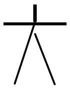 -Designs

(Simple t-Designs with large t: A survey)

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A t–design or t–(v,k,λ) design is a pair $(\mathcal{X},\mathcal{B})$ where: $\mathcal{X}$ is a v–element set of *points*; $\mathcal{B}$ is a family of k–elements subsets of $\mathcal{X}$, called *blocks* and every t–element subset $T \subseteq \mathcal{X}$ is contained in exactly λ blocks. It is said to be *simple* or to have *no repeated blocks* if all the members of $\mathcal{B}$ are distinct. This survey is concerned only with simple designs.

For example a 2–(7,3,1) design $(\mathcal{X},\mathcal{B})$ is given by:

$$\mathcal{X} = \{0,1,2,3,4,5,6\} \text{ and } \mathcal{B} = \{126, 235, 346, 056, 145, 024, 013\}.$$

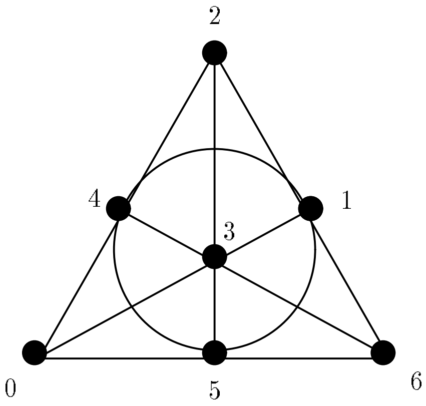

Figure 1: the 2–(7,3,1) design

This design is represented by the picture found in figure 1.

An elementary counting argument shows that it is necessary that

$$\binom{k-i}{t-i} \text{ divides } \binom{v-i}{t-i}\lambda \text{ for all } i = 0,1,\ldots,t.$$

These conditions are referred to as the *necessary* conditions and if the parameters t, v, k, λ satisfy them we say that they are *admissible*. Perhaps it was Denniston in 1980 who first discussed the problems that arise when t is large. What do I mean by large? Well, careful examination of the settling of admissible parameters, Tables I,II,III and IV, and also of the techniques used has made me realize that there is a definite change in amount settled and a definite change in the techniques available when t changes from less than 4, to 4 and 5 and to greater than 5. Thus I classify t as *small* when $t \leq 3$, *middle* when $t \in \{4, 5\}$ and *large* when $t \geq 6$. In this survey then, I will concentrate only on t–designs with $t \geq 6$.

Table I: A summary of the parameters settled when $k = t + 1$ and $\lambda = 1$

$t = 1$: exist if and only if $v \equiv 0 \bmod 2$.

$t = 2$: exist if and only if $v \equiv 1$ or $3 \bmod 6$ (Kirkman 1847).

$t = 3$: exist if and only if $v \equiv 2$ or $4 \bmod 6$ (Hanani 1960).

$t = 4$: Only ones known have:
$v = 23, 47$ and 83 (Denniston 1976);
$v = 71$ (Mills 1978) and
$v = 107, 131$ (Grannell, Griggs and Mathon 1993).

The smallest unsettled parameter set is 4–(17,5,1).

$t = 5$: Only ones known have:
$v = 12$ (Carmichael 1937, Witt 1938);
$v = 24, 48$ and 84 (Denniston 1976);
$v = 72$ (Mills 1978) and
$v = 108, 132$ (Grannell, Griggs and Mathon 1993).

The smallest unsettled parameter set is 5–(18,6,1).

$t \geq 6$: None are known.

The smallest unsettled parameter set is 6–(19,7,1).

Table II: A summary of the parameters settled when $k \neq t+1$ and $\lambda = 1$

