Lecture #12

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Hypothesis Testing - Example #3

Because of our continuing concern about the temperature of soup at the MUB - let’s devise a scheme to continuously monitor it.

Let’s perform a hypothesis test for each $\bar{X}$
Hypothesis Testing of Multiple Sample Means

Recall that the claimed average soup temperature is 160 deg.

Mean = 160
S.D. = 10
X’s normal

Under $H_0$
Hypothesis Testing -- Example #3

Mean = 160

\[ \sigma_\bar{x} = \frac{\sigma_x}{\sqrt{n}} = \frac{10}{2} = 5 \]

X’s normal -- Xbars normal

Under H₀

Distribution of sample means

\[ \bar{x} \]
Example #3

\[ H_0: \mu_x = 160 \quad H_A: \mu_x \neq 160 \quad \alpha = 0.05 \]

So, for every sample mean we obtain from the MUB (say 175), we must:

- Calculate Z
- Find probability of getting a value more extreme than that Z
- Compare the calculated probability to \( \alpha/2 \) - reject claim if calc. prob. is less than \( \alpha/2 \).
Example #3 - comments

What a hassle! There is no way we wish to calculate a separate Z for each sample mean and then look up the probability for each Z.

Another approach: let’s define limits for distribution of sample means - if an Xbar is beyond the limits - reject claim. If an Xbar is within limits, can’t reject claim.
Rejection Limits

\[ \bar{x} \pm \sigma_{\bar{x}} = 150 \pm 5 \]

\begin{align*}
\text{Reject } H_0 & \quad \text{Under } H_0 \\
\text{Can’t Reject } H_0 & \quad \text{Reject } H_0
\end{align*}
Rejection Limits - continued

We want to pick values for $\bar{X}_{lo}$ and $\bar{X}_{hi}$.

Any sample mean larger than $\bar{X}_{hi}$ should fail the hypothesis test, i.e., the probability of a value this large or larger is less than $\alpha/2$.

So, let’s place $\bar{X}_{hi}$ so that it places $\alpha/2$ in the tail of the distribution.

$$\Pr(\bar{X} \geq \bar{X}_{hi}) = \alpha/2 = 0.025$$

$\bar{X}_{hi}$ corresponds to $z_{0.975}$.
Example #3 -- continued

\( \bar{X}_{hi} \) corresponds to \( Z_{1 - \alpha/2} = Z_{0.975} = 1.96 \)

\( \bar{X}_{lo} \) corresponds to \( Z_{\alpha/2} = Z_{0.025} = -1.96 \)

Given the above,

\[
\frac{\bar{X}_{hi} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = Z_{1 - \alpha/2} \quad \frac{\bar{X}_{lo} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = Z_{\alpha/2}
\]

Solving for Xlo and Xhi gives 150.2 and 169.8
Rejection Limits

Rejection limits are shown with values under H₀ and above H₀ indicating when to reject or not reject the null hypothesis. The graph shows a normal distribution curve with rejection limits set at specific points.

- Reject H₀ at 150.2
- Can’t Reject H₀ in the central region
- Reject H₀ at 169.8
Chapter #4

- The origins and nature of variability
- Process evolution over time
- Shewhart’s concept of statistical control
- Managing variability using control charts
- The process of statistical process control
Origin and Nature of Variability

We focus on variation in product function

- Outer Noise - external sources or environmental effects
- Inner Noise - internal changes (wear, aging, etc.)
- Variational Noise - uncertainties due to manufacturing

Consider a baseball bat . . . .