-47</td>
</tr>
<tr>
<td>( l_{12} ) est 12+4+37+56</td>
<td>( l_{12} ) est 12-4+37+56</td>
</tr>
<tr>
<td>( l_{13} ) est 13+5+27+46</td>
<td>( l_{13} ) est 13-5+27+46</td>
</tr>
<tr>
<td>( l_{23} ) est 23+6+17+45</td>
<td>( l_{23} ) est 23-6+17+45</td>
</tr>
<tr>
<td>( l_{123} ) est 34+25+16+7</td>
<td>( l_{123} ) est -34-25-16+7</td>
</tr>
</tbody>
</table>
By combining the two designs, we can obtain

\[
\frac{1}{2} (l_1 + l_2) \text{ est } 1 \quad \frac{1}{2} (l_3 + l_3) \text{ est } 3
\]

\[
\frac{1}{2} (l_{12} + l_{12}) \text{ est } 12 + 37 + 56 
\]

\[
\frac{1}{2} (l_{13} + l_{13}) \text{ est } 13 + 27 + 46 
\]

\[
\frac{1}{2} (l_{23} + l_{23}) \text{ est } 23 + 17 + 45 
\]

\[
\frac{1}{2} (l_{123} + l_{123}) \text{ est } 7
\]

\[
\frac{1}{2} (l_1 - l_1) \text{ est Higher Order Eff. } 
\]

\[
\frac{1}{2} (l_1 - l_1) \text{ est } 24 + 35 + 67 
\]

\[
\frac{1}{2} (l_1 - l_1) \text{ est } +14 + 36 + 57 
\]

\[
\frac{1}{2} (l_3 - l_3) \text{ est } 15 + 26 + 47 
\]

\[
\frac{1}{2} (l_{12} - l_{12}) \text{ est } 4 
\]

\[
\frac{1}{2} (l_{13} - l_{13}) \text{ est } 5 
\]

\[
\frac{1}{2} (l_{23} - l_{23}) \text{ est } 6 
\]

\[
\frac{1}{2} (l_{123} - l_{123}) \text{ est } 34 + 25 + 16 
\]