$t = 1$:	exist if and only if $v \equiv 0 \bmod k$.
$t = 2$:	For all k there is a v_0 such that for all $v > v_0$ there exists a 2–$(v, k, 1)$ whenever $v(v-1) \equiv 0 \bmod k$ and $(v-1) \equiv 0 \bmod (k-1)$ (Wilson 1975). The necessary conditions are sufficient for $k = 4$ and $k = 5$ (Hanani 1961, 1972). The case $k = 6$ is studied in (Mullin 1987 & 1989, Zhu, Du and Yin 1987, Mills 1987, and Greig 1992) The cases $k = 7, 8, 9$ were studied by M. Greig (Greig 1992). The case $k = 8$ was studied independently by B. Du and L. Zhu in 1988. The smallest unsettled parameter set is 2–(46,6,1).
$t = 3$:	Only ones known have: $(v, k) = (q^n + 1, q + 1)$, $n \geq 2$, q a prime power; $(v, k) = (22, 6)$ (Carmichael 1937, Witt 1938) and $(v, k) = (25, 5)$ (Denniston 1976). There are also recursions which give further infinite classes (Hanani 1979). The smallest unsettled parameter set is 3–(42,6,1).
$t = 4$:	Only ones known have: $(v, k) = (23, 7)$ (Carmichael 1937, Witt 1938) and $(v, k) = (27, 6)$ (Denniston 1976). The smallest unsettled parameter set is 4–(42,6,1).
$t = 5$:	Only ones known have: $(v, k) = (24, 8)$ (Carmichael 1937, Witt 1938) and $(v, k) = (28, 7)$ (Denniston 1976). The smallest unsettled parameter set is 5–(43,7,1).
$t \geq 6$:	None are known. The smallest unsettled parameter set is 6–(29,8,1).

Table III: A summary of the parameters settled when $k = t + 1$ and $\lambda \neq 1$

$t = 1$:	The necessary conditions are sufficient.
$t = 2$:	The necessary conditions are sufficient (Dehon 1983).
$t = 3$:	The necessary conditions are know to be sufficient when: $\lambda = 2$ (Hartman and Phelps 1990); $\lambda = 3$ (Phelps, Stinson, Vanstone 1989); $\lambda \equiv 0 \bmod 3$ and $\lambda = \gcd(v-3, 12)$ (Teirlinck 1984); for all λ when $v = 5 \cdot 2^n$ (Etzion, Hartman 1990). Other sporadic results are also known. The smallest unknown is 3–(25,4,4).
$t = 4$:	The only known infinite families are: 4–$(4 + 8u, 5, 4u)$ for all $u > 0$ (Teirlinck 1989) 4–$(v, 5, \lambda)$ for all $v \equiv 4 \bmod \lambda$, where $\lambda = 2641807540224$ (Teirlinck 1987). Sporadic examples are also known. The smallest unknown is 4–(15,5,2).
$t = 5$:	The only known infinite families known are: 5–$(5 + 8u, 6, 4u)$ for all $u > 0$ (Teirlinck 1989, using a result of Kreher and Radziszowski 1986) 5–$(v, 6, \lambda)$ for all $v \equiv 5 \bmod \lambda$, where $\lambda = 7430083706880000000000$ (Teirlinck 1987). Sporadic examples are also known. The smallest unknown is 5–(16,6,2).
$t = 6$:	The only known infinite families are: 6–$(6 + 8u, 7, 4u)$ for all $u > 0$ (Teirlinck 1989, using a result of Kreher and Radziszowski 1986) 6–$(v, 7, \lambda)$ $\forall v \equiv 6 \bmod \lambda$, where $\lambda = 1397405517247104682033152000000000000000$ (Teirlinck 1987). No other 6–$(v, 7, \lambda)$ designs are known. The smallest unknown is 6–(15,7,3).
$t \geq 7$:	The only known infinite family is: t–$(v, t+1, \lambda)$ for all $v \equiv t \bmod \lambda$ where $\lambda = (t+1)!^{2t+1}$ (Teirlinck 1987). No other t–$(v, t + 1, \lambda)$ designs are known. The smallest unknown is 7–(16,8,3).

Table IV: A summary of the parameters settled when $k \neq t+1$ and $\lambda \neq 1$

$t = 1$: The necessary conditions are sufficient.

$t = 2$: The case $k = 4$ and $\lambda = 2$ is completely settled (Rosa 1993).

The smallest unsettled parameter set is 2–(22,8,4).

$t = 3$: Several infinite families and many sporadic examples are known.

The smallest unsettled parameter set is 3–(16,7,5).

$t = 4$: There are twenty infinite families known, see Table V.
Many sporadic examples are known.

The smallest unsettled parameter set is 4–(12,6,6).

$t = 5$: There are seven infinite families known., see Table V
Many sporadic examples are known.

The smallest unsettled parameter set is 5–(13,7,6).

$t = 6$: The only known infinite family is: 6–$(23+16m, 8, 4(m+1)(16m+17)$ for $m \geq 0$, (Kreher 1993).
The only other known examples are:
6–(33,8,36) (Leavitt and Magliveras 1984);
6–(20,9,112) (Kramer, Leavitt and Magliveras 1982);
6–(22,8,60) (Kreher and DeCaen 1992) and
6–$(28, 8, \lambda)$, for $\lambda = 63, 84, 105$, (Schmalz 1993).

