Small orthogonal main effect plans with four factors

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ABSTRACT

In this paper we study the orthogonal main effect plans with four factors, each with at most 10 levels, and with at most 50 runs. We determine the spectrum of the degrees of freedom of pure error for these designs.

1 Introduction

An $s_1 \times s_2 \times \cdots \times s_k//n$ *orthogonal main effect plan* (OMEP) is a k by n array such that

1. the i-th row contains entries from an s_i element set S_i; and
2. given rows $i \neq j$, and symbols $x \in S_i$, $y \in S_j$, if $N_i[x]$, and $N_j[y]$, denote the number of times x appears in row i, and y appears in row j, respectively, then the number, $N_{ij}[x, y]$, of columns that contain x in row i and y in row j is exactly $N_i[x]N_j[y]/n$.

The rows are the *factors* and the columns are the *runs* of an asymmetric fractional factorial design and the above conditions guarantee orthogonality of the main effects in the experiment [5, 1, 3]. A run is said to be *repeated* if it occurs more then once as a column of the array. Pigeon and McAllister [4] discussed the advantages of having some repeated runs in an OMEPto provide an estimate of pure error. If the array has m_i runs that are each repeated i times, then the *repeated run sequence* (RRS) of the array is $\text{RRS} = 1^{m_1}2^{m_2}\cdots n^{m_n}$. Note that $\sum_i im_i = n$. Then the *degrees of freedom of pure error* (DFPE) of the experimental design represented by the array is

$$\text{DFPE} = \sum_{i=1}^{n} m_i(i-1). \tag{1}$$

Example 1. Some $2 \times 2 \times 3 \times 4//16$ OMEP.

1 1 2 2 1 1 2 2 2 2 2 2 1 1 1 1	1 1 2 2 1 2 2 1 2 2 1 2 1 1 1 2
1 1 2 2 2 2 1 1 1 2 1 2 1 2 1 2	1 2 1 2 1 1 2 2 2 2 2 1 1 2 1 1
1 1 1 1 2 2 2 2 3 3 3 3 3 3 3 3	1 1 1 1 2 2 2 2 3 3 3 3 3 3 3 3
1 2 3 4 1 2 3 4 1 1 2 2 3 3 4 4	1 2 3 4 1 2 3 4 1 1 2 2 3 3 4 4
RRS $= 1^{16}$, DFPE $= 0$.	RRS $= 1^{14}2^1$, DFPE $= 1$
(a)	(b)
1 1 2 2 1 1 2 2 2 2 2 2 1 1 1 1	1 1 2 2 1 1 2 2 2 2 2 2 1 1 1 1
1 2 1 2 1 2 2 1 2 2 1 1 1 2 1 2	1 2 1 2 1 2 1 2 2 2 1 1 2 2 1 1
1 1 1 1 2 2 2 2 3 3 3 3 3 3 3 3	1 1 1 1 2 2 2 2 3 3 3 3 3 3 3 3
1 2 3 4 1 2 3 4 1 1 2 2 3 3 4 4	1 2 3 4 1 2 3 4 1 1 2 2 3 3 4 4
RRS $= 1^{12}2^2$, DFPE $= 2$	RRS $= 1^{8}2^4$, DFPE $= 4$
(c)	(d)

An *orthogonal array* of strength 2, degree k, order s and index λ, denoted by $\mathrm{OA}_\lambda(k, s)$, is a $\underbrace{s \times s \times \cdots \times s}_{k}//s^2\lambda$ OMEP.

In this paper we give a recursive construction for OMEPs and a construction based on incomplete mutually orthogonal Latin squares. Using these results, and the techniques of [6, 2], we give a tabulation of four factor minimal OMEPs, in which each factor has at most 10 levels, and with at most 50 runs, for all possible values of DFPE. We also assume for the table that the OMEPS will have leval replication as equal as possible in each of the factors.

Lewis and John (Testing main effects in fractions of asymmetrical factorial experiments, Biometrika 63, 678-680) have shown that in an OMEP with unequal replication the hypotheses that are orthogonally estimated are weighted hypotheses that depend on the particular fraction that is used. In an equally replicated experiment the orthogonal hypotheses are the unweighted hypotheses which are of most interest to experimenters. The weights depend on the replication and so the more equal the replication the less weighted are the comparisons. For this reason we have concentrated on constructing designs in which the replication is as equal as possible.

2 Constructions

One well-known method of obtaining one OMEPfrom another is by 'collapsing levels', introduced in [1]. In its simplest form this is a many-to-one correspondence which maps the s levels of a factor onto $t(< s)$ levels.

Theorem 2.1 *Let $S_i = \{1, 2, ..., s_i\}$, $1 \le i \le k$, be the level sets corresponding to the factors of a $s_1 \times s_2 \times \cdots \times s_k//n$ OMEPA. If, for each i, a map $f_i : S_i \mapsto T_i$ is chosen such that $t_i = |T_i| \le s_i = |S_i|$, then applying f_i to the levels in factor i of A, $1 \le i \le k$, gives a $t_1 \times t_2 \times \cdots \times t_k//n$ OMEPB.*

To obtain the OMEPB constructed in 2.1 we would say:

Collapse A via $f_1(1), \cdots, f_1(s_1)/f_2(1), \cdots, f_2(s_2)/\cdots/f_k(1), \cdots, f_k(s_k)$.

Example 2. The $2 \times 2 \times 2 \times 3//16$ OMEPbelow has RRS $= 1^8 2^4$ and DFPE $= 4$. It was obtained by performing a collapse of example 1 (a) via 12/12/122/1123.

1 1 2 2 1 1 2 2 2 2 2 2 1 1 1 1
1 1 2 2 2 2 1 1 1 2 1 2 1 2 1 2
1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2
1 1 2 3 1 1 2 3 1 1 1 1 2 2 3 3

In the next result we establish conditions under which two OMEPcan be juxtaposed to get an OMEPwith more runs and with more levels for one of the factors.

Theorem 2.2 *If there is a $s_1 \times s_2 \times \cdots \times s_k//n_1$ OMEPA and a $t_1 \times t_2 \times \cdots \times t_k//n_2$ OMEPB such that*

1. *$s_i = t_i$ for all i except possibly for some $i = i_0$,*
2. *$n_2 = \mu n_1$,*
3. *$N_{Bi}[x] = \mu N_{Ai}[x]$ for all x and all $i \ne i_0$*
4. *for all x, $N_{Ai_0}[x]$ and N_{Bi_0} are multiples of some r_{i_0}.*

Then there is a $s_1 \times s_2 \times \cdots \times (s_{i_0} + t_{i_0}) \times \cdots \times s_k//(n_1 + n_2)$.

Proof: Use the same set of s_i symbols in each row $i \ne i_0$ of both A and B, and use disjoint sets of symbols in row i_0. Juxtapose A and B to obtain the desired OMEP.

We now show that the second point in the definition is satisfied by the combined array. In the combined array, for $i, j \ne i_0$, $N_{ij}[x, y] = N_{Aij}[x, y] + N_{Bij}[x, y]$, where $N_{Aij}[x, y] = N_{Ai}[x]N_{Aj}[y]/n_1$ and $N_{Bij}[x, y] = N_{Bi}[x]N_{Bj}[y]/n_2$. Now $N_i[x] = N_{Ai}[x] + N_{Bi}[x] = (1 + \mu)N_{Ai}[x]$ and $N_j[y] = N_{Aj}[y] + N_{Bj}[y] = (1 + \mu)N_{Aj}[y]$. Thus

$$\begin{aligned}
\frac{N_i[x]N_j[y]}{(n_1 + n_2)} &= \frac{(1+\mu)^2 N_{Ai}[x]N_{Aj}[y]}{(1+\mu)n_1} \\
&= \frac{(1+\mu)N_{Ai}[x]N_{Aj}[y]}{n_1} \\
&= \frac{N_{Ai}[x]N_{Aj}[y]}{n_1} + \frac{\mu N_{Ai}[x]N_{Aj}[y]}{n_1} \\
&= N_{Aij}[x, y] + \frac{\mu^2 N_{Ai}[x]N_{Aj}[y]}{n_2} \\
&= N_{Aij}[x, y] + \frac{N_{Bi}[x]N_{Bj}[y]}{n_2} = N_{Aij}[x, y] + N_{Bij}[x, y]
\end{aligned}$$

as required.

The proof for row $i = i_0$ is similar.

□

Example 3. Consider a $2 \times 2 \times 2 \times 3//8$ OMEPand a $2 \times 2 \times 2 \times 4//8$ OMEP. Apply Theorem 2.2 with $i_0 = 4$. Then $\mu = 1$ and in the first three rows $N_{Bi}[x] = N_{Ai}[x]$. In the fourth row all the $N_{A4}[x]$'s and $N_{B4}[x]$'s are multiples of 2. Hence we get a $2\times2\times2\times7//16$ OMEP. The DFPEis the sum of the DFPEin each of the original OMEP.

1 1 2 2 2 2 1 1 1 1 2 2 2 2 1 1
1 2 1 2 2 1 2 1 1 2 1 2 2 1 2 1
1 1 1 1 2 2 2 2 1 1 1 1 2 2 2 2
1 2 3 3 1 2 3 3 4 5 6 7 4 5 6 7

This result can be applied to pairs of mutually orthogonal Latin squares of order s_1 to give $s_1 \times s_1 \times s_1 \times s_4//{s_1}^2$ minimal OMEPs where $s_4 \leq 2s_1$ and with a range of values for DFPE.

Theorem 2.3 *Suppose there exists a pair of MOLS of order s_1. Then there exists a $s_1 \times s_1 \times s_1 \times (2s_1 - i)//2{s_1}^2$ OMEP with* DFPE$= jf$, *for $1 \leq j \leq i$ and $0 \leq f \leq s_1, f \neq s_1 - 1$.*

Proof: A pair of MOLS of order s_1 is a $s_1 \times s_1 \times s_1 \times s_1//{s_1}^2$ OMEP. Using Theorem 2.2 with two of these gives a $s_1 \times s_1 \times s_1 \times 2s_1//2{s_1}^2$. It is possible to collapse levels so that there are repeated runs. Let σ be a permutation of s_1 symbols and suppose that σ has f fixed points. Apply σ to the symbols in the first Latin square in the second $s_1 \times s_1 \times s_1 \times s_1//{s_1}^2$ OMEP. Now perform the collapsing of levels by collapsing $x + s_1$ to x on j occasions and for the remaining $i - j$ levels collapse $x + s_1$ to some y with $x \neq y$. The fixed points of σ appear with all the symbols in the first square equally often and so for each level which is collapsed by mapping $x + s_1$ to x there are f repeated pairs of runs. Since σ is a permutation of s_1 things there can be any number of fixed points between 0 and s_1 except $s_1 - 1$. □

Example 4. Let $s_1 = 5$ and consider the two MOLS:

1 2 3 4 5		1 3 5 2 4
2 3 4 5 1		2 4 1 3 5
3 4 5 1 2	and	3 5 2 4 1
4 5 1 2 3		4 1 3 5 2
5 1 2 3 4		5 2 4 1 3

They can be used to construct the $5 \times 5 \times 5 \times 10//50$ OMEPgiven below.

1 1 1 1 1 2 2 2 2 2 3 3 3 3 3 4 4 4 4 4 5 5 5 5 5 1 1 1 1 1 2 2 2 2 2 3 3 3 3 3 3 4 4 4 4 4 5 5 5 5 5
1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5
1 2 3 4 5 2 3 4 5 1 3 4 5 1 2 4 5 1 2 3 5 1 2 3 4 1 2 3 4 5 2 3 4 5 1 3 4 5 1 2 4 5 1 2 3 5 1 2 3 4
1 3 5 2 4 2 4 1 3 5 3 5 2 4 1 4 1 3 5 2 5 2 4 1 3 6 8 10 7 9 7 9 6 8 10 8 10 7 9 6 9 6 8 10 7 10 7 9 6 8

Now apply the permutation (12) to the third row of the OMEP. Thus there are 3 fixed points Now collapse by equating 6 and 1 in row 4. The runs with 6 in row 4 and symbols 3, 4 or 5 in row 3 are all repeats. Thus we have constructed a $5\times5\times5\times9//50$ OMEPwith DFPE= 3. We can get a $5\times5\times5\times8//50$ OMEPwith 4 repeated runs by using the same initial OMEPand the permutation (123) and collapsing 6 to 1 and 7 to 2. To get 2 repeated runs we could collapse 6 to 1 and 7 to 3.

It is possible to extend the previous result to adjoining more than 2 copies of the OMEP. Conditions for these OMEPs to be minimal still need to be established.

Theorem 2.4 *Suppose there exists a pair of MOLS of order s_1. Then there exists a $s_1\times s_1\times s_1\times(ms_1-i)//{ms_1}^2$* OMEP, *$1\le i\le s_1-1$, with* DFPE$= jf$, *$1\le j\le i$ and $0\le f\le s_1$, $f\ne s_1-1$.*

The designs of the previous result could never have DFPEequal to one less than the upper bound. The next result shows that this observation is independent of the method of construction of the designs.

Theorem 2.5 *Consider a minimal $s_1\times s_2\times s_3\times s_4//n$* OMEP*where $n=s_3(s_4+x)$ say. Then the upper bound for* DFPE *is s_3x and there is no design with* DFPE$= s_3x-1$.

Proof: The replication vector for the third factor has all entries equal to s_4+x and the replication vector for the fourth factor has all entries multiples of s_3 (because of the form of n and the proportionality requirement) so there are s_4-x entries equal to s_3 and x entries equal to $2s_3$ (if we want the replication to be as equal as possible).

Repeated runs must come from those levels of the fourth factor with replication $2s_3$. Each of these levels have two occurrences of each level of the third factor and so all repeated runs occur in pairs.

Consider a design with DFPE$= s_3x-1$. In $x-1$ of the levels of the fourth factor all levels of the third factor are involved in repeated runs. In the xth level there are only s_3-1 levels of the third factor involved in repeated runs. The final level of the third factor can only not be involved in repeated runs if there are different levels of the second and first factors available to fill the places. But the proportionality requirement means that each of the x levels of the fourth factor has the same number of the occurrences of each level of each of the other factors. So if there are s_3-1 repeats in a level of factor 4 then the only levels left must correspond to a repeated pair. □

The same argument can be used to establish the following result.

Theorem 2.6 *Consider a minimal $s_1\times s_2\times s_3\times s_4//n$* OMEP*with $n=(s_3+x)s_4$ say. Then the upper bound for the* DFPE *is s_4x and there is no design with* DFPE$= s_4x-1$.

There are relationships between OMEPs and incomplete arrays or Latin squares with holes.

Using the notation of Horton (JCTA 16 (1974)) an IA(n, k, s) is an $s \times (n^2 - k^2)$ array on n objects with a distinguished subset of k objects such that the ordered pairs obtained by superimposing any two rows gives the n^2 ordered pairs except for the k^2 ordered pairs from the distinguished subset. So when $k = 2$ and $s = 4$ these things are a pair of MOLS of order n with a 2×2 subsquare missing.

Theorem 2.7 *An IA$(n, 2, 4)$ is equivalent to an $n-1 \times n-1 \times n-1 \times n//n^2$ OMEP with* DFPE=*2.*

Proof: In one square we put 4 $n-1$'s in the 2×2 hole and change all n's to $n-1$'s. In the other square we put a 2×2 Latin square on the symbols $n-1$ and n. Then we label the rows and columns so that we can write a $(n-1) \times (n-1) \times (n-1) \times n//n^2$ OMEP. The OMEPwill always have DFPE $= 2$ which is maximum possible for these parameters. □

Example 5. For example, $n = 6$, $k = 2$, gives the IMOLS

5	6	3	4	1	2
2	1	6	5	3	4
6	5	1	2	4	3
4	3	5	6	2	1
1	4	2	3		
3	2	4	1		

and

1	2	5	6	3	4
6	5	1	2	4	3
4	3	6	5	1	2
5	6	4	3	2	1
2	4	3	1		
3	1	2	4		

In the first square put $\begin{array}{|cc|}\hline 5 & 6 \\ 6 & 5 \\ \hline \end{array}$ in the hole and fill the second hole with 5's and change all the 6's to 5's. This gives the OMEP:

1 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 3 3 4 4 4 4 4 4 5 5 5 5 5 5 5 5 5 5 5 5
1 2 3 4 5 5 1 2 3 4 5 5 1 2 3 4 5 5 1 2 3 4 5 5 1 2 3 4 1 2 3 4 5 5 5 5
1 2 5 5 3 4 5 5 1 2 4 3 4 3 5 5 1 2 5 5 4 3 2 1 2 4 3 1 3 1 2 4 5 5 5 5
5 6 3 4 1 2 2 1 6 5 3 4 6 5 1 2 4 3 4 3 5 6 2 1 1 4 2 3 3 2 4 1 5 5 6 6

This also effectively establishes that there are no 3x3x3x4//16, 4x4x4x5//25 with dfpe=2.

In general a IA$(n, k, 4)$ gives a $(n-k+1)x(n-k+1)x(n-k+1)xn//n^2$ OMEPwith DFPE $= k(k-1)$ (and $k < n$ of course). When are these minimal?

3 Table of OMEPs

In this section minimal orthogonal main effect plans with four factors, each with at most 10 levels, and at most 50 runs, are listed.

1. $2 \times 2 \times 2 \times 3//8$, DFPE $= 0$:

11222211
12122112
11112222
12331233

2. $2 \times 2 \times 2 \times 3//8$, DFPE $= 1$: Dose not exist by Theorem 2.5.
3. $2 \times 2 \times 2 \times 3//8$, DFPE $= 2$: Dose not exist by exhaustive computer search.
4. $2 \times 2 \times 2 \times 4//8$, DFPE $= 0$:

11222211
12122121
11112222
12341234

5. $2 \times 2 \times 3 \times 3//9$, DFPE $= 0$:

122212221
122221212
123123123
111222333

6. $2 \times 3 \times 3 \times 3//9$, DFPE $= 0$:

122212221
123231312
123123123
111222333

7. $3 \times 3 \times 3 \times 3//9$, DFPE $= 0$:

123312231
123231312
123123123
111222333

8. $2 \times 2 \times 2 \times 7//16$, DFPE $= 0$: Collapse 11 via 12/12/12/12345676.
9. $2 \times 2 \times 2 \times 7//16$, DFPE $= 1$: Dose not exist by Theorem 2.5.
10. $2 \times 2 \times 2 \times 7//16$, DFPE $= 2$: Collapse 11 via 12/12/12/12345677.
11. $2 \times 2 \times 2 \times 8//16$, DFPE $= 0$:

1111222222221111
1122112222112211
1111111122222222
1234567812345678

12. $2 \times 2 \times 3 \times 4//16$, DFPE $= 0$: Collapse 30 via 1234/1233/1222/1221.
13. $2 \times 2 \times 3 \times 4//16$, DFPE $= 1$: Collapse 30 via 1234/1233/1222/1222.
14. $2 \times 2 \times 3 \times 4//16$, DFPE $= 2$: Collapse 30 via 1234/1233/1222/1212.
15. $2 \times 2 \times 3 \times 4//16$, DFPE $= 3$: Dose not exist by Theorem 2.6.
16. $2 \times 2 \times 3 \times 4//16$, DFPE $= 4$: Collapse 30 via 1234/1233/1221/1212.
17. $2 \times 2 \times 4 \times 4//16$, DFPE $= 0$: Collapse 30 via 1234/1234/1222/1222.
18. $2 \times 3 \times 3 \times 4//16$, DFPE $= 0$: Collapse 30 via 1234/1233/1233/1222.
19. $2 \times 3 \times 3 \times 4//16$, DFPE $= 1$: Collapse 30 via 1234/1233/1231/1222.
20. $2 \times 3 \times 3 \times 4//16$, DFPE $= 2$: Collapse 30 via 1234/1233/1231/1212.
21. $2 \times 3 \times 3 \times 4//16$, DFPE $= 3$: Dose not exist by Theorem 2.6.

22. $2 \times 3 \times 4 \times 4//16$, DFPE $= 0$: Collapse 30 via 1234/1234/1233/1222.
23. $2 \times 4 \times 4 \times 4//16$, DFPE $= 0$: Collapse 30 via 1234/1234/1234/1222.
24. $3 \times 3 \times 3 \times 4//16$, DFPE $= 0$: Collapse 30 via 1234/1233/1233/1233.
25. $3 \times 3 \times 3 \times 4//16$, DFPE $= 1$: Collapse 30 via 1234/1233/1231/1232.
26. $3 \times 3 \times 3 \times 4//16$, DFPE $= 2$: Dose not exist by exhaustive computer search.
27. $3 \times 3 \times 3 \times 4//16$, DFPE $= 3$: Dose not exist by Theorem 2.6.
28. $3 \times 3 \times 4 \times 4//16$, DFPE $= 0$: Collapse 30 via 1234/1234/1233/1233.
29. $3 \times 4 \times 4 \times 4//16$, DFPE $= 0$: Collapse 30 via 1234/1234/1234/1233.
30. $4 \times 4 \times 4 \times 4//16$, DFPE $= 0$:

1342421324313124
1234214334124321
1234123412341234
1111222233334444

31. $2 \times 2 \times 3 \times 5//18$, DFPE $= 0$: Collapse 45 via 123/122/122/123454.
32. $2 \times 2 \times 3 \times 5//18$, DFPE $= 1$: Collapse 45 via 123/122/122/123455.
33. $2 \times 2 \times 3 \times 5//18$, DFPE $= 2$: Dose not exist by Theorem 2.5.
34. $2 \times 2 \times 3 \times 5//18$, DFPE $= 3$: Collapse 45 via 123/122/122/123453.
35. $2 \times 2 \times 3 \times 6//18$, DFPE $= 0$: Collapse 45 via 123/122/122/123456.
36. $2 \times 3 \times 3 \times 5//18$, DFPE $= 0$: Collapse 45 via 123/123/122/123455.
37. $2 \times 3 \times 3 \times 5//18$, DFPE $= 1$: Collapse 42 via 123/123/122/12345.
38. $2 \times 3 \times 3 \times 5//18$, DFPE $= 2$: Dose not exist by Theorem 2.5.
39. $2 \times 3 \times 3 \times 5//18$, DFPE $= 3$: Collapse 45 via 123/123/122/123453.
40. $2 \times 3 \times 3 \times 6//18$, DFPE $= 0$: Collapse 45 via 123/123/122/123456.
41. $3 \times 3 \times 3 \times 5//18$, DFPE $= 0$: Collapse 45 via 123/123/123/123455.
42. $3 \times 3 \times 3 \times 5//18$, DFPE $= 1$:

12312312312311233
23132131213212 1233
32113223131221 1233
11122233344455555

43. $3 \times 3 \times 3 \times 5//18$, DFPE $= 2$: Dose not exist by Theorem 2.5.
44. $3 \times 3 \times 3 \times 5//18$, DFPE $= 3$: Collapse 45 via 123/123/123/123453.
45. $3 \times 3 \times 3 \times 6//18$, DFPE $= 0$: Juxtapose 7 and 7 with distinct symbols on row 1.
46. $2 \times 2 \times 2 \times 9//20$, DFPE $= 0$:

12121212121212121122
12121212212121212121
12211221122112211221
11223344556677889999

47. $2 \times 2 \times 2 \times 9//20$, DFPE $= 1$: Dose not exist by Theorem 2.5.
48. $2 \times 2 \times 2 \times 9//20$, DFPE $= 2$: Dose not exist by exhaustive computer search.
49. $2 \times 2 \times 4 \times 5//24$, DFPE $= 0$: Juxtapose 4 and 12 with distinct symbols on row 3.
50. $2 \times 2 \times 4 \times 5//24$, DFPE $= 1$: Juxtapose 4 and 13 with distinct symbols on row 3.
51. $2 \times 2 \times 4 \times 5//24$, DFPE $= 2$: Juxtapose 4 and 14 with distinct symbols on row 3.
52. $2 \times 2 \times 4 \times 5//24$, DFPE $= 3$: Dose not exist by Theorem 2.5.
53. $2 \times 2 \times 4 \times 5//24$, DFPE $= 4$: Juxtapose 4 and 16 with distinct symbols on row 3.
54. $2 \times 2 \times 4 \times 6//24$, DFPE $= 0$: Juxtapose 4 and 17 with distinct symbols on row 3.
55. $2 \times 2 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12222/12222.

