MA4310: Abstract Algebra

(http://www.math.mtu.edu/ kreher/ABOUTME/syllabus/MA4310.html)

Course description

Detailed study of abstract algebra: elementary number theory (congruences, quadratic residues, arithmetic functions), group theory (monoids, permutation groups, homomorphisms, quotients, Lagrange's theorem, finite Abelian groups, Sylow's theorems), ring theory (domains, prime and maximal ideals, quotients, PID's), splitting fields, finite fields.
Credits: 3.0
Lec-Rec-Lab: (0-3-0)
Semesters Offered: Spring
Pre-Requisite(s): MA 3310

Text

Here are my lecture notes for Group Theory: GTN.pdf. We will likely follow these notes until they run out. They cover the same essential material as Chapters 3 through 15 of the back-up text, but are a little dense. I will probably add material as I go and send you updates.
After surveying the class I may alter what we actually cover, depending on what you have learned in MA 3310, what your future plans are and what you would like me to cover.
I was originally trained in group theory and much of my early focus was the applications of group theory to combinatorics.
If I understand correctly what was covered in MA3310 Fall 2019. We may just begin with group action,

- chapter 3 of my notes GTN.pdf.
- chapter 14 of Thomas W. Judson, Abstract Algebra: Theory and Applications http://abstract.ups.edu/aata/aata.html.
- Its also in Gallian's book I think.

Backup Text

T.W. Judson, *Abstract Algebra*, (Version 2.22)

- This is a free open-source textbook available from the website: http://abstract.ups.edu/.
- It was originally published in 1994 by PWS-Kent and is thus well put together and complete. It is 454 pages enough for 2 semesters. It contains all that we need and the price is right!
- I suggest you download the PDF file labeled "Full Text, 2010 edition" with file name aata-20100827.pdf and keep it on your computers just occasionally printing out small portions as you need.
- This download is: http://abstract.ups.edu/download/aata-20100827.pdf
- You can purchase a nicely printed and bound (hard-back) copy of AATA can be purchased from Amazon.com for $17.95 via the link http://www.amazon.com/Abstract-Algebra-Applications-Thomas-Judson/dp/0982406223/.
- See http://abstract.ups.edu/purchase.html for more details.
- You will read on your own Chapters 1 and 2. We will at least cover Chapters 3 to Chapter 15 in class and maybe if there is time we will do more.

Second backup Text

J. Gallian, Contemporary Abstract Algebra.
I think many of you have this text, so we may use it.

Grading

Your grade will be based on homework assignments and midterm examinations.

Homework

Don't panic I may rethink how we do homework, but below is what I did in 2013.
All home work and take-home examinations are to be typeset in LaTeX and stored in a single file, that will be handed in periodically. You are expected to have correctly solved all assigned problems by the end of the semester. The file name must be: MA4310DLKreher.pdf, except "D" is your first initial, "L" is your middle initial and "Kreher" is your last name. Here are the home work exercises

- `http://www.math.mtu.edu/ kreher/ABOUTME/syllabus/GTNHomework.pdf`
- `http://www.math.mtu.edu/ kreher/ABOUTME/syllabus/GTNHomework.tex`.

Using LaTex

LaTeX is available from the mathlab computers. You can also download your own personal version for your PC or Laptop. Here are some instructions I found on the Internet:
http://www.ling.upenn.edu/advice/latex/pc-setup.html.

Sample Exercise solution

Here is a sample LaTeX file:

```
\documentclass{article}
\usepackage{fullpage,amssymb}

\newcommand{\N}{\mbox{$\mathbb N$}}%Natural numbers
\newcommand{\R}{\mbox{$\mathbb R$}}%Reals
\newcommand{\Z}{\mbox{$\mathbb Z$}}%Integers
\newcommand{\C}{\mbox{$\mathbb C$}}%Complex
\newcommand{\F}{\mbox{$\mathbb F$}}%Field

\newenvironment{proof}%
{\medskip\quad\newline\noindent{\bf Proof.}\rm}%
{\hfill $\square$\medskip}

\newenvironment{solution}%
{\medskip\quad\newline\noindent{\bf Solution.}\rm}%
{\hfill $\square$\medskip}

\newenvironment{exercise}[1]%
{\begin{description}\item[#1]}
{\end{description}}
```

```
\renewcommand{\div}%Divides
{\ensuremath{\makebox[4pt][c]{\rule[-1ex]{.5pt}{3ex}}}}

\newcommand{\nodiv}%Does not divide
{\ensuremath{\not\mbox{\hspace*{.2ex}\rule[-1ex]{.5pt}{3ex}}\hspace*{1ex}}}

\title{MA 4310: Abstact Algebra Homework Solutions}
\author{Donald L. Kreher}

\begin{document}
\maketitle
\section*{AATA: Chapter 1 exercises}
%AATA referes to Judson's online source book.
\begin{exercise}{1}
Suppose that
\begin{eqnarray*}
A&=&\{x : x \in \N \mbox{ and $x$ is even}\},\\
B&=&\{x : x \in \N \mbox{ and $x$ is prime}\},\\
C&=&\{x : x \in \N \mbox{ and $x$ is a multiple of 5}\}
\end{eqnarray*}
Describe each of the following sets.\newline
\begin{tabular}{lcl}
\\
(a) $A\cap B$&\hspace*{1.5in}&(c) $A\cup B$\\
\\
(b) $A\cap C$&&(d) $A\cap (B \cup C)$\\
\end{tabular}
\end{exercise}
\begin{solution}
\begin{itemize}
\item[(a)] $A\cap B = \{ 2 \}$ the set of all natural  numbers that are both even and prime.
\item[(b)] $B\cap C = \{5\}$ the set of all primes that are multiples of $5$.
\item[(c)] $A \cup B$ is the set of all natural numbers that are
either even or prime.
\item[(d)] $A\cap (B \cup C) = \{2\}$. Because 2 is the only number that
is both even and either a prime or multiple of 5.
\end{itemize}
\end{solution}
\begin{exercise}{8}
Prove $A \subset B$ if and only if $A \cap B = A$.
\end{exercise}
\begin{proof}
Suppose $A \subset B$ and let $a \in A$. Then $a \in b$
and thus $a \in A \cap B$. Hence $A \subset A \cap B$.
Therefore $A\cap B = A$, because
by definition $A \cap B \subset A$.

Conversly suppose $A \cap B = A$. By definition $A \cap B \subset B$. Hence $A = A \cap B \subset B$.
\end{proof}
\section*{GTN: Chapter 1 exercises}
%GTN referes to Kreher's Group Theory Notes
```

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\end{document}
```

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After running pdflatex on this file it produced the pdf file which is shown on the next page.

MA 4310: Abstact Algebra Homework Solutions

Donald L. Kreher

January 12, 2011

AATA: Chapter 1 exercises

1 Suppose that

$$\begin{aligned} A &= \{x : x \in \mathbb{N} \text{ and } x \text{ is even}\}, \\ B &= \{x : x \in \mathbb{N} \text{ and } x \text{ is prime}\}, \\ C &= \{x : x \in \mathbb{N} \text{ and } x \text{ is a multiple of } 5\} \end{aligned}$$

Describe each of the following sets.

(a) $A \cap B$

(b) $A \cap C$

(c) $A \cup B$

(d) $A \cap (B \cup C)$

Solution.

(a) $A \cap B = \{2\}$ the set of all natural numbers that are both even and prime.

(b) $B \cap C = \{5\}$ the set of all primes that are multiples of 5.

(c) $A \cup B$ is the set of all natural numbers that are either even or prime.

(d) $A \cap (B \cup C) = \{2\}$. Because 2 is the only number that is both even and either a prime or multiple of 5.

□

8 Prove $A \subset B$ if and only if $A \cap B = A$.

Proof. Suppose $A \subset B$ and let $a \in A$. Then $a \in b$ and thus $a \in A \cap B$. Hence $A \subset A \cap B$. Therefore $A \cap B = A$, because by definition $A \cap B \subset A$.

Conversly suppose $A \cap B = A$. By definition $A \cap B \subset B$. Hence $A = A \cap B \subset B$. □

GTN: Chapter 1 exercises

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