The smallest unsettled parameter set is 6–(16,8,15).

$t \geq 7$: None are known. The smallest unsettled parameter set is 7–(18,9,5).

Table V: The known infinite families of t designs with $t \geq 4$.

4–$(2^n+1, 2^m, (2^m-3)\Pi_{i=2}^{m-1}\frac{2^{n-i}-1}{2^{m-i}-1})$ designs exist provided $2 < m < n$ (Hubaut 1974).

4–$(2^n+1, 2^{n-1}+1, (2^{n-1}-3)(2^{n-2}-1)(2^{n-1}-4))$ designs exist provided $n \geq 4$ (Driessen 1978).

4–$(2^n+1, 2^m+1, (2^m+1)\Pi_{i=2}^{m-1}\frac{2^{n-i}-1}{2^{m-i}-1})$ designs exist provided $2 < m < n$ and m does not divide n (Hubaut 1974).

4–$(2^n+1+s, 2^{n-1}-1, \binom{2^n+s-3}{s}(2^{n-1}-1)(2^{n-2}-1)(2^{n-1}-4))$ designs exist for each $s \geq 2$ such that $n \geq 6$ is large enough so that $\frac{2^{n-1}-2}{n-1} > s+6$ (Magliveras 1987).

4–$(2^n+1+s, 2^m, \binom{2^n+s-3}{s}(2^m-3)\mu)$ designs exist for m sufficiently close to n, with m large enough so that $\binom{v}{k}/\binom{v+s}{s} > \lambda_0(\lambda_0-\lambda_1)$ where $\mu = \Pi_{i=2}^{m-1}\frac{2^{n-i}-1}{2^{m-i}-1}$ and λ_0, λ_1 are the number of blocks and replication number respectively (Magliveras 1987).

4–$(2^n+1+s, 2^m+1, \binom{2^n+s-3}{s}(2^m+1)\mu)$ designs exist for m sufficiently close to n, with m large enough so that $\binom{v}{k}/\binom{v+s}{s} > \lambda_0(\lambda_0-\lambda_1)$ where $\mu = \Pi_{i=2}^{m-1}\frac{2^{n-i}-1}{2^{m-i}-1}$ and λ_0, λ_1 are the number of blocks and replication number respectively (Magliveras 1987).

4–$(4+8u, 5, 4u)$ designs exist for each $u > 0$ (Tierlinck 1989, using a result of Kreher and Radziszowski 1986).

4–$(9m+5, 6, (27m^2+3m)/2)$ (Kramer, Magliveras, O'Brien 1993).

4–$(2^f+1, 6, 10)$ designs exist for all odd $f \geq 5$ (Bierbrauer 1989a).

4–$(2^f+1, 9, 84)$ designs exist for all $f \geq 5$ (Bierbrauer 1989b).

4–$(2^f+1, 8, 35)$, 4–$(2^f+1, 6, 60)$, 4–$(2^f+1, 6, 90)$, 4–$(2^f+1, 6, 150)$, 4–$(2^f+1, 6, 70)$,
4–$(2^f+1, 6, 100)$, 4–$(2^f+1, 6, 160)$, 4–$(2^f+1, 9, 63)$ and 4–$(2^f+1, 9, 147)$ designs exist for all f relatively prime to 6 (Bierbrauer 1989c).

4–$(4+8u, 5, 4u)$ designs exist for each $u > 0$ (Tierlinck 1989, using a result of Kreher and Radziszowski 1986).

5–$(2^n+2, 2^{n-1}+1, (2^{n-1}-3)(2^{n-2}-1))$ designs exist provided $n \geq 4$ (Alltop 1972).

5–$(2^n+3, 2^{n-1}+1, (2^n-2)(2^{n-1}-3)(2^{n-2}-1))$, 5–$(2^n+4, 2^{n-1}+2, (2^n-1)(2^n-2)(2^{n-2}-1))$, 5–$(2^n+5, 2^{n-1}+2, 2^n(2^n-1)(2^n-2)(2^{n-2}-1))$ and 5–$(2^n+6, 2^{n-1}+3, 2^{n-1}(2^n+1)(2^n-1)(2^n-2))$ designs exist provided $n \geq 6$ (van Trung 1986).

5–$(2^n+2+s, 2^n+1, \binom{2^n+s-3}{s}(2^{n-1}-3)(2^{n-2}-1))$ designs exist for each $n \geq N$ such that $s > 0$, $N \geq 4$ and $\frac{2^N-N}{N-1} > s+4$ (Magliveras 1987).