56. $2 \times 3 \times 4 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12344/12333/12122.
57. $2 \times 3 \times 4 \times 5//25$, DFPE $= 1$: Collapse 83 via 12345/12344/12333/12222.
58. $2 \times 3 \times 4 \times 5//25$, DFPE $= 2$: Collapse 83 via 12345/12344/12333/11122.
59. $2 \times 3 \times 4 \times 5//25$, DFPE $= 3$:

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11222122122221122121122121
12233231322321333213322211
11111222223333344444444444
12345123451234511223344455
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60. $2 \times 3 \times 4 \times 5//25$, DFPE $= 4$: Dose not exist by Theorem 2.6.
61. $2 \times 3 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12333/12222.
62. $2 \times 4 \times 4 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12344/12344/12122.
63. $2 \times 4 \times 4 \times 5//25$, DFPE $= 1$: Collapse 83 via 12345/12344/12344/12222.
64. $2 \times 4 \times 4 \times 5//25$, DFPE $= 2$:

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11222112222221122222111212
12344342142414344134423112
11111222223333344444444444
12345123451234511223344455
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65. $2 \times 4 \times 4 \times 5//25$, DFPE $= 4$: Dose not exist by Theorem 2.6.
66. $2 \times 4 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12344/12222.
67. $2 \times 5 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12345/12222.
68. $3 \times 3 \times 4 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12344/12333/12333.
69. $3 \times 3 \times 4 \times 5//25$, DFPE $= 1$: Collapse 83 via 12345/12344/12333/12233.
70. $3 \times 3 \times 4 \times 5//25$, DFPE $= 2$: Collapse 83 via 12345/12344/12333/11123.
71. $3 \times 3 \times 4 \times 5//25$, DFPE $= 4$: Dose not exist by Theorem 2.6.
72. $3 \times 3 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12333/12333.
73. $3 \times 4 \times 4 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12344/12344/12333.
74. $3 \times 4 \times 4 \times 5//25$, DFPE $= 1$: Collapse 83 via 12345/12344/12344/12233.
75. $3 \times 4 \times 4 \times 5//25$, DFPE $= 4$: Dose not exist by Theorem 2.6.
76. $3 \times 4 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12344/12333.
77. $3 \times 5 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12345/12333.
78. $4 \times 4 \times 4 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12344/12344/12344.
79. $4 \times 4 \times 4 \times 5//25$, DFPE $= 1$: Collapse 83 via 12345/12344/12344/12234.
80. $4 \times 4 \times 4 \times 5//25$, DFPE $= 4$: Dose not exist by Theorem 2.6.
81. $4 \times 4 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12344/12344.
82. $4 \times 5 \times 5 \times 5//25$, DFPE $= 0$: Collapse 83 via 12345/12345/12345/12344.
83. $5 \times 5 \times 5 \times 5//25$, DFPE $= 0$:

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11111222223333344444555555
12345123451234512345123451
12345234513451245123512341
15432215433215443215543211
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84. $2 \times 2 \times 3 \times 7//27$, DFPE $= 0$: Collapse 119 via 112/122/123/123456775.
85. $2 \times 2 \times 3 \times 7//27$, DFPE $= 1$: Collapse 119 via 122/122/123/123456777.
86. $2 \times 2 \times 3 \times 7//27$, DFPE $= 2$: Collapse 119 via 121/122/123/123456773.
87. $2 \times 2 \times 3 \times 7//27$, DFPE $= 3$: Collapse 119 via 122/122/123/123456776.
88. $2 \times 2 \times 3 \times 7//27$, DFPE $= 4$: Collapse 119 via 121/122/123/123456776.
89. $2 \times 2 \times 3 \times 7//27$, DFPE $= 5$: Dose not exist by Theorem 2.5.

90. $2 \times 2 \times 3 \times 7//27$, DFPE $= 6$: Collapse 119 via 122/122/123/123456756.
91. $2 \times 2 \times 3 \times 8//27$, DFPE $= 0$: Collapse 119 via 121/122/123/123456788.
92. $2 \times 2 \times 3 \times 8//27$, DFPE $= 1$: Collapse 119 via 122/122/123/123456788.
93. $2 \times 2 \times 3 \times 8//27$, DFPE $= 2$: Dose not exist by Theorem 2.5.
94. $2 \times 2 \times 3 \times 8//27$, DFPE $= 3$: Collapse 119 via 122/122/123/123456786.
95. $2 \times 2 \times 3 \times 9//27$, DFPE $= 0$: Collapse 119 via 122/122/123/123456789.
96. $2 \times 3 \times 3 \times 7//27$, DFPE $= 0$: Collapse 119 via 122/123/123/123456777.
97. $2 \times 3 \times 3 \times 7//27$, DFPE $= 1$: Collapse 119 via 121/123/123/123456773.
98. $2 \times 3 \times 3 \times 7//27$, DFPE $= 2$: Collapse 119 via 123/122/123/123456733.
99. $2 \times 3 \times 3 \times 7//27$, DFPE $= 3$: Collapse 119 via 122/123/123/123456776.
100. $2 \times 3 \times 3 \times 7//27$, DFPE $= 4$: Collapse 119 via 121/123/123/123456753.
101. $2 \times 3 \times 3 \times 7//27$, DFPE $= 5$: Dose not exist by Theorem 2.5.
102. $2 \times 3 \times 3 \times 7//27$, DFPE $= 6$: Collapse 119 via 122/123/123/123456756.
103. $2 \times 3 \times 3 \times 8//27$, DFPE $= 0$: Juxtapose 6 and 36 with distinct symbols on row 4.
104. $2 \times 3 \times 3 \times 8//27$, DFPE $= 1$: Juxtapose 6 and 37 with distinct symbols on row 4.
105. $2 \times 3 \times 3 \times 8//27$, DFPE $= 2$: Dose not exist by Theorem 2.5.
106. $2 \times 3 \times 3 \times 8//27$, DFPE $= 3$: Juxtapose 6 and 39 with distinct symbols on row 4.
107. $2 \times 3 \times 3 \times 9//27$, DFPE $= 0$: Juxtapose 6 and 40 with distinct symbols on row 4.
108. $3 \times 3 \times 3 \times 7//27$, DFPE $= 0$: Collapse 119 via 123/123/123/123456777.
109. $3 \times 3 \times 3 \times 7//27$, DFPE $= 1$: Collapse 119 via 123/123/123/123456773.
110. $3 \times 3 \times 3 \times 7//27$, DFPE $= 2$: Collapse 119 via 123/123/123/123456723.
111. $3 \times 3 \times 3 \times 7//27$, DFPE $= 3$: Collapse 119 via 123/123/123/123456776.
112. $3 \times 3 \times 3 \times 7//27$, DFPE $= 4$: Collapse 119 via 123/123/123/123456753.
113. $3 \times 3 \times 3 \times 7//27$, DFPE $= 5$: Dose not exist by Theorem 2.5.
114. $3 \times 3 \times 3 \times 7//27$, DFPE $= 6$: Collapse 119 via 123/123/123/123456756.
115. $3 \times 3 \times 3 \times 8//27$, DFPE $= 0$: Juxtapose 7 and 41 with distinct symbols on row 4.
116. $3 \times 3 \times 3 \times 8//27$, DFPE $= 1$: Juxtapose 7 and 42 with distinct symbols on row 4.
117. $3 \times 3 \times 3 \times 8//27$, DFPE $= 2$: Dose not exist by Theorem 2.5.
118. $3 \times 3 \times 3 \times 8//27$, DFPE $= 3$: Juxtapose 7 and 44 with distinct symbols on row 4.
119. $3 \times 3 \times 3 \times 9//27$, DFPE $= 0$: Juxtapose 7 and 45 with distinct symbols on row 4.
120. $2 \times 2 \times 4 \times 7//32$, DFPE $= 0$: Juxtapose 8 and 8 with distinct symbols on row 1.
121. $2 \times 2 \times 4 \times 7//32$, DFPE $= 1$: Juxtapose 13 and 17 with distinct symbols on row 3.
122. $2 \times 2 \times 4 \times 7//32$, DFPE $= 2$: Juxtapose 8 and 10 with distinct symbols on row 1.
123. $2 \times 2 \times 4 \times 7//32$, DFPE $= 3$: Dose not exist by Theorem 2.5.
124. $2 \times 2 \times 4 \times 7//32$, DFPE $= 4$: Juxtapose 10 and 10 with distinct symbols on row 1.
125. $2 \times 2 \times 4 \times 8//32$, DFPE $= 0$: Juxtapose 11 and 11 with distinct symbols on row 1.
126. $2 \times 3 \times 4 \times 6//32$, DFPE $= 0$: Juxtapose 18 and 18 with distinct symbols on row 2.
127. $2 \times 3 \times 4 \times 6//32$, DFPE $= 1$: Juxtapose 18 and 19 with distinct symbols on row 2.
128. $2 \times 3 \times 4 \times 6//32$, DFPE $= 2$: Juxtapose 18 and 20 with distinct symbols on row 2.
129. $2 \times 3 \times 4 \times 6//32$, DFPE $= 3$: Juxtapose 19 and 20 with distinct symbols on row 2.
130. $2 \times 3 \times 4 \times 6//32$, DFPE $= 4$: Juxtapose 20 and 20 with distinct symbols on row 2.
131. $2 \times 3 \times 4 \times 6//32$, DFPE $= 5$: Collapse 140 via 12/123/1234/12345654.
132. $2 \times 3 \times 4 \times 6//32$, DFPE $= 6$: Collapse 157 via 112/123/1234/123456.
133. $2 \times 3 \times 4 \times 6//32$, DFPE $= 7$: Dose not exist by Theorem 2.5.
134. $2 \times 3 \times 4 \times 6//32$, DFPE $= 8$: Collapse 140 via 12/123/1234/12345634.
135. $2 \times 3 \times 4 \times 7//32$, DFPE $= 0$: Juxtapose 18 and 22 with distinct symbols on row 3.
136. $2 \times 3 \times 4 \times 7//32$, DFPE $= 1$: Juxtapose 19 and 22 with distinct symbols on row 3.

137. $2 \times 3 \times 4 \times 7//32$, DFPE $= 2$: Juxtapose 20 and 22 with distinct symbols on row 3.
138. $2 \times 3 \times 4 \times 7//32$, DFPE $= 3$: Dose not exist by Theorem 2.5.
139. $2 \times 3 \times 4 \times 7//32$, DFPE $= 4$: Collapse 140 via 12/123/1234/12345674.
140. $2 \times 3 \times 4 \times 8//32$, DFPE $= 0$: Juxtapose 22 and 22 with distinct symbols on row 3.
141. $2 \times 4 \times 4 \times 6//32$, DFPE $= 0$: Juxtapose 17 and 23 with distinct symbols on row 2.
142. $2 \times 4 \times 4 \times 6//32$, DFPE $= 1$: Collapse 147 via 12/1234/1234/1234526.
143. $2 \times 4 \times 4 \times 6//32$, DFPE $= 2$: Collapse 147 via 12/1234/1234/1234566.
144. $2 \times 4 \times 4 \times 6//32$, DFPE $= 4$: Collapse 151 via 12/1234/1234/12345664.
145. $2 \times 4 \times 4 \times 6//32$, DFPE $= 7$: Dose not exist by Theorem 2.5.
146. $2 \times 4 \times 4 \times 6//32$, DFPE $= 8$: Collapse 151 via 12/1234/1234/12345634.
147. $2 \times 4 \times 4 \times 7//32$, DFPE $= 0$: Juxtapose 22 and 23 with distinct symbols on row 2.
148. $2 \times 4 \times 4 \times 7//32$, DFPE $= 1$: Collapse 173 via 122/1234/1234/1234567.
149. $2 \times 4 \times 4 \times 7//32$, DFPE $= 3$: Dose not exist by Theorem 2.5.
150. $2 \times 4 \times 4 \times 7//32$, DFPE $= 4$: Collapse 151 via 12/1234/1234/12345674.
151. $2 \times 4 \times 4 \times 8//32$, DFPE $= 0$: Juxtapose 23 and 23 with distinct symbols on row 2.
152. $3 \times 3 \times 4 \times 6//32$, DFPE $= 0$: Juxtapose 24 and 24 with distinct symbols on row 1.
153. $3 \times 3 \times 4 \times 6//32$, DFPE $= 1$: Juxtapose 24 and 25 with distinct symbols on row 1.
154. $3 \times 3 \times 4 \times 6//32$, DFPE $= 2$: Juxtapose 25 and 25 with distinct symbols on row 1.
155. $3 \times 3 \times 4 \times 6//32$, DFPE $= 3$: Collapse 173 via 123/1233/1234/1234566.
156. $3 \times 3 \times 4 \times 6//32$, DFPE $= 4$: Collapse 165 via 123/123/1234/12345664.
157. $3 \times 3 \times 4 \times 6//32$, DFPE $= 5$: Collapse 165 via 123/123/1234/12345644.
158. $3 \times 3 \times 4 \times 6//32$, DFPE $= 7$: Dose not exist by Theorem 2.5.
159. $3 \times 3 \times 4 \times 6//32$, DFPE $= 8$: Collapse 165 via 123/123/1234/12345634.
160. $3 \times 3 \times 4 \times 7//32$, DFPE $= 0$: Juxtapose 24 and 28 with distinct symbols on row 3.
161. $3 \times 3 \times 4 \times 7//32$, DFPE $= 1$: Juxtapose 25 and 28 with distinct symbols on row 3.
162. $3 \times 3 \times 4 \times 7//32$, DFPE $= 2$: Collapse 173 via 123/1233/1234/1234567.
163. $3 \times 3 \times 4 \times 7//32$, DFPE $= 3$: Dose not exist by Theorem 2.5.
164. $3 \times 3 \times 4 \times 7//32$, DFPE $= 4$: Collapse 165 via 123/123/1234/12345674.
165. $3 \times 3 \times 4 \times 8//32$, DFPE $= 0$: Juxtapose 28 and 28 with distinct symbols on row 3.
166. $3 \times 4 \times 4 \times 6//32$, DFPE $= 0$: Juxtapose 28 and 28 with distinct symbols on row 1.
167. $3 \times 4 \times 4 \times 6//32$, DFPE $= 1$: Collapse 172 via 123/1234/1234/1234526.
168. $3 \times 4 \times 4 \times 6//32$, DFPE $= 2$: Collapse 172 via 123/1234/1234/1234565.
169. $3 \times 4 \times 4 \times 6//32$, DFPE $= 4$: Collapse 175 via 123/1234/1234/1234566.
170. $3 \times 4 \times 4 \times 6//32$, DFPE $= 7$: Dose not exist by Theorem 2.5.
171. $3 \times 4 \times 4 \times 6//32$, DFPE $= 8$: Collapse 175 via 123/1234/1234/1234536.
172. $3 \times 4 \times 4 \times 7//32$, DFPE $= 0$: Juxtapose 28 and 29 with distinct symbols on row 2.
173. $3 \times 4 \times 4 \times 7//32$, DFPE $= 1$: Collapse 184 via 1233/1234/1234/1234567.
174. $3 \times 4 \times 4 \times 7//32$, DFPE $= 3$: Dose not exist by Theorem 2.5.
175. $3 \times 4 \times 4 \times 7//32$, DFPE $= 4$: Collapse 176 via 123/1234/1234/12345674.
176. $3 \times 4 \times 4 \times 8//32$, DFPE $= 0$: Juxtapose 29 and 29 with distinct symbols on row 2.
177. $4 \times 4 \times 4 \times 6//32$, DFPE $= 0$: Juxtapose 23 and 30 with distinct symbols on row 1.
178. $4 \times 4 \times 4 \times 6//32$, DFPE $= 1$: Collapse 183 via 1234/1234/1234/1234566.
179. $4 \times 4 \times 4 \times 6//32$, DFPE $= 2$: Collapse 184 via 1234/1234/1234/1234564.
180. $4 \times 4 \times 4 \times 6//32$, DFPE $= 4$: Collapse 187 via 1234/1234/1234/12345664.
181. $4 \times 4 \times 4 \times 6//32$, DFPE $= 7$: Dose not exist by Theorem 2.5.
182. $4 \times 4 \times 4 \times 6//32$, DFPE $= 8$: Collapse 187 via 1234/1234/1234/12345634.
183. $4 \times 4 \times 4 \times 7//32$, DFPE $= 0$: Juxtapose 29 and 30 with distinct symbols on row 1.

184. $4 \times 4 \times 4 \times 7//32$, DFPE $= 1$: First an 4x4x4x4//32 OMEP was constructed by juxtaposing 2 pairs of MOLS(16) and then a 4x4x4x8//32 OMEP was obtained by changeing the symbols of the the last square to be disjoint from the other three. Then the last row was collapsed to 7 symbols.
185. $4 \times 4 \times 4 \times 7//32$, DFPE $= 3$: Dose not exist by Theorem 2.5.
186. $4 \times 4 \times 4 \times 7//32$, DFPE $= 4$: Collapse 187 via 1234/1234/1234/12345674.
187. $4 \times 4 \times 4 \times 8//32$, DFPE $= 0$: Juxtapose 30 and 30 with distinct symbols on row 1.
188. $2 \times 2 \times 3 \times 10//36$, DFPE $= 0$: Juxtapose 31 and 31 with distinct symbols on row 4.
189. $2 \times 2 \times 3 \times 10//36$, DFPE $= 1$: Juxtapose 31 and 32 with distinct symbols on row 4.
190. $2 \times 2 \times 3 \times 10//36$, DFPE $= 2$: Juxtapose 32 and 32 with distinct symbols on row 4.
191. $2 \times 2 \times 3 \times 10//36$, DFPE $= 3$: Juxtapose 31 and 34 with distinct symbols on row 4.
192. $2 \times 2 \times 3 \times 10//36$, DFPE $= 4$: Juxtapose 32 and 34 with distinct symbols on row 4.
193. $2 \times 2 \times 3 \times 10//36$, DFPE $= 5$: Dose not exist by Theorem 2.5.
194. $2 \times 2 \times 3 \times 10//36$, DFPE $= 6$: Juxtapose 34 and 34 with distinct symbols on row 4.
195. $2 \times 2 \times 5 \times 6//36$, DFPE $= 0$: Juxtapose 31 and 31 with distinct symbols on row 3.
196. $2 \times 2 \times 5 \times 6//36$, DFPE $= 1$: Juxtapose 31 and 32 with distinct symbols on row 3.
197. $2 \times 2 \times 5 \times 6//36$, DFPE $= 2$: Juxtapose 32 and 32 with distinct symbols on row 3.
198. $2 \times 2 \times 5 \times 6//36$, DFPE $= 3$: Juxtapose 31 and 34 with distinct symbols on row 3.
199. $2 \times 2 \times 5 \times 6//36$, DFPE $= 4$: Juxtapose 32 and 34 with distinct symbols on row 3.
200. $2 \times 2 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
201. $2 \times 2 \times 5 \times 6//36$, DFPE $= 6$: Juxtapose 34 and 34 with distinct symbols on row 3.
202. $2 \times 2 \times 6 \times 6//36$, DFPE $= 0$: Juxtapose 35 and 35 with distinct symbols on row 3.
203. $2 \times 3 \times 3 \times 10//36$, DFPE $= 0$: Juxtapose 36 and 36 with distinct symbols on row 4.
204. $2 \times 3 \times 3 \times 10//36$, DFPE $= 1$: Juxtapose 36 and 37 with distinct symbols on row 4.
205. $2 \times 3 \times 3 \times 10//36$, DFPE $= 2$: Juxtapose 37 and 37 with distinct symbols on row 4.
206. $2 \times 3 \times 3 \times 10//36$, DFPE $= 3$: Juxtapose 36 and 39 with distinct symbols on row 4.
207. $2 \times 3 \times 3 \times 10//36$, DFPE $= 4$: Juxtapose 37 and 39 with distinct symbols on row 4.
208. $2 \times 3 \times 3 \times 10//36$, DFPE $= 5$: Dose not exist by Theorem 2.5.
209. $2 \times 3 \times 3 \times 10//36$, DFPE $= 6$: Juxtapose 39 and 39 with distinct symbols on row 4.
210. $2 \times 3 \times 5 \times 6//36$, DFPE $= 0$: Juxtapose 35 and 40 with distinct symbols on row 2.
211. $2 \times 3 \times 5 \times 6//36$, DFPE $= 1$: Juxtapose 36 and 37 with distinct symbols on row 2.
212. $2 \times 3 \times 5 \times 6//36$, DFPE $= 2$: Juxtapose 37 and 37 with distinct symbols on row 2.
213. $2 \times 3 \times 5 \times 6//36$, DFPE $= 3$: Juxtapose 36 and 39 with distinct symbols on row 2.
214. $2 \times 3 \times 5 \times 6//36$, DFPE $= 4$: Juxtapose 37 and 39 with distinct symbols on row 2.
215. $2 \times 3 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
216. $2 \times 3 \times 5 \times 6//36$, DFPE $= 6$: Juxtapose 39 and 39 with distinct symbols on row 2.
217. $2 \times 3 \times 6 \times 6//36$, DFPE $= 0$: Juxtapose 40 and 40 with distinct symbols on row 2.
218. $2 \times 4 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
219. $2 \times 5 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
220. $3 \times 3 \times 3 \times 10//36$, DFPE $= 0$: Juxtapose 41 and 41 with distinct symbols on row 4.
221. $3 \times 3 \times 3 \times 10//36$, DFPE $= 1$: Juxtapose 41 and 42 with distinct symbols on row 4.
222. $3 \times 3 \times 3 \times 10//36$, DFPE $= 2$: Juxtapose 42 and 42 with distinct symbols on row 4.
223. $3 \times 3 \times 3 \times 10//36$, DFPE $= 3$: Juxtapose 41 and 44 with distinct symbols on row 4.
224. $3 \times 3 \times 3 \times 10//36$, DFPE $= 4$: Juxtapose 42 and 44 with distinct symbols on row 4.
225. $3 \times 3 \times 3 \times 10//36$, DFPE $= 5$: Dose not exist by Theorem 2.5.
226. $3 \times 3 \times 3 \times 10//36$, DFPE $= 6$: Juxtapose 44 and 44 with distinct symbols on row 4.
227. $3 \times 3 \times 5 \times 6//36$, DFPE $= 0$: Juxtapose 40 and 45 with distinct symbols on row 1.

228. $3 \times 3 \times 5 \times 6//36$, DFPE $= 1$: Juxtapose 41 and 42 with distinct symbols on row 1.
229. $3 \times 3 \times 5 \times 6//36$, DFPE $= 2$: Juxtapose 42 and 42 with distinct symbols on row 1.
230. $3 \times 3 \times 5 \times 6//36$, DFPE $= 3$: Juxtapose 41 and 44 with distinct symbols on row 1.
231. $3 \times 3 \times 5 \times 6//36$, DFPE $= 4$: Juxtapose 42 and 44 with distinct symbols on row 1.
232. $3 \times 3 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
233. $3 \times 3 \times 5 \times 6//36$, DFPE $= 6$: Juxtapose 44 and 44 with distinct symbols on row 1.
234. $3 \times 3 \times 6 \times 6//36$, DFPE $= 0$: Juxtapose 45 and 45 with distinct symbols on row 1.
235. $3 \times 4 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
236. $3 \times 5 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
237. $4 \times 4 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
238. $4 \times 5 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
239. $5 \times 5 \times 5 \times 6//36$, DFPE $= 5$: Dose not exist by Theorem 2.6.
240. $6 \times 6 \times 6 \times 6//36$, DFPE $= 0$: This would coresponds to a pair of orthoganal latin squares of order 6, which does not exist.
241. $2 \times 2 \times 4 \times 10//40$, DFPE $= 0$:

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1122221111222211112222111122221111222 2 2 1 1
1212212112122121121221211212212112122 1 2 1 2
1234123412341234123412341234123412341234 1 2 3 4
1111222233334444555566667777888899991010101010
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242. $2 \times 2 \times 4 \times 9//40$, DFPE $= 0$: Juxtapose 46 and 46 with distinct symbols on row 1.
243. $2 \times 2 \times 4 \times 9//40$, DFPE $= 3$: Dose not exist by Theorem 2.5.
244. $2 \times 2 \times 4 \times 9//40$, DFPE $= 4$: Collapse 241 via 12/12/1234/1234567898.
245. $2 \times 2 \times 5 \times 7//48$, DFPE $= 0$: Collapse 267 via 12/12/123455/1234567.
246. $2 \times 2 \times 5 \times 7//48$, DFPE $= 1$: Collapse 258 via 12/12/12345/12345677.
247. $2 \times 2 \times 5 \times 7//48$, DFPE $= 10$: Collapse 264 via 12/12/12345/12345674.
248. $2 \times 2 \times 5 \times 7//48$, DFPE $= 11$: Collapse 264 via 12/12/12345/12345637.
249. $2 \times 2 \times 5 \times 7//48$, DFPE $= 13$: Collapse 266 via 12/12/12345/12345674.
250. $2 \times 2 \times 5 \times 7//48$, DFPE $= 2$: Collapse 258 via 12/12/12345/12345675.
251. $2 \times 2 \times 5 \times 7//48$, DFPE $= 3$: Collapse 259 via 12/12/12345/12345677.
252. $2 \times 2 \times 5 \times 7//48$, DFPE $= 4$: Collapse 259 via 12/12/12345/12345627.
253. $2 \times 2 \times 5 \times 7//48$, DFPE $= 5$: Collapse 261 via 12/12/12345/12345676.
254. $2 \times 2 \times 5 \times 7//48$, DFPE $= 6$: Collapse 258 via 12/12/12345/12345674.
255. $2 \times 2 \times 5 \times 7//48$, DFPE $= 7$: Collapse 261 via 12/12/12345/12341567.
256. $2 \times 2 \times 5 \times 7//48$, DFPE $= 8$: Collapse 261 via 12/12/12345/12345674.
257. $2 \times 2 \times 5 \times 7//48$, DFPE $= 9$: Collapse 263 via 12/12/12345/12345674.
258. $2 \times 2 \times 5 \times 8//48$, DFPE $= 0$: Juxtapose 49 and 49 with distinct symbols on row 3.
259. $2 \times 2 \times 5 \times 8//48$, DFPE $= 1$: Juxtapose 49 and 50 with distinct symbols on row 3.
260. $2 \times 2 \times 5 \times 8//48$, DFPE $= 2$: Juxtapose 49 and 51 with distinct symbols on row 3.
261. $2 \times 2 \times 5 \times 8//48$, DFPE $= 3$: Juxtapose 50 and 51 with distinct symbols on row 3.
262. $2 \times 2 \times 5 \times 8//48$, DFPE $= 4$: Juxtapose 49 and 53 with distinct symbols on row 3.
263. $2 \times 2 \times 5 \times 8//48$, DFPE $= 5$: Juxtapose 50 and 53 with distinct symbols on row 3.
264. $2 \times 2 \times 5 \times 8//48$, DFPE $= 6$: Juxtapose 51 and 53 with distinct symbols on row 3.
265. $2 \times 2 \times 5 \times 8//48$, DFPE $= 7$: Dose not exist by Theorem 2.6.
266. $2 \times 2 \times 5 \times 8//48$, DFPE $= 8$: Juxtapose 53 and 53 with distinct symbols on row 3.
267. $2 \times 2 \times 6 \times 7//48$, DFPE $= 0$: Juxtapose 8 and 120 with distinct symbols on row 3.
268. $2 \times 2 \times 6 \times 7//48$, DFPE $= 1$: Juxtapose 8 and 121 with distinct symbols on row 3.
269. $2 \times 2 \times 6 \times 7//48$, DFPE $= 2$: Juxtapose 8 and 122 with distinct symbols on row 3.

270. $2 \times 2 \times 6 \times 7//48$, DFPE $= 3$: Juxtapose 10 and 121 with distinct symbols on row 3.
271. $2 \times 2 \times 6 \times 7//48$, DFPE $= 4$: Juxtapose 8 and 124 with distinct symbols on row 3.
272. $2 \times 2 \times 6 \times 7//48$, DFPE $= 5$: Dose not exist by Theorem 2.5.
273. $2 \times 2 \times 6 \times 7//48$, DFPE $= 6$: Juxtapose 10 and 124 with distinct symbols on row 3.
274. $2 \times 2 \times 6 \times 8//48$, DFPE $= 0$: Juxtapose 11 and 125 with distinct symbols on row 3.
275. $2 \times 3 \times 4 \times 10//48$, DFPE $= 0$: Juxtapose 22 and 126 with distinct symbols on row 4.
276. $2 \times 3 \times 4 \times 10//48$, DFPE $= 1$: Juxtapose 22 and 127 with distinct symbols on row 4.
277. $2 \times 3 \times 4 \times 10//48$, DFPE $= 2$: Juxtapose 22 and 128 with distinct symbols on row 4.
278. $2 \times 3 \times 4 \times 10//48$, DFPE $= 3$: Juxtapose 22 and 129 with distinct symbols on row 4.
279. $2 \times 3 \times 4 \times 10//48$, DFPE $= 4$: Juxtapose 22 and 130 with distinct symbols on row 4.
280. $2 \times 3 \times 4 \times 10//48$, DFPE $= 5$: Juxtapose 22 and 131 with distinct symbols on row 4.
281. $2 \times 3 \times 4 \times 10//48$, DFPE $= 6$: Juxtapose 22 and 132 with distinct symbols on row 4.
282. $2 \times 3 \times 4 \times 10//48$, DFPE $= 7$: Dose not exist by Theorem 2.5.
283. $2 \times 3 \times 4 \times 10//48$, DFPE $= 8$: Juxtapose 22 and 134 with distinct symbols on row 4.
284. $2 \times 3 \times 4 \times 9//48$, DFPE $= 0$: Collapse 353 via 1221/1233/1234/1234567899.
285. $2 \times 3 \times 4 \times 9//48$, DFPE $= 1$: Collapse 353 via 1222/1232/1234/1234567897.
286. $2 \times 3 \times 4 \times 9//48$, DFPE $= 10$: Collapse 279 via 12/123/1234/1234567899.
287. $2 \times 3 \times 4 \times 9//48$, DFPE $= 11$: Dose not exist by Theorem 2.5.
288. $2 \times 3 \times 4 \times 9//48$, DFPE $= 12$: Collapse 283 via 12/123/1234/1234567689.
289. $2 \times 3 \times 4 \times 9//48$, DFPE $= 2$: Collapse 353 via 1222/1233/1234/1234567899.
290. $2 \times 3 \times 4 \times 9//48$, DFPE $= 3$: Collapse 353 via 1211/1233/1234/1234567894.
291. $2 \times 3 \times 4 \times 9//48$, DFPE $= 4$: Collapse 353 via 1122/1233/1234/1234567899.
292. $2 \times 3 \times 4 \times 9//48$, DFPE $= 5$: Collapse 353 via 1122/1233/1234/1234567894.
293. $2 \times 3 \times 4 \times 9//48$, DFPE $= 6$: Collapse 353 via 1222/1233/1234/1234567898.
294. $2 \times 3 \times 4 \times 9//48$, DFPE $= 7$: Collapse 280 via 12/123/1234/1234567894.
295. $2 \times 3 \times 4 \times 9//48$, DFPE $= 8$: Collapse 353 via 1122/1233/1234/1234567898.
296. $2 \times 3 \times 4 \times 9//48$, DFPE $= 9$: Collapse 280 via 12/123/1234/1234567879.
297. $2 \times 4 \times 4 \times 10//48$, DFPE $= 0$: Juxtapose 23 and 141 with distinct symbols on row 4.
298. $2 \times 4 \times 4 \times 10//48$, DFPE $= 1$: Juxtapose 23 and 142 with distinct symbols on row 4.
299. $2 \times 4 \times 4 \times 10//48$, DFPE $= 2$: Juxtapose 23 and 143 with distinct symbols on row 4.
300. $2 \times 4 \times 4 \times 10//48$, DFPE $= 4$: Juxtapose 23 and 144 with distinct symbols on row 4.
301. $2 \times 4 \times 4 \times 10//48$, DFPE $= 7$: Dose not exist by Theorem 2.5.
302. $2 \times 4 \times 4 \times 10//48$, DFPE $= 8$: Juxtapose 23 and 146 with distinct symbols on row 4.
303. $2 \times 4 \times 4 \times 9//48$, DFPE $= 0$: Collapse 353 via 1222/1234/1234/1234567899.
304. $2 \times 4 \times 4 \times 9//48$, DFPE $= 1$: Collapse 353 via 1212/1234/1234/1234567894.
305. $2 \times 4 \times 4 \times 9//48$, DFPE $= 10$: Collapse 358 via 1234/1234/1222/1234567892.
306. $2 \times 4 \times 4 \times 9//48$, DFPE $= 11$: Dose not exist by Theorem 2.5.
307. $2 \times 4 \times 4 \times 9//48$, DFPE $= 12$: Collapse 302 via 12/1234/1234/1234567689.
308. $2 \times 4 \times 4 \times 9//48$, DFPE $= 2$: Collapse 353 via 1222/1234/1234/1234567894.
309. $2 \times 4 \times 4 \times 9//48$, DFPE $= 3$: Collapse 297 via 12/1234/1234/1234567899.
310. $2 \times 4 \times 4 \times 9//48$, DFPE $= 4$: Collapse 353 via 1222/1234/1234/1234567898.
311. $2 \times 4 \times 4 \times 9//48$, DFPE $= 5$: Collapse 298 via 12/1234/1234/1234156789.
312. $2 \times 4 \times 4 \times 9//48$, DFPE $= 6$: Collapse 299 via 12/1234/1234/1234156789.
313. $2 \times 4 \times 4 \times 9//48$, DFPE $= 8$: Collapse 300 via 12/1234/1234/1234567897.
314. $2 \times 4 \times 4 \times 9//48$, DFPE $= 9$: Collapse 302 via 12/1234/1234/1234567893.
315. $3 \times 3 \times 4 \times 10//48$, DFPE $= 0$: Juxtapose 28 and 152 with distinct symbols on row 4.
316. $3 \times 3 \times 4 \times 10//48$, DFPE $= 1$: Juxtapose 28 and 153 with distinct symbols on row 4.

317. $3 \times 3 \times 4 \times 10//48$, DFPE $= 2$: Juxtapose 28 and 154 with distinct symbols on row 4.
318. $3 \times 3 \times 4 \times 10//48$, DFPE $= 3$: Juxtapose 28 and 155 with distinct symbols on row 4.
319. $3 \times 3 \times 4 \times 10//48$, DFPE $= 4$: Juxtapose 28 and 156 with distinct symbols on row 4.
320. $3 \times 3 \times 4 \times 10//48$, DFPE $= 5$: Juxtapose 28 and 157 with distinct symbols on row 4.
321. $3 \times 3 \times 4 \times 10//48$, DFPE $= 7$: Dose not exist by Theorem 2.5.
322. $3 \times 3 \times 4 \times 10//48$, DFPE $= 8$: Juxtapose 28 and 159 with distinct symbols on row 4.
323. $3 \times 3 \times 4 \times 9//48$, DFPE $= 0$: Collapse 353 via 1232/1233/1234/1234567899.
324. $3 \times 3 \times 4 \times 9//48$, DFPE $= 1$: Collapse 353 via 1232/1232/1234/1234567897.
325. $3 \times 3 \times 4 \times 9//48$, DFPE $= 10$: Collapse 341 via 123/1232/1234/1234567893.
326. $3 \times 3 \times 4 \times 9//48$, DFPE $= 11$: Dose not exist by Theorem 2.5.
327. $3 \times 3 \times 4 \times 9//48$, DFPE $= 12$: Collapse 322 via 123/123/1234/1234567689.
328. $3 \times 3 \times 4 \times 9//48$, DFPE $= 2$: Collapse 353 via 1233/1233/1234/1234567899.
329. $3 \times 3 \times 4 \times 9//48$, DFPE $= 3$: Collapse 353 via 1233/1233/1234/1234567894.
330. $3 \times 3 \times 4 \times 9//48$, DFPE $= 4$: Collapse 353 via 1232/1233/1234/1234567898.
331. $3 \times 3 \times 4 \times 9//48$, DFPE $= 5$: Collapse 316 via 123/123/1234/1234567849.
332. $3 \times 3 \times 4 \times 9//48$, DFPE $= 6$: Collapse 353 via 1233/1233/1234/1234567898.
333. $3 \times 3 \times 4 \times 9//48$, DFPE $= 7$: Collapse 339 via 123/1232/1234/1234567893.
334. $3 \times 3 \times 4 \times 9//48$, DFPE $= 8$: Collapse 315 via 123/123/1234/1234567899.
335. $3 \times 3 \times 4 \times 9//48$, DFPE $= 9$: Collapse 317 via 123/123/1234/1234567899.
336. $3 \times 4 \times 4 \times 10//48$, DFPE $= 0$: Juxtapose 29 and 166 with distinct symbols on row 4.
337. $3 \times 4 \times 4 \times 10//48$, DFPE $= 1$: Juxtapose 29 and 167 with distinct symbols on row 4.
338. $3 \times 4 \times 4 \times 10//48$, DFPE $= 2$: Juxtapose 29 and 168 with distinct symbols on row 4.
339. $3 \times 4 \times 4 \times 10//48$, DFPE $= 4$: Juxtapose 29 and 169 with distinct symbols on row 4.
340. $3 \times 4 \times 4 \times 10//48$, DFPE $= 7$: Dose not exist by Theorem 2.5.
341. $3 \times 4 \times 4 \times 10//48$, DFPE $= 8$: Juxtapose 29 and 171 with distinct symbols on row 4.
342. $3 \times 4 \times 4 \times 9//48$, DFPE $= 0$: Collapse 353 via 1233/1234/1234/1234567899.
343. $3 \times 4 \times 4 \times 9//48$, DFPE $= 1$: Collapse 360 via 1233/1234/1234/123456789.
344. $3 \times 4 \times 4 \times 9//48$, DFPE $= 11$: Dose not exist by Theorem 2.5.
345. $3 \times 4 \times 4 \times 9//48$, DFPE $= 12$: Collapse 341 via 123/1234/1234/1234567689.
346. $3 \times 4 \times 4 \times 9//48$, DFPE $= 2$: Collapse 360 via 1223/1234/1234/123456789.
347. $3 \times 4 \times 4 \times 9//48$, DFPE $= 3$: Collapse 337 via 123/1234/1234/1234567898.
348. $3 \times 4 \times 4 \times 9//48$, DFPE $= 4$: Collapse 353 via 1233/1234/1234/1234567898.
349. $3 \times 4 \times 4 \times 9//48$, DFPE $= 5$: Collapse 339 via 123/1234/1234/1234567893.
350. $3 \times 4 \times 4 \times 9//48$, DFPE $= 6$: Collapse 339 via 123/1234/1234/1234567891.
351. $3 \times 4 \times 4 \times 9//48$, DFPE $= 8$: Collapse 336 via 123/1234/1234/1234567899.
352. $3 \times 4 \times 4 \times 9//48$, DFPE $= 9$: Collapse 341 via 123/1234/1234/1234567893.
353. $4 \times 4 \times 4 \times 10//48$, DFPE $= 0$: Juxtapose 30 and 177 with distinct symbols on row 4.
354. $4 \times 4 \times 4 \times 10//48$, DFPE $= 1$: Juxtapose 30 and 178 with distinct symbols on row 4.
355. $4 \times 4 \times 4 \times 10//48$, DFPE $= 2$: Juxtapose 30 and 179 with distinct symbols on row 4.
356. $4 \times 4 \times 4 \times 10//48$, DFPE $= 4$: Juxtapose 30 and 180 with distinct symbols on row 4.
357. $4 \times 4 \times 4 \times 10//48$, DFPE $= 7$: Dose not exist by Theorem 2.5.
358. $4 \times 4 \times 4 \times 10//48$, DFPE $= 8$: Juxtapose 30 and 182 with distinct symbols on row 4.
359. $4 \times 4 \times 4 \times 9//48$, DFPE $= 0$: Collapse 353 via 1234/1234/1234/1234567899.
360. $4 \times 4 \times 4 \times 9//48$, DFPE $= 1$: Collapse 353 via 1234/1234/1234/1234567893.
361. $4 \times 4 \times 4 \times 9//48$, DFPE $= 11$: Dose not exist by Theorem 2.5.
362. $4 \times 4 \times 4 \times 9//48$, DFPE $= 12$: Collapse 358 via 1234/1234/1234/1234567689.
363. $4 \times 4 \times 4 \times 9//48$, DFPE $= 2$: Collapse 354 via 1234/1234/1234/1234567899.

364. $4 \times 4 \times 4 \times 9//48$, DFPE $= 3$: Collapse 355 via 1234/1234/1234/1234567898.
365. $4 \times 4 \times 4 \times 9//48$, DFPE $= 4$: Collapse 353 via 1234/1234/1234/1234567898.
366. $4 \times 4 \times 4 \times 9//48$, DFPE $= 5$: Collapse 356 via 1234/1234/1234/1234567894.
367. $4 \times 4 \times 4 \times 9//48$, DFPE $= 8$: Collapse 356 via 1234/1234/1234/1234567897.
368. $4 \times 4 \times 4 \times 9//48$, DFPE $= 9$: Collapse 358 via 1234/1234/1234/1234567892.
369. $2 \times 2 \times 7 \times 7//49$, DFPE $= 0$: Collapse 386 via 12/122/1234567/1234567.
370. $2 \times 3 \times 5 \times 7//49$, DFPE $= 0$: Collapse 380 via 12/123/123445/1234567.
371. $2 \times 3 \times 5 \times 7//49$, DFPE $= 1$: Collapse 380 via 12/123/123435/1234567.
372. $2 \times 3 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
373. $2 \times 3 \times 5 \times 7//49$, DFPE $= 2$: Collapse 380 via 12/123/123425/1234567.
374. $2 \times 3 \times 5 \times 7//49$, DFPE $= 3$: Collapse 380 via 12/123/123455/1234567.
375. $2 \times 3 \times 5 \times 7//49$, DFPE $= 4$: Collapse 380 via 12/123/123454/1234567.
376. $2 \times 3 \times 5 \times 7//49$, DFPE $= 5$: Collapse 380 via 12/123/123452/1234567.
377. $2 \times 3 \times 5 \times 7//49$, DFPE $= 6$: Collapse 381 via 12/123/123452/1234567.
378. $2 \times 3 \times 5 \times 7//49$, DFPE $= 7$: Collapse 383 via 12/123/123451/1234567.
379. $2 \times 3 \times 5 \times 7//49$, DFPE $= 8$: Collapse 384 via 12/123/123453/1234567.
380. $2 \times 3 \times 6 \times 7//49$, DFPE $= 0$: Collapse 395 via 12/1223/123456/1234567.
381. $2 \times 3 \times 6 \times 7//49$, DFPE $= 1$: Collapse 386 via 12/123/1234556/1234567.
382. $2 \times 3 \times 6 \times 7//49$, DFPE $= 2$: Collapse 386 via 12/123/1234566/1234567.
383. $2 \times 3 \times 6 \times 7//49$, DFPE $= 3$: Collapse 386 via 12/123/1234564/1234567.
384. $2 \times 3 \times 6 \times 7//49$, DFPE $= 4$: Collapse 386 via 12/123/1234567/1234561.
385. $2 \times 3 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
386. $2 \times 3 \times 7 \times 7//49$, DFPE $= 0$: Collapse 400 via 12/1233/1234567/1234567.
387. $2 \times 4 \times 5 \times 7//49$, DFPE $= 0$: Collapse 401 via 12/12334/12345/1234567.
388. $2 \times 4 \times 5 \times 7//49$, DFPE $= 1$: Collapse 401 via 12/12314/12345/1234567.
389. $2 \times 4 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
390. $2 \times 4 \times 5 \times 7//49$, DFPE $= 2$: Collapse 401 via 12/12344/12345/1234567.
391. $2 \times 4 \times 5 \times 7//49$, DFPE $= 3$: Collapse 401 via 12/12343/12345/1234567.
392. $2 \times 4 \times 5 \times 7//49$, DFPE $= 4$: Collapse 402 via 12/12343/12345/1234567.
393. $2 \times 4 \times 5 \times 7//49$, DFPE $= 5$: Collapse 405 via 12/12342/12345/1234567.
394. $2 \times 4 \times 5 \times 7//49$, DFPE $= 6$: Collapse 406 via 12/12234/12345/1234567.
395. $2 \times 4 \times 6 \times 7//49$, DFPE $= 0$: Collapse 400 via 12/1234/1234562/1234567.
396. $2 \times 4 \times 6 \times 7//49$, DFPE $= 1$: Collapse 400 via 12/1234/1234566/1234567.
397. $2 \times 4 \times 6 \times 7//49$, DFPE $= 2$: Collapse 400 via 12/1234/1234563/1234567.
398. $2 \times 4 \times 6 \times 7//49$, DFPE $= 3$: Collapse 400 via 12/1234/1234564/1234567.
399. $2 \times 4 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
400. $2 \times 4 \times 7 \times 7//49$, DFPE $= 0$: Collapse 411 via 12/12344/1234567/1234567.
401. $2 \times 5 \times 5 \times 7//49$, DFPE $= 0$: Collapse 407 via 12/12345/123455/1234567.
402. $2 \times 5 \times 5 \times 7//49$, DFPE $= 1$: Collapse 407 via 12/12345/123452/1234567.
403. $2 \times 5 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
404. $2 \times 5 \times 5 \times 7//49$, DFPE $= 2$: Collapse 407 via 12/12345/123454/1234567.
405. $2 \times 5 \times 5 \times 7//49$, DFPE $= 3$: Collapse 408 via 12/12345/123451/1234567.
406. $2 \times 5 \times 5 \times 7//49$, DFPE $= 4$: Collapse 409 via 12/12345/123451/1234567.
407. $2 \times 5 \times 6 \times 7//49$, DFPE $= 0$: Collapse 412 via 12/123455/123456/1234567.
408. $2 \times 5 \times 6 \times 7//49$, DFPE $= 1$: Collapse 412 via 12/123454/123456/1234567.
409. $2 \times 5 \times 6 \times 7//49$, DFPE $= 2$: Collapse 413 via 12/123455/123456/1234567.
410. $2 \times 5 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.

411. $2 \times 5 \times 7 \times 7//49$, DFPE $= 0$: Collapse 415 via 12/123455/1234567/1234567.
412. $2 \times 6 \times 6 \times 7//49$, DFPE $= 0$: Collapse 415 via 12/123456/1234566/1234567.
413. $2 \times 6 \times 6 \times 7//49$, DFPE $= 1$: Collapse 415 via 12/123456/1234564/1234567.
414. $2 \times 6 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
415. $2 \times 6 \times 7 \times 7//49$, DFPE $= 0$: Collapse 416 via 12/1234566/1234567/1234567.
416. $2 \times 7 \times 7 \times 7//49$, DFPE $= 0$: Collapse 461 via 122/1234567/1234567/1234567.
417. $3 \times 3 \times 5 \times 7//49$, DFPE $= 0$: Collapse 433 via 123/1223/12345/1234567.
418. $3 \times 3 \times 5 \times 7//49$, DFPE $= 1$: Collapse 426 via 123/123/123435/1234567.
419. $3 \times 3 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
420. $3 \times 3 \times 5 \times 7//49$, DFPE $= 2$: Collapse 426 via 123/123/123445/1234567.
421. $3 \times 3 \times 5 \times 7//49$, DFPE $= 3$: Collapse 426 via 123/123/123455/1234567.
422. $3 \times 3 \times 5 \times 7//49$, DFPE $= 4$: Collapse 426 via 123/123/123451/1234567.
423. $3 \times 3 \times 5 \times 7//49$, DFPE $= 5$: Collapse 427 via 123/123/123451/1234567.
424. $3 \times 3 \times 5 \times 7//49$, DFPE $= 6$: Collapse 429 via 123/123/123453/1234567.
425. $3 \times 3 \times 5 \times 7//49$, DFPE $= 7$: Collapse 430 via 123/123/123445/1234567.
426. $3 \times 3 \times 6 \times 7//49$, DFPE $= 0$: Collapse 432 via 123/123/1234456/1234567.
427. $3 \times 3 \times 6 \times 7//49$, DFPE $= 1$: Collapse 432 via 123/123/1234565/1234567.
428. $3 \times 3 \times 6 \times 7//49$, DFPE $= 2$: Collapse 432 via 123/123/1234566/1234567.
429. $3 \times 3 \times 6 \times 7//49$, DFPE $= 3$: Collapse 432 via 123/123/1234561/1234567.
430. $3 \times 3 \times 6 \times 7//49$, DFPE $= 4$: Collapse 456 via 123/11223/1234567/1234561.
431. $3 \times 3 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
432. $3 \times 3 \times 7 \times 7//49$, DFPE $= 0$: Collapse 445 via 123/1233/1234567/1234567.
433. $3 \times 4 \times 5 \times 7//49$, DFPE $= 0$: Collapse 440 via 123/1234/123445/1234567.
434. $3 \times 4 \times 5 \times 7//49$, DFPE $= 1$: Collapse 440 via 123/1234/123435/1234567.
435. $3 \times 4 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
436. $3 \times 4 \times 5 \times 7//49$, DFPE $= 2$: Collapse 440 via 123/1234/123455/1234567.
437. $3 \times 4 \times 5 \times 7//49$, DFPE $= 3$: Collapse 440 via 123/1234/123452/1234567.
438. $3 \times 4 \times 5 \times 7//49$, DFPE $= 4$: Collapse 440 via 123/1234/123451/1234567.
439. $3 \times 4 \times 5 \times 7//49$, DFPE $= 5$: Collapse 443 via 123/1234/123345/1234567.
440. $3 \times 4 \times 6 \times 7//49$, DFPE $= 0$: Collapse 445 via 123/1234/1234562/1234567.
441. $3 \times 4 \times 6 \times 7//49$, DFPE $= 1$: Collapse 445 via 123/1234/1234566/1234567.
442. $3 \times 4 \times 6 \times 7//49$, DFPE $= 2$: Collapse 445 via 123/1234/1234564/1234567.
443. $3 \times 4 \times 6 \times 7//49$, DFPE $= 3$: Collapse 454 via 123/12343/123456/1234567.
444. $3 \times 4 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
445. $3 \times 4 \times 7 \times 7//49$, DFPE $= 0$: Collapse 456 via 123/12344/1234567/1234567.
446. $3 \times 5 \times 5 \times 7//49$, DFPE $= 0$: Collapse 452 via 123/12345/123455/1234567.
447. $3 \times 5 \times 5 \times 7//49$, DFPE $= 1$: Collapse 452 via 123/12345/123415/1234567.
448. $3 \times 5 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
449. $3 \times 5 \times 5 \times 7//49$, DFPE $= 2$: Collapse 452 via 123/12345/123454/1234567.
450. $3 \times 5 \times 5 \times 7//49$, DFPE $= 3$: Collapse 454 via 123/12345/123455/1234567.
451. $3 \times 5 \times 5 \times 7//49$, DFPE $= 4$: Collapse 454 via 123/12345/122345/1234567.
452. $3 \times 5 \times 6 \times 7//49$, DFPE $= 0$: Collapse 457 via 123/123455/123456/1234567.
453. $3 \times 5 \times 6 \times 7//49$, DFPE $= 1$: Collapse 457 via 123/123454/123456/1234567.
454. $3 \times 5 \times 6 \times 7//49$, DFPE $= 2$: Collapse 458 via 123/123451/123456/1234567.
455. $3 \times 5 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
456. $3 \times 5 \times 7 \times 7//49$, DFPE $= 0$: Collapse 460 via 123/123455/1234567/1234567.
457. $3 \times 6 \times 6 \times 7//49$, DFPE $= 0$: Collapse 460 via 123/123456/1234566/1234567.

