Lecture # 28

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Another Example

Three identical parts

Manufacturing process -- $C_p = 1.5$ (at least)
What will the individual process $\sigma_x$ be, if tolerances are obtained by simple division?
Find (i) standard deviation for the processes (ii) tolerances for individual parts
Another Example

For now, let’s assume hole & pin producing processes are centered at the nominal values and that processes have Cp values of 1.0.

What does clearance dist. look like??
Clearance Distribution

\[ C = H - P \]
\[ \mu_C = \mu_H - \mu_P = 0.035 \]
\[ \sigma_C^2 = \sigma_H^2 + \sigma_P^2 \quad ---- \quad \sigma_C = 0.007 \]
What can go Wrong?

• We have already seen that if our individual processes (in this case pin and hole) do not remain centered - the results can be disastrous.

• What if our processes are not maintained in a state of statistical control?
Histogram - Hole Dimension

Frequency

Hole Diameter

0.9865 0.9895 0.9925 0.9955 0.9985 1.0015 1.0045 1.0075 1.0105 1.0135
Histogram - Pin Dimension

Pin Diameter

Frequency

0.9515 0.9545 0.9575 0.9605 0.9635 0.9665 0.9695 0.9725 0.9755 0.9785
Histogram - Clearance Dimension

About 91.4% Capable
Capability Assessment via a Loss Function

Quality loss

Quadratic Loss Function

Nominal Quality characteristic

x
Defining the Loss Function

\[ L = L_0 + k(x - m)^2 \]

- for an \( x \) value of 0, the loss, \( L \), is $0. This means that \( m = 0 \) and \( L_0 = 0 \)

- when \( x = 1.5 \), the loss, \( L \), is $2.25
  \[ \frac{L}{(x-m)^2} = k \quad k = \frac{2.25}{(1.5 - 0)^2} = 1 \]

  The $2.25 loss is obtained from the fact that when \( x = 1.5 \), the probability of a $45 complaint is 5%, and
  \[ $45 \times (0.05) = $2.25 \]
Loss Function Interpretation of Engineering Specifications

Cost to Rework/Replace

Quality loss

LS  m  US

x
More on Loss / Specifications

So, we want to set our specifications at the point of indifference: where the loss = cost of replacement

If cost of replacement = $2 for our previous example,

\[ L = 1 \ (x - 0)^2 = 2 \quad \text{-----} \quad x = \pm 1.414 \]
More on the Loss Function

Can we come up with a loss associated with this distribution?

Quality loss

$m \quad 1.5$

$\$2.25$

$x$

Can we come up with a loss associated with this distribution.
Expected Process Loss

\[ E[L(X)] = \int L(t)f(t) \, dt \]

where,

\[ L(x) = k(x - m)^2 \]

and \( f(t) \) is the pdf

after simplification,

\[ E[L(X)] = k \left\{ (\mu_X - m)^2 + \sigma_X^2 \right\} \]
Loss Function: Example #1

$m=0$, $k=1.0$, LSL=$-1$, USL=1,
$X$ normal w/ mean 0 and $\sigma_x = 0.33$

$E[L] = 1 \left[ 0.33^2 + (0-0)^2 \right] = 0.11$

How to interpret??
Loss Function: Example #2

\[ \sigma_x = 0.58 \]

\[ E[L] = 1 \left[ 0.58^2 + (0-0)^2 \right] = $0.33 \]
Filling in a Past Blank

Should know from prior Statistics Class
For a Uniform Distribution between a & b

\[ f(x) = \frac{1}{(b - a)} \]

\[ \mu_x = \int_a^b x f(x) \, dx = \int_a^b \frac{x}{b - a} \, dx = \frac{x^2}{2(b - a)} \bigg|_a^b = \frac{b + a}{2} \]

\[ \sigma_x^2 = \int_a^b (x - \mu_x)^2 f(x) \, dx = \frac{(b - a)^2}{12} \]
Loss Function: Example #3

Say we reduce $\sigma_x = 0.33$ to $\sigma_x = 0.10$

$E[L] = 1 \left[ 0.10^2 + (0-0)^2 \right] = 0.01$
Loss Function: Example #4

Can save $0.10 by reducing process mean from 0 to -0.7

$$E[L] = 1 \left[0.10^2 + (-0.7 - 0)^2\right] = $0.50 \quad \text{WOW!!}$$

Taguchi’s comment: