From t-wise balanced designs to orthogonal arrays

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An *orthogonal array of size* N, *degree* k, *order* s and *strength* t is a k by N array with entries from a set of $s \geq 2$ symbols, having the property that in every t by N subarray, every t by 1 column array appears the same number $\lambda = N/s^t$ times. We denote such an array by $\mathrm{OA}_\lambda(t,k,s)$. The parameter λ is called the *index* of the array.

Example: An $\mathrm{OA}_1(2,3,4)$

$$\begin{array}{cccccccccccccccc}
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 3 \\
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\
0 & 1 & 2 & 3 & 3 & 0 & 1 & 2 & 2 & 3 & 0 & 1 & 1 & 2 & 3 & 0
\end{array}$$

Applications of orthogonal arrays include, the construction of combinatorial configurations, the design of error correcting codes, the detection of communication errors and the testing of software.

Existence results for orthogonal arrays of strength greater than or equal to three are few and far between. A summary can be found in the handbook of combinatorial designs [3].

A Collection of subsets of a finite set X is said to be t-wise balanced if every t-element subset of X is contained in the same number λ of them.

Example: A 2-wise balanced collection of subsets, with $\lambda = 2$.

$$\{\{0,1,2,3\},\{1,2,3,4\},\{0,1,4\},\{0,2,4\},\{0,3,4\}\}$$

Starting from t-wise balanced designs we show how to construct orthogonal arrays of strength t, with $t = 2$ or 3. Difference matrices and t-homogeneous groups, are employed. The results discussed can be found in [2, 4, 1].

References

[1] M.A. Chateauneuf, C.J. Colbourn and D.L.Kreher, Covering arrays of strength 3, *Designs Codes and Cryptography*, 16, 235-242 (1999).

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[3] J. Bierbrauer and C.J. Colbourn, "Orthogonal arrays of strength more than two", *The CRC handbook of Combinatorial Designs*, C.J. Colbourn and J.H. Dinitz) (Editors), CRC Press, Boca Raton, 1996.

[4] D.L. Kreher, Orthogonal arrays of strength 3. *Journal of Combinatorial Designs*, **4** 67-69 (1996)