458. $3 \times 6 \times 6 \times 7//49$, DFPE $= 1$: Collapse 460 via 123/123456/1234564/1234567.
459. $3 \times 6 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
460. $3 \times 6 \times 7 \times 7//49$, DFPE $= 0$: Collapse 461 via 123/1234566/1234567/1234567.
461. $3 \times 7 \times 7 \times 7//49$, DFPE $= 0$: Collapse 490 via 1233/1234567/1234567/1234567.
462. $4 \times 4 \times 5 \times 7//49$, DFPE $= 0$: Collapse 475 via 1234/12342/12345/1234567.
463. $4 \times 4 \times 5 \times 7//49$, DFPE $= 1$: Collapse 475 via 1234/12344/12345/1234567.
464. $4 \times 4 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
465. $4 \times 4 \times 5 \times 7//49$, DFPE $= 2$: Collapse 475 via 1234/12345/12343/1234567.
466. $4 \times 4 \times 5 \times 7//49$, DFPE $= 3$: Collapse 476 via 1234/12345/12342/1234567.
467. $4 \times 4 \times 5 \times 7//49$, DFPE $= 4$: Collapse 478 via 1234/12345/12234/1234567.
468. $4 \times 4 \times 5 \times 7//49$, DFPE $= 5$: Collapse 479 via 1234/12345/12234/1234567.
469. $4 \times 4 \times 6 \times 7//49$, DFPE $= 0$: Collapse 474 via 1234/1234/1234565/1234567.
470. $4 \times 4 \times 6 \times 7//49$, DFPE $= 1$: Collapse 474 via 1234/1234/1234566/1234567.
471. $4 \times 4 \times 6 \times 7//49$, DFPE $= 2$: Collapse 474 via 1234/1234/1234356/1234567.
472. $4 \times 4 \times 6 \times 7//49$, DFPE $= 3$: Collapse 474 via 1234/1234/1232456/1234567.
473. $4 \times 4 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
474. $4 \times 4 \times 7 \times 7//49$, DFPE $= 0$: Collapse 485 via 1234/12344/1234567/1234567.
475. $4 \times 5 \times 5 \times 7//49$, DFPE $= 0$: Collapse 481 via 1234/12345/123455/1234567.
476. $4 \times 5 \times 5 \times 7//49$, DFPE $= 1$: Collapse 481 via 1234/12345/123454/1234567.
477. $4 \times 5 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
478. $4 \times 5 \times 5 \times 7//49$, DFPE $= 2$: Collapse 481 via 1234/12345/123435/1234567.
479. $4 \times 5 \times 5 \times 7//49$, DFPE $= 3$: Collapse 482 via 1234/12345/123452/1234567.
480. $4 \times 5 \times 5 \times 7//49$, DFPE $= 4$: Collapse 483 via 1234/12345/122345/1234567.
481. $4 \times 5 \times 6 \times 7//49$, DFPE $= 0$: Collapse 485 via 1234/12345/1234566/1234567.
482. $4 \times 5 \times 6 \times 7//49$, DFPE $= 1$: Collapse 485 via 1234/12345/1234546/1234567.
483. $4 \times 5 \times 6 \times 7//49$, DFPE $= 2$: Collapse 485 via 1234/12345/1234356/1234567.
484. $4 \times 5 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
485. $4 \times 5 \times 7 \times 7//49$, DFPE $= 0$: Collapse 489 via 1234/123455/1234567/1234567.
486. $4 \times 6 \times 6 \times 7//49$, DFPE $= 0$: Collapse 489 via 1234/123456/1234566/1234567.
487. $4 \times 6 \times 6 \times 7//49$, DFPE $= 1$: Collapse 489 via 1234/123456/1234546/1234567.
488. $4 \times 6 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
489. $4 \times 6 \times 7 \times 7//49$, DFPE $= 0$: Collapse 490 via 1234/1234566/1234567/1234567.
490. $4 \times 7 \times 7 \times 7//49$, DFPE $= 0$: Collapse 505 via 12344/1234567/1234567/1234567.
491. $5 \times 5 \times 5 \times 7//49$, DFPE $= 0$: Collapse 496 via 12345/12345/123455/1234567.
492. $5 \times 5 \times 5 \times 7//49$, DFPE $= 1$: Collapse 496 via 12345/12345/123454/1234567.
493. $5 \times 5 \times 5 \times 7//49$, DFPE $= 13$: Dose not exist by Theorem 2.6.
494. $5 \times 5 \times 5 \times 7//49$, DFPE $= 2$: Collapse 496 via 12345/12345/123435/1234567.
495. $5 \times 5 \times 5 \times 7//49$, DFPE $= 3$: Collapse 497 via 12345/12345/123345/1234567.
496. $5 \times 5 \times 6 \times 7//49$, DFPE $= 0$: Collapse 500 via 12345/12345/1234566/1234567.
497. $5 \times 5 \times 6 \times 7//49$, DFPE $= 1$: Collapse 500 via 12345/12345/1234546/1234567.
498. $5 \times 5 \times 6 \times 7//49$, DFPE $= 2$: Collapse 500 via 12345/12345/1234356/1234567.
499. $5 \times 5 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
500. $5 \times 5 \times 7 \times 7//49$, DFPE $= 0$: Collapse 504 via 12345/123455/1234567/1234567.
501. $5 \times 6 \times 6 \times 7//49$, DFPE $= 0$: Collapse 504 via 12345/123456/1234566/1234567.
502. $5 \times 6 \times 6 \times 7//49$, DFPE $= 1$: Collapse 504 via 12345/123456/1234546/1234567.
503. $5 \times 6 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
504. $5 \times 6 \times 7 \times 7//49$, DFPE $= 0$: Collapse 505 via 12345/1234566/1234567/1234567.

505. $5 \times 7 \times 7 \times 7//49$, DFPE $= 0$: Collapse 510 via 123455/1234567/1234567/1234567.
506. $6 \times 6 \times 6 \times 7//49$, DFPE $= 0$: Collapse 509 via 123456/123456/1234566/1234567.
507. $6 \times 6 \times 6 \times 7//49$, DFPE $= 1$: Collapse 509 via 123456/123456/1234546/1234567.
508. $6 \times 6 \times 6 \times 7//49$, DFPE $= 6$: Dose not exist by Theorem 2.6.
509. $6 \times 6 \times 7 \times 7//49$, DFPE $= 0$: Collapse 510 via 123456/1234566/1234567/1234567.
510. $6 \times 7 \times 7 \times 7//49$, DFPE $= 0$: Collapse 511 via 1234566/1234567/1234567/1234567.
511. $7 \times 7 \times 7 \times 7//49$, DFPE $= 0$:

```
11111112222222333333344444445555555666666677777777
12345671234567123456712345671234567123456712345671234567
12345672345671345671245671235671234671234571234567
17654322176543321765443217655432176654321776543321
```

512. $2 \times 2 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 55 and 55 with distinct symbols on row 3.
513. $2 \times 2 \times 5 \times 9//50$, DFPE $= 0$: Collapse 653 via 12112/12222/12345/1234567899.
514. $2 \times 2 \times 5 \times 9//50$, DFPE $= 1$: Collapse 653 via 12212/12222/12345/1234567899.
515. $2 \times 2 \times 5 \times 9//50$, DFPE $= 2$: Collapse 653 via 12222/12222/12345/1234567899.
516. $2 \times 2 \times 5 \times 9//50$, DFPE $= 3$: Collapse 653 via 12211/12222/12345/1234567899.
517. $2 \times 2 \times 5 \times 9//50$, DFPE $= 4$: Dose not exist by Theorem 2.5.
518. $2 \times 2 \times 5 \times 9//50$, DFPE $= 5$: Collapse 621 via 122/12222/12345/123456789.
519. $2 \times 3 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 61 and 61 with distinct symbols on row 3.
520. $2 \times 3 \times 5 \times 8//50$, DFPE $= 0$: Juxtapose 56 and 56 with distinct symbols on row 3.
521. $2 \times 3 \times 5 \times 8//50$, DFPE $= 1$: Juxtapose 56 and 57 with distinct symbols on row 3.
522. $2 \times 3 \times 5 \times 8//50$, DFPE $= 10$: Collapse 608 via 122/12333/12345/12345678.
523. $2 \times 3 \times 5 \times 8//50$, DFPE $= 2$: Juxtapose 56 and 58 with distinct symbols on row 3.
524. $2 \times 3 \times 5 \times 8//50$, DFPE $= 3$: Juxtapose 56 and 59 with distinct symbols on row 3.
525. $2 \times 3 \times 5 \times 8//50$, DFPE $= 4$: Juxtapose 57 and 59 with distinct symbols on row 3.
526. $2 \times 3 \times 5 \times 8//50$, DFPE $= 5$: Juxtapose 58 and 59 with distinct symbols on row 3.
527. $2 \times 3 \times 5 \times 8//50$, DFPE $= 6$: Juxtapose 59 and 59 with distinct symbols on row 3.
528. $2 \times 3 \times 5 \times 8//50$, DFPE $= 7$: Collapse 612 via 112/12331/12345/12345678.
529. $2 \times 3 \times 5 \times 8//50$, DFPE $= 8$: Collapse 657 via 12345/12333/12221/12345678.
530. $2 \times 3 \times 5 \times 8//50$, DFPE $= 9$: Dose not exist by Theorem 2.5.
531. $2 \times 3 \times 5 \times 9//50$, DFPE $= 0$: Juxtapose 56 and 61 with distinct symbols on row 3.
532. $2 \times 3 \times 5 \times 9//50$, DFPE $= 1$: Juxtapose 57 and 61 with distinct symbols on row 3.
533. $2 \times 3 \times 5 \times 9//50$, DFPE $= 2$: Juxtapose 58 and 61 with distinct symbols on row 3.
534. $2 \times 3 \times 5 \times 9//50$, DFPE $= 3$: Juxtapose 59 and 61 with distinct symbols on row 3.
535. $2 \times 3 \times 5 \times 9//50$, DFPE $= 4$: Dose not exist by Theorem 2.5.
536. $2 \times 3 \times 5 \times 9//50$, DFPE $= 5$: Collapse 621 via 122/12333/12345/123456789.
537. $2 \times 4 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 66 and 66 with distinct symbols on row 3.
538. $2 \times 4 \times 5 \times 8//50$, DFPE $= 0$: Juxtapose 62 and 62 with distinct symbols on row 2.
539. $2 \times 4 \times 5 \times 8//50$, DFPE $= 1$: Juxtapose 62 and 63 with distinct symbols on row 2.
540. $2 \times 4 \times 5 \times 8//50$, DFPE $= 10$: Collapse 608 via 122/12344/12345/12345678.
541. $2 \times 4 \times 5 \times 8//50$, DFPE $= 2$: Juxtapose 62 and 64 with distinct symbols on row 2.
542. $2 \times 4 \times 5 \times 8//50$, DFPE $= 3$: Juxtapose 63 and 64 with distinct symbols on row 2.
543. $2 \times 4 \times 5 \times 8//50$, DFPE $= 4$: Juxtapose 64 and 64 with distinct symbols on row 2.
544. $2 \times 4 \times 5 \times 8//50$, DFPE $= 5$: Collapse 612 via 121/12344/12345/12345678.
545. $2 \times 4 \times 5 \times 8//50$, DFPE $= 6$: Collapse 612 via 122/12344/12345/12345678.
546. $2 \times 4 \times 5 \times 8//50$, DFPE $= 8$: Collapse 657 via 12345/12344/12221/12345678.
547. $2 \times 4 \times 5 \times 8//50$, DFPE $= 9$: Dose not exist by Theorem 2.5.