5–$(5+8u, 6, 4u)$ designs exist for each $u > 0$ (Tierlinck 1989, using a result of Kreher and Radziszowski 1986).

6–$(6+8u, 7, 4u)$ designs exist for each $u > 0$ (Tierlinck 1989, using a result of Kreher and Radziszowski 1986).

6–$(23+16m, 8, \frac{1}{2}\binom{16m+1}{2})$ designs exist for each $m > 0$ (Kreher 1993).

t–$(v, t+1, (t+1)!^{2t+1})$ designs exist provided $v \equiv t \bmod (t+1)!^{2t+1})$ and $v \geq t+1$ (Teirlinck 1987).

Table VI

	t	What's known	Techniques used
小	2	lots	Various algebraic , geometric
Small	3	lots	and recursive constructions.
	4	something	Use of 4 and 5 transitive groups,
中			union of orbits under other groups
Middle	5	something	recursive constructions and coding theory[1].
大	6	a little	Union of orbits under some group
Large	≥ 7	almost nothing	and 'large set recursion'.

Let $g \in$ Sym$(\mathcal{X})$, the full symmetric group on $\mathcal{X}$. For each $x \in \mathcal{X}$, the image of x under g is denoted by x^g. If $K \subseteq \mathcal{X}$, then $K^g = \{x^g : x \in \mathcal{X}\}$. An *automorphism* of a t–(v,k,λ) design $(\mathcal{X}, \mathcal{B})$ is an element $g \in$ Sym$(\mathcal{X})$, such that $K^g \in \mathcal{B}$ whenever $K \in \mathcal{B}$. The set of all automorphisms of a t–(v,k,λ) design is said to be the *full automorphism group* of the design. Any subgroup of the full automorphism group is simply said to be *an automorphism group* of the design. For example $G = \langle\ (1,4,5)(0,6,2)\ ,\ (2,6)(4,5)\ \rangle$ is an obvious automorphism group of the 2–(7,3,1) design given in figure 1.

[1] Simple 5-designs were constructed using coding theory for example in the 1969 seminal paper of Assmus and Mattson.

If $K \subseteq \mathcal{X}$, and G is a subgroup of $\mathrm{Sym}(\mathcal{X})$, then the *orbit* of K under G is $\{K^g : g \in G\}$.

Given integers $0 < t < k < v$, v–set $\mathcal{X}$ and $G \leq Sym(\mathcal{X})$ let Δ_1 , Δ_2 , . . . , Δ_{N_t} be the distinct orbits of t–subsets and let Γ_1 , Γ_2 , . . . , Γ_{N_k} be the distinct orbits of k–subsets. The N_t by N_k incidence matrix A_{tk} is defined by $A_{tk}[\Delta_i, \Gamma_j] = |\{K \in \Gamma_j : K \supseteq T\}$, $T \in \Delta_i$ fixed.

For example when $G = \langle (1,4,5)(2,0,6), (2,6)(4,5) \rangle$, then the A_{23} matrix is:

	123 340 136 356 350 234	124 450 156	125 140 146 456 150 245	126 240 560	120 460 160 256 250 246	134 345 135	130 346 235	145	236 230 360	260
12 40 16 56 50 24	1	1	1	1	1	0	0	0	0	0
13 34 35	2	0	0	0	0	2	1	0	0	0
14 45 15	0	1	2	0	0	1	0	1	0	0
10 46 25	0	0	2	0	2	0	1	0	0	0
23 30 36	2	0	0	0	0	0	1	0	2	0
26 20 60	0	0	0	1	2	0	0	0	1	1
		↑					↑			↑

In 1973 Kramer and Mesner made the following fundamental observation:

> A t–(v, k, λ) design exists with $G \leq Sym(\mathcal{X})$ as an automorphism group if and only if there is a (0,1)–solution U to the matrix equation
> $$A_{tk}U = \lambda J,$$
> where: $J = [1, 1, 1, \ldots, 1]^T$.

This observation lead to the investigation of several algorithms for solving $A_{tk}U = \lambda J$ and hence for finding t–designs. Besides ordinary backtracking the most successful such algorithms are: Leavitt's Algorithm[1], Schmalz's Algorithm[2] and Basis Reduction[3]. In general *the method* now commonly used to find t–designs with large t is:

- Choose parameters t, k, v, and λ;
- Find a candidate for an automorphism group G;

[1] Found in the 1985 paper of Kramer, Leavitt and Magliveras.
[2] Described in Schmalz's 1993 paper
[3] Best description is in the 1990 paper of Kreher and Radziszowski.

- Generate the incidence matrix A_{tk};
- Solve the system of equations $A_{tk}U = \lambda J$ for one, some or all (0,1)–vectors U;
- Check for any special properties you may require of the found solutions;
- Apply any known recursive methods to the solutions found to construct more designs.

Almost every known 6–design was either found using the method or obtained from a 6–design found with the method.

Table VII: The known 6 designs

Parameters	Aut. Group.	Size of A_{tk}	Method
6–(14,7,4)	C_{13}	99 by 132	Basis reduction[3]
6–$(6+8u,7,4u)$ for all $u>0$	?	?	L.S. recursion[4]
6–(20,9,112)	$PSL(2,19)$	19 by 52	Leavitt's Alg.[1]
6–(22,8,60)	$PSL(2,19)pp$	36 by 120	Basis Reduction[5]
6–(23,8,68)	?	?	Cleverness[6]
6–$(23+16m,8,\frac{1}{2}\binom{16m+1}{2})$ for all $m\geq 0$	?	?	L.S. recursion[6]
6–$(28,8,\lambda)$, for $\lambda=63,84,105$	$P\Gamma L(2,27)$	14 by 72	Schmalz's Alg.[2]
6–(33,8,36)	$P\Gamma L(2,32)$	13 by 97	Leavitt's Alg.[1]
6–$(v,7,\lambda)$, where $v\equiv 6 \bmod \lambda$ & $\lambda=(7!)^{13}$	?	?	Magic[7] and L.S. rec.

The only theoretical methods that have been successful in constructing t–designs with large t is the method of *large set recursion* due to Teirlinck 1986 & 1989. An LS$[n](t,k,v)$ is a partition of the k–subsets of a v–element set into n disjoint t–(v,k,λ) designs; where $\lambda = \frac{1}{n}\binom{v-t}{k-t}$.

[4] This method of using large sets to recursively construct designs was first described by Teirlinck in 1987.

[5] Described in the 1992 tech. report of Kreher and DeCaen.

[6] Details appear in the 1993 paper of Kreher.

[7] A remarkable result by Teirlinck in 1984.

Example: A LS[7](2, 3, 9), here $\lambda = 1$.

Design 1	Design 2	Design 3	Design 4	Design 5	Design 6	Design 7
024	035	046	057	061	072	013
136	247	351	462	573	614	725
857	861	872	813	824	835	846
018	028	038	048	058	068	078
235	346	457	561	672	713	124
467	571	612	723	134	245	356
037	041	052	063	074	015	026
268	378	418	528	638	748	158
415	526	637	741	152	263	374
056	067	071	012	023	034	045
127	231	342	453	564	675	716
348	458	568	678	718	128	238

A survey of large sets is given by Teirlinck in the 1992 collection of surveys edited by Dinitz and Stinson. We have at our disposal two important results.

Theorem 1 (Khosrovshahi and Ajoodani–Namini 1991) *If there is an* LS$[N](t, t+1, v)$ *and an* $LS[N](t, t+1, w)$*, then there is also an* LS$[N](t, t+1, v+w-t)$.

The case when $v = w$ in Theorem 1 was first established by Teirlinck 1987 & 1989. Teirlinck also established for all t the existence of an LS$[(v-t)/\lambda](t, t+1, v)$ for all $v \equiv t \bmod \lambda$; where $\lambda = (t+1)!^{2t+1}$. This result in particular implies the existence of simple t-designs for all values of t. A very useful generalization is the Corollary below.

Theorem 2 (Qiu–rong Wu 1991) *If there are large sets with parameters* LS$[N](t-1, t+1, v)$*,* $LS[N](t-1, t+1, w)$*,* LS$[N](t-1, t, v-1)$ *and* $LS[N](t-1, t, w-1)$*, then there is also an* LS$[N](t-1, t+1, v+w-t)$.

A LS$[2](t, k, v)$ is also known as a *halving of the complete design* as first discussed by Hartman in 1987. It is interesting to note that other than Teirlinck's family described above the only known examples of large sets of 6–designs are in fact halvings of the complete design: LS$[2](6, 7, 6+8u)$ for $u > 0$, LS$[2](6, 8, 60)$ and LS$[2](6, 8, 23+16m)$ $m \geq 0$, see Table VI.

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