Last time:

Let’s say we draw 5 items.

\[ P(5D) = 0.0000003 \]
\[ P(4D,1N) = 0.0000297 \]
\[ P(3D,2N) = 0.0011 \]
\[ P(2D,3N) = 0.0214 \]
\[ P(1D,4N) = 0.2036 \]
\[ P(5N) = 0.7735 \]
The General Form

\[ P(d) = \binom{n}{d} (p')^d (1 - p')^{n-d} \]

where, \( \binom{n}{d} = \frac{n!}{d!(n-d)!} \)

what if \( \binom{1000000}{2} = ? \)
So, when $n = 5$, $p' = .10$, $P(d=1) = ??$

When $n = 10$, $p' = .10$, $P(d=3) = ??$
b(d; n=5, p' = 0.05)
Mean and Variance

\[ \mu_d = np' \]

\[ \sigma^2_d = np'(1 - p') \]

\[ \mu_p = E[p] = \]

\[ \sigma^2_p = Var[p] \]
Control Limits - p Chart

In principle, $\mu_p \pm 3\sigma_p$

We don’t know $\mu_p$ or $\sigma_p$ so we must estimate them.

Form of the control limits:

$\hat{\mu}_p \pm \hat{\sigma}_p$

Estimating $\mu_p$ for case of constant sample size, n
Estimating $\mu_p$ for case of varying sample size, $n$

Estimating $\sigma_p$
p (fraction defective) Chart

• Return to Injection molding example (n=100); instrument panels (flash, splay, voids, & short shots)

• 30 samples collected (100 panels per shift)

• d recorded for each sample; p = d/n calculated

\[ \bar{p} = 0.079 \]

\[ \sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.027 \]
LCL, UCL

Use 4 rules to interpret chart.

Process stable!!

But average fraction defective is high -- 8%.

Common cause problem. Flash accounts for 50% of the defects.

Mold pressure identified as the problem -- process settings adjusted.
30 addl. samples collected

Data plotted on existing charts -- interpretation

\[ \bar{p} = 0.038 \]

\[ \sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0191 \]

LCL, UCL
Control Chart for Number of Defectives

- Sometimes, more convenient to make chart by plotting $d$ rather than $p$.

- In particular, plotting $d$ appropriate if
  - $n$ is constant
  - $p$ is very small

- Referred to as “np” chart. Basically same as $p$ chart except for scaling factor, $n$. 
• np chart requires one less calculation since \( p = d/n \) need not be calculated for each sample

• Recall that

\[
\mu_d = np'
\]

\[
\sigma_d^2 = np'(1 - p') \quad \text{or} \quad \sigma_d = \sqrt{np'(1 - p')}
\]

Form of the control limits (use \( \bar{p} \) to estimate \( p' \)):

\[
np \pm 3\sqrt{np(1 - \bar{p})}
\]
Revisit the instrument panel assembly example.

\[
\bar{p} = \frac{237}{3000} = 0.079
\]

\[
n\bar{p} = 100(0.079) = 7.9
\]

\[
n\bar{p} \pm 3 \sqrt{n\bar{p}(1 - \bar{p})}
\]

\[
7.9 \pm 3 \sqrt{7.9(1 - 0.079)}
\]

\[
\text{UCL}_{np} = 16.00
\]

\[
\text{LCL}_{np} = -0.20 \quad \text{so use a lower limit of 0.0}
\]
Problems with np Chart

- If sample size varies, control limits will vary from sample to sample
- If sample size varies, centerline will vary as well from sample to sample
- This makes chart interpretation difficult
- d value (easier calculations) easy to communicate -- but what does a value of d = 12 mean?? Doesn’t mean much unless you also know sample size
Variable-Sample-Size $p$ Charts

- Speaking of dealing with varying sample sizes, how do we manage this for our $p$ Chart????

- Good news, $p$ will not change from sample to sample, but control limits depend on $n$. How to handle:
  - Separate limits for each subgroup
  - Limits based on $n$-bar
  - Standardized $p$-chart
Manufacture of a Package Tray

Wood fiber/polymer composite extruded into thin sheets for use in hatchbacks. Sheets heated then molded into shape of tray. Carpet applied to sheet w/adhesive. Carpeted tray trimmed to size.

200 - 300 trays per shift.

Defects observed: cloth coverage on hinge, carpet coverage problem, bleed-through, soiled carpet, improper flex on hinge, (chips, scratches, abrasions on back surface), wrinkles
Data for Package Tray Example

<table>
<thead>
<tr>
<th>Samp.</th>
<th>n</th>
<th>d</th>
<th>p</th>
<th>( \hat{\sigma}_p )</th>
<th>LCL(_p)</th>
<th>UCL(_p)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>238</td>
<td>11</td>
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<td></td>
<td></td>
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<tr>
<td>2</td>
<td>245</td>
<td>18</td>
<td>0.073</td>
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<tr>
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<td>0.063</td>
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<td>:</td>
<td>:</td>
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<td>:</td>
</tr>
</tbody>
</table>
p Chart - Individual Limits

sum of the n’s = 7433

\[
p = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i} = \frac{582}{7433} = 0.0783
\]

\[
\bar{p} \pm 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}
\]

For sample 1, 0.0783 ± 0.0527 : 0.026, 0.131
p Chart --- n-bar Limits

- Plot the p’s and \( \bar{p} \). Put limits on the chart based on \( \bar{n} \).

\[
\bar{n} = \left( \frac{\sum_{i=1}^{k} n_i}{k} \right) / k = \frac{7433}{30} = \text{approx. 248}
\]

\[
\bar{p} \pm 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0783 \pm 3 \sqrt{\frac{0.0783(1-0.0783)}{248}}
\]

Limits are 0.0271 & 0.1295
Pts near the limits must be interpreted separately.
### p Chart --- Standardized Limits

- Calc. standardized p value for each sample

<table>
<thead>
<tr>
<th>Sample</th>
<th>p</th>
<th>p - pbar</th>
<th>n</th>
<th>( \sigma_p )</th>
<th>( Z=(p - pbar)/\sigma_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.046</td>
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<td>238</td>
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