Lecture #11

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A Quick Note on Notation
Decision Errors in Hyp. Testing
(Reconsider the Dist. of Individuals - X’s)
Hypothesis Testing - Cutoff Values

\[ Z_{\alpha/2} = Z_{0.025} = -1.96 \]
\[ Z_{1 - \alpha/2} = Z_{0.975} = 1.96 \]

\[ Z_{\alpha/2} = \frac{(X_L - \mu_x)}{\sigma_x} = \frac{(X_L - 160)}{10} = -1.96 \]
\[ -19.6 + 160 = X_L = 140.4 \]

\[ Z_{1 - \alpha/2} = \frac{(X_U - \mu_x)}{\sigma_x} = \frac{(X_U - 160)}{10} = 1.96 \]
\[ 19.6 + 160 = X_U = 179.6 \]
Beta Errors

![Diagram showing Beta Errors]

- True State of Affairs
- $X_L$ and $X_U$
- Reject $H_0$
- Can’t Reject $H_0$
- $\beta$

130 140 150 160 170 180 190
Risk & Errors

\( \alpha = \) Producer’s Risk

Wrongly Reject Ho: Type 1 Error

\( \beta = \) Consumer’s Risk

Don’t Reject Ho when we should: Type 2 Error
"Wrongly Accepting Ho"
## Decision Errors (Summary)

<table>
<thead>
<tr>
<th>True State</th>
<th>Claim True</th>
<th>Claim False</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can’t Reject Ho</td>
<td>No Error</td>
<td>β Risk Type 2</td>
</tr>
<tr>
<td>Reject Ho</td>
<td>α Risk Type 1</td>
<td>No Error</td>
</tr>
</tbody>
</table>

- **α Risk**: Type 1
- **β Risk**: Type 2
Judicial System Analogy

<table>
<thead>
<tr>
<th>True State</th>
<th>Innocent</th>
<th>Guilty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Guilty</td>
<td>No Error</td>
<td>$\beta$ Risk</td>
</tr>
<tr>
<td>Guilty</td>
<td>$\alpha$ Risk</td>
<td>No Error</td>
</tr>
</tbody>
</table>

Trial -- Jury's Decision

- No Error
- $\alpha$ Risk
- Type 1
- $\beta$ Risk
- Type 2
More on alpha/beta risk

Reject $H_0$

Can’t Reject $H_0$

Reject $H_0$
Let’s Make a Graph

Probability - No Rejection

True Process Mean
Completed Graph

[Graph showing the probability of no rejection against the true mean. The X-axis represents the true mean ranging from -3 to 3, and the Y-axis represents the probability ranging from 0 to 1. The graph is a bell-shaped curve, indicating a normal distribution.]
Hypothesis Testing - Example #3

Because of our continuing concern about the temperature of soup at the MUB - let’s devise a scheme to continuously monitor it.

Let’s perform a hypothesis test for each $\bar{X}$
Hypothesis Testing of Multiple Sample Means

Recall that the claimed average soup temperature is 160 deg.

Mean = 160
S.D. = 10
X’s normal
Hypothesis Testing -- Example #3

Mean = 160

\[ \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{10}{2} = 5 \]

X’s normal -- Xbars normal

Under H_0

Distribution of sample means

\[ \bar{x} \]
Example #3

\[ H_0 : \mu_x = 160 \quad H_A : \mu_x \neq 160 \quad \alpha = 0.05 \]

So, for every sample mean we obtain from the MUB (say 175), we must:

- Calculate Z
- Find probability of getting a value more extreme than that Z
- Compare the calculated probability to \( \alpha/2 \) - reject claim if calc. prob. is less than \( \alpha/2 \).
Example #3 - comments

What a hassle! There is no way we wish to calculate a separate Z for each sample mean and then look up the probability for each Z.

Another approach: let’s define limits for distribution of sample means - if an Xbar is beyond the limits - reject claim. If an Xbar is within limits, can’t reject claim.
Rejection Limits

\[ \bar{x} \pm 3 \bar{\sigma}_x = 5 \]

\( \bar{x}_{lo} \) and \( \bar{x}_{hi} \) are the rejection limits under \( H_0 \).

Cannot Reject \( H_0 \)

Reject \( H_0 \)

\( \mu_\bar{x} \) is the mean of the distribution.
Rejection Limits - continued

We want to pick values for $\bar{X}_\text{lo}$ and $\bar{X}_\text{hi}$.

Any sample mean larger than $\bar{X}_\text{hi}$ should fail the hypothesis test, i.e., the probability of a value this large or larger is less than $\alpha/2$.

So, let’s place $\bar{X}_\text{hi}$ so that it places $\alpha/2$ in the tail of the distribution.

$$\Pr(\bar{X} \geq \bar{X}_\text{hi}) = \alpha/2 = 0.025$$

$\bar{X}_\text{hi}$ corresponds to $Z_{0.975}$
Example #3 -- continued

\( \bar{X}_{hi} \) corresponds to \( Z_{1 - \alpha/2} = Z_{0.975} = 1.96 \)

\( \bar{X}_{lo} \) corresponds to \( Z_{\alpha/2} = Z_{0.025} = -1.96 \)

Given the above,

\[
\frac{\bar{X}_{hi} - \mu}{\sigma_{\bar{x}}} = Z_{1 - \alpha/2} \quad \frac{\bar{X}_{lo} - \mu}{\sigma_{\bar{x}}} = Z_{\alpha/2}
\]

Solving for \( X_{lo} \) and \( X_{hi} \) gives 150.2 and 169.8
Rejection Limits

Under $H_0$

Reject $H_0$

Can’t Reject $H_0$

145 150 155 160 165 170 175

150.2 169.8

$\bar{x}$