548. $2 \times 4 \times 5 \times 9//50$, DFPE $= 0$: Juxtapose 62 and 66 with distinct symbols on row 3.
549. $2 \times 4 \times 5 \times 9//50$, DFPE $= 1$: Juxtapose 63 and 66 with distinct symbols on row 3.
550. $2 \times 4 \times 5 \times 9//50$, DFPE $= 2$: Juxtapose 64 and 66 with distinct symbols on row 3.
551. $2 \times 4 \times 5 \times 9//50$, DFPE $= 3$: Collapse 661 via 12122/12343/12345/123456789.
552. $2 \times 4 \times 5 \times 9//50$, DFPE $= 4$: Dose not exist by Theorem 2.5.
553. $2 \times 4 \times 5 \times 9//50$, DFPE $= 5$: Collapse 621 via 122/12344/12345/123456789.
554. $2 \times 5 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 67 and 67 with distinct symbols on row 2.
555. $2 \times 5 \times 5 \times 8//50$, DFPE $= 0$: Juxtapose 61 and 67 with distinct symbols on row 2.
556. $2 \times 5 \times 5 \times 8//50$, DFPE $= 1$: Collapse 607 via 121/12345/12345/12345678.
557. $2 \times 5 \times 5 \times 8//50$, DFPE $= 10$: Collapse 608 via 122/12345/12345/12345678.
558. $2 \times 5 \times 5 \times 8//50$, DFPE $= 2$: Collapse 607 via 122/12345/12345/12345678.
559. $2 \times 5 \times 5 \times 8//50$, DFPE $= 3$: Collapse 657 via 12345/12222/12345/12345678.
560. $2 \times 5 \times 5 \times 8//50$, DFPE $= 4$: Collapse 657 via 12345/12345/12122/12345678.
561. $2 \times 5 \times 5 \times 8//50$, DFPE $= 5$: Collapse 612 via 122/12345/12345/12345678.
562. $2 \times 5 \times 5 \times 8//50$, DFPE $= 6$: Collapse 657 via 12345/12345/12222/12345678.
563. $2 \times 5 \times 5 \times 8//50$, DFPE $= 8$: Collapse 657 via 12345/12345/12221/12345678.
564. $2 \times 5 \times 5 \times 8//50$, DFPE $= 9$: Dose not exist by Theorem 2.5.
565. $2 \times 5 \times 5 \times 9//50$, DFPE $= 0$: Juxtapose 66 and 67 with distinct symbols on row 2.
566. $2 \times 5 \times 5 \times 9//50$, DFPE $= 1$: Collapse 661 via 12222/12345/12345/123456789.
567. $2 \times 5 \times 5 \times 9//50$, DFPE $= 2$: Collapse 662 via 12222/12345/12345/123456789.
568. $2 \times 5 \times 5 \times 9//50$, DFPE $= 3$: Collapse 663 via 12222/12345/12345/123456789.
569. $2 \times 5 \times 5 \times 9//50$, DFPE $= 4$: Dose not exist by Theorem 2.5.
570. $2 \times 5 \times 5 \times 9//50$, DFPE $= 5$: Collapse 621 via 122/12345/12345/123456789.
571. $3 \times 3 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 72 and 72 with distinct symbols on row 3.
572. $3 \times 3 \times 5 \times 8//50$, DFPE $= 0$: Juxtapose 68 and 68 with distinct symbols on row 3.
573. $3 \times 3 \times 5 \times 8//50$, DFPE $= 1$: Juxtapose 68 and 69 with distinct symbols on row 3.
574. $3 \times 3 \times 5 \times 8//50$, DFPE $= 10$: Collapse 608 via 123/12333/12345/12345678.
575. $3 \times 3 \times 5 \times 8//50$, DFPE $= 2$: Juxtapose 68 and 70 with distinct symbols on row 3.
576. $3 \times 3 \times 5 \times 8//50$, DFPE $= 3$: Juxtapose 69 and 70 with distinct symbols on row 3.
577. $3 \times 3 \times 5 \times 8//50$, DFPE $= 4$: Juxtapose 70 and 70 with distinct symbols on row 3.
578. $3 \times 3 \times 5 \times 8//50$, DFPE $= 5$: Collapse 612 via 123/12333/12345/12345678.
579. $3 \times 3 \times 5 \times 8//50$, DFPE $= 6$: Collapse 612 via 123/12331/12345/12345678.
580. $3 \times 3 \times 5 \times 8//50$, DFPE $= 7$: Collapse 612 via 123/11231/12345/12345678.
581. $3 \times 3 \times 5 \times 8//50$, DFPE $= 9$: Dose not exist by Theorem 2.5.
582. $3 \times 3 \times 5 \times 9//50$, DFPE $= 0$: Juxtapose 68 and 72 with distinct symbols on row 3.
583. $3 \times 3 \times 5 \times 9//50$, DFPE $= 1$: Juxtapose 69 and 72 with distinct symbols on row 3.
584. $3 \times 3 \times 5 \times 9//50$, DFPE $= 2$: Juxtapose 70 and 72 with distinct symbols on row 3.
585. $3 \times 3 \times 5 \times 9//50$, DFPE $= 3$: Collapse 661 via 12133/12333/12345/123456789.
586. $3 \times 3 \times 5 \times 9//50$, DFPE $= 4$: Dose not exist by Theorem 2.5.
587. $3 \times 3 \times 5 \times 9//50$, DFPE $= 5$: Collapse 571 via 123/123/12345/1234567895.
588. $3 \times 4 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 76 and 76 with distinct symbols on row 3.
589. $3 \times 4 \times 5 \times 8//50$, DFPE $= 0$: Juxtapose 73 and 73 with distinct symbols on row 2.
590. $3 \times 4 \times 5 \times 8//50$, DFPE $= 1$: Juxtapose 73 and 74 with distinct symbols on row 2.
591. $3 \times 4 \times 5 \times 8//50$, DFPE $= 10$: Collapse 608 via 123/12344/12345/12345678.
592. $3 \times 4 \times 5 \times 8//50$, DFPE $= 2$: Juxtapose 74 and 74 with distinct symbols on row 2.
593. $3 \times 4 \times 5 \times 8//50$, DFPE $= 3$: Collapse 607 via 123/12134/12345/12345678.
594. $3 \times 4 \times 5 \times 8//50$, DFPE $= 4$: Collapse 660 via 12333/12334/12345/123456788.

595. $3 \times 4 \times 5 \times 8//50$, DFPE $= 5$: Collapse 612 via 123/12344/12345/12345678.
596. $3 \times 4 \times 5 \times 8//50$, DFPE $= 6$: Collapse 612 via 123/12341/12345/12345678.
597. $3 \times 4 \times 5 \times 8//50$, DFPE $= 7$: Collapse 657 via 12345/12344/12231/12345678.
598. $3 \times 4 \times 5 \times 8//50$, DFPE $= 9$: Dose not exist by Theorem 2.5.
599. $3 \times 4 \times 5 \times 9//50$, DFPE $= 0$: Juxtapose 73 and 76 with distinct symbols on row 3.
600. $3 \times 4 \times 5 \times 9//50$, DFPE $= 1$: Juxtapose 74 and 76 with distinct symbols on row 3.
601. $3 \times 4 \times 5 \times 9//50$, DFPE $= 2$: Collapse 662 via 12333/12344/12345/123456789.
602. $3 \times 4 \times 5 \times 9//50$, DFPE $= 3$: Collapse 661 via 12133/12343/12345/123456789.
603. $3 \times 4 \times 5 \times 9//50$, DFPE $= 4$: Dose not exist by Theorem 2.5.
604. $3 \times 4 \times 5 \times 9//50$, DFPE $= 5$: Collapse 621 via 123/12344/12345/123456789.
605. $3 \times 5 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 77 and 77 with distinct symbols on row 2.
606. $3 \times 5 \times 5 \times 8//50$, DFPE $= 0$: Juxtapose 72 and 77 with distinct symbols on row 2.
607. $3 \times 5 \times 5 \times 8//50$, DFPE $= 1$: Collapse 616 via 123/12345/12345/123456788.
608. $3 \times 5 \times 5 \times 8//50$, DFPE $= 10$: Collapse 621 via 123/12345/12345/123456748.
609. $3 \times 5 \times 5 \times 8//50$, DFPE $= 2$: Collapse 660 via 12333/12345/12345/123456788.
610. $3 \times 5 \times 5 \times 8//50$, DFPE $= 3$: Collapse 657 via 12345/12345/12323/12345678.
611. $3 \times 5 \times 5 \times 8//50$, DFPE $= 4$: Collapse 657 via 12345/12345/12333/12345678.
612. $3 \times 5 \times 5 \times 8//50$, DFPE $= 5$: Collapse 621 via 123/12345/12345/123456788.
613. $3 \times 5 \times 5 \times 8//50$, DFPE $= 6$: Collapse 657 via 12345/12345/12331/12345678.
614. $3 \times 5 \times 5 \times 8//50$, DFPE $= 7$: Collapse 657 via 12345/12345/12231/12345678.
615. $3 \times 5 \times 5 \times 8//50$, DFPE $= 9$: Dose not exist by Theorem 2.5.
616. $3 \times 5 \times 5 \times 9//50$, DFPE $= 0$: Juxtapose 76 and 77 with distinct symbols on row 2.
617. $3 \times 5 \times 5 \times 9//50$, DFPE $= 1$: Collapse 661 via 12333/12345/12345/123456789.
618. $3 \times 5 \times 5 \times 9//50$, DFPE $= 2$: Collapse 662 via 12333/12345/12345/123456789.
619. $3 \times 5 \times 5 \times 9//50$, DFPE $= 3$: Collapse 663 via 12333/12345/12345/123456789.
620. $3 \times 5 \times 5 \times 9//50$, DFPE $= 4$: Dose not exist by Theorem 2.5.
621. $3 \times 5 \times 5 \times 9//50$, DFPE $= 5$: Collapse 605 via 123/12345/12345/1234567895.
622. $4 \times 4 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 81 and 81 with distinct symbols on row 3.
623. $4 \times 4 \times 5 \times 8//50$, DFPE $= 0$: Juxtapose 78 and 78 with distinct symbols on row 1.
624. $4 \times 4 \times 5 \times 8//50$, DFPE $= 1$: Juxtapose 78 and 79 with distinct symbols on row 1.
625. $4 \times 4 \times 5 \times 8//50$, DFPE $= 10$: Collapse 641 via 1234/12344/12345/12345678.
626. $4 \times 4 \times 5 \times 8//50$, DFPE $= 2$: Juxtapose 79 and 79 with distinct symbols on row 1.
627. $4 \times 4 \times 5 \times 8//50$, DFPE $= 3$: Collapse 660 via 12343/12343/12345/123456788.
628. $4 \times 4 \times 5 \times 8//50$, DFPE $= 4$: Collapse 657 via 12345/12344/12334/12345678.
629. $4 \times 4 \times 5 \times 8//50$, DFPE $= 5$: Collapse 622 via 1234/1234/12345/1234567885.
630. $4 \times 4 \times 5 \times 8//50$, DFPE $= 6$: Collapse 622 via 1234/1234/12345/1234567855.
631. $4 \times 4 \times 5 \times 8//50$, DFPE $= 9$: Dose not exist by Theorem 2.5.
632. $4 \times 4 \times 5 \times 9//50$, DFPE $= 0$: Juxtapose 78 and 81 with distinct symbols on row 3.
633. $4 \times 4 \times 5 \times 9//50$, DFPE $= 1$: Juxtapose 79 and 81 with distinct symbols on row 3.
634. $4 \times 4 \times 5 \times 9//50$, DFPE $= 2$: Collapse 662 via 12344/12344/12345/123456789.
635. $4 \times 4 \times 5 \times 9//50$, DFPE $= 3$: Collapse 661 via 12134/12343/12345/123456789.
636. $4 \times 4 \times 5 \times 9//50$, DFPE $= 4$: Dose not exist by Theorem 2.5.
637. $4 \times 4 \times 5 \times 9//50$, DFPE $= 5$: Collapse 622 via 1234/1234/12345/1234567895.
638. $4 \times 5 \times 5 \times 10//50$, DFPE $= 0$: Juxtapose 82 and 82 with distinct symbols on row 2.
639. $4 \times 5 \times 5 \times 8//50$, DFPE $= 0$: Juxtapose 81 and 81 with distinct symbols on row 1.
640. $4 \times 5 \times 5 \times 8//50$, DFPE $= 1$: Collapse 660 via 12344/12345/12345/123456788.
641. $4 \times 5 \times 5 \times 8//50$, DFPE $= 10$: Collapse 638 via 1234/12345/12345/1234567845.

642. $4 \times 5 \times 5 \times 8//50$, DFPE = 2: Collapse 660 via 12343/12345/12345/123456788.
643. $4 \times 5 \times 5 \times 8//50$, DFPE = 3: Collapse 657 via 12345/12345/12344/12345678.
644. $4 \times 5 \times 5 \times 8//50$, DFPE = 4: Collapse 657 via 12345/12345/12334/12345678.
645. $4 \times 5 \times 5 \times 8//50$, DFPE = 5: Collapse 638 via 1234/12345/12345/1234567885.
646. $4 \times 5 \times 5 \times 8//50$, DFPE = 9: Dose not exist by Theorem 2.5.
647. $4 \times 5 \times 5 \times 9//50$, DFPE = 0: Juxtapose 81 and 82 with distinct symbols on row 2.
648. $4 \times 5 \times 5 \times 9//50$, DFPE = 1: Collapse 661 via 12344/12345/12345/123456789.
649. $4 \times 5 \times 5 \times 9//50$, DFPE = 2: Collapse 662 via 12344/12345/12345/123456789.
650. $4 \times 5 \times 5 \times 9//50$, DFPE = 3: Collapse 663 via 12344/12345/12345/123456789.
651. $4 \times 5 \times 5 \times 9//50$, DFPE = 4: Dose not exist by Theorem 2.5.
652. $4 \times 5 \times 5 \times 9//50$, DFPE = 5: Collapse 638 via 1234/12345/12345/1234567895.
653. $5 \times 5 \times 5 \times 10//50$, DFPE = 0: Juxtapose 83 and 83 with distinct symbols on row 1.
654. $5 \times 5 \times 5 \times 8//50$, DFPE = 0: Juxtapose 77 and 83 with distinct symbols on row 1.
655. $5 \times 5 \times 5 \times 8//50$, DFPE = 1: Collapse 660 via 12345/12345/12345/123456788.
656. $5 \times 5 \times 5 \times 8//50$, DFPE = 10: Collapse 653 via 12345/12345/12345/1234567845.
657. $5 \times 5 \times 5 \times 8//50$, DFPE = 3: Collapse 661 via 12345/12345/12345/123456748.
658. $5 \times 5 \times 5 \times 8//50$, DFPE = 5: Collapse 653 via 12345/12345/12345/1234567885.
659. $5 \times 5 \times 5 \times 8//50$, DFPE = 9: Dose not exist by Theorem 2.5.
660. $5 \times 5 \times 5 \times 9//50$, DFPE = 0: Juxtapose 82 and 83 with distinct symbols on row 1.
661. $5 \times 5 \times 5 \times 9//50$, DFPE = 1:

```
11111222223333344444555551111122222333334444455555
12345123451234512345123451234512345123451234512345
53142142532531431425425315142323514142353514242351
41239123942394139412941238567956798679857985698567
```

662. $5 \times 5 \times 5 \times 9//50$, DFPE = 2:

```
11111222223333344444555551111122222333334444455555
12345123451234512345123451234512345123451234512345
53142142532531431425425315142323514142353514242351
91234123492349134912491239567856789678957895689567
```

663. $5 \times 5 \times 5 \times 9//50$, DFPE = 3:

```
11111222223333344444555551111122222333334444455555
12345123451234512345123451234512345123451234512345
52413135242413535241413525142323514142353514242351
91234123492349134912491239567856789678957895689567
```

664. $5 \times 5 \times 5 \times 9//50$, DFPE = 4: Dose not exist by Theorem 2.5.
665. $5 \times 5 \times 5 \times 9//50$, DFPE = 5: Collapse 653 via 12345/12345/12345/1234567895.

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