So, where do we stand??

Monitor changes in $\bar{X}$ and $R$ as a function of time

Samples of size n

11 AM  10 AM  9 AM  8 AM  ....

$\bar{X}$  $\bar{X}$  $\bar{X}$  $\bar{X}$

$R$  $R$  $R$  $R$
Shewhart Control Chart Model

\[ \overline{X} = \frac{1}{n} \sum X \]

\[ R = \text{Range} \]

\[ \text{UCL}_{X} = \overline{X} + 3\overline{\sigma} \]
\[ \text{LCL}_{X} = \overline{X} - 3\overline{\sigma} \]

\[ \text{UCL}_{R} = D_4 \overline{R} \]
\[ \text{LCL}_{R} = D_3 \overline{R} \]

\[ 3\overline{\sigma} = R \]

Where:
- \( \overline{X} \) is the mean of the sample means.
- \( R \) is the range of the samples.
- \( \overline{\sigma} \) is the average standard deviation of the samples.
- \( D_4 \) and \( D_3 \) are constants based on sample size.

These charts are used to monitor process stability and identify patterns or trends that may indicate a change in the process.

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Control Chart - Hypothesis Test

Control limits depend, of course, on the risk level, $\alpha$.
Where do we put the limits??

Centerline

Put the limits 3 standard deviations from the mean
Limits??

Centerline

Put the limits 3 standard deviations from the mean
Chapter 6 - Making Control Charts

<table>
<thead>
<tr>
<th>Sample</th>
<th>$X_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>k</td>
<td>5</td>
</tr>
</tbody>
</table>

\[
X = \frac{1}{k} \sum_{i=1}^{k} \bar{X}_i/k
\]

\[
\bar{R} = \frac{1}{k} \sum_{i=1}^{k} R_i/k
\]
Chart Guidelines

- \( \bar{x} \) = \( x \) - Range

- \( R \) = UCL - LCL

UCL
LCL
Xbar Chart Details

\[
\mu \overline{X} \pm 3\sigma \overline{X} = \overline{X} \pm 3\sigma \overline{X} \quad \sigma \overline{X} = ??
\]

Control Limits

\[
\overline{X} \pm A_2 \overline{R} = \overline{X} \pm A_2 \overline{R}
\]
R Chart Details

\[
\mu_R \pm 3\sigma_R \quad \bar{R} \pm 3\sigma_R
\]

Control Limits

\[UCL : D_4 \bar{R} \quad LCL : D_3 \bar{R}\]
Control Charts (cont.)

Now we know how to construct control charts!!

We know to look for points beyond the limits -- is this enough?

How do we interpret the control charts??

Remember, our goal is to learn about the process -- we want to extract as much information as possible from the charts.
Interpreting Control Charts

• We will look for more than just points beyond the limits, i.e., extreme points.

• Behavior that suggests presence of special causes.
  - Trends / cyclic behavior
  - High proportion of points near / beyond the limits
  - Sudden shifts in level
  - In general, any non-random behavior
8 Rules for Chart Interpretation

- Test 1: Extreme points
- Test 2: 2 out of 3 points in zone A or beyond
- Test 3: 4 out of 5 points in zone B or beyond
- Test 4: Runs above / below the centerline
- Test 5: Linear trend
- Test 6: Oscillatory trend
- Test 7: Avoidance of zone C
- Test 8: Run in zone C
Figure 6.2 Appearance of a Process in Good Statistical Control
Figure 6.3 Control Chart Zones to Aid Chart Interpretation
Figure 6.4 Examples of Test 1: Extreme Points