Chapter 4: Learning Objectives

1. Understand why and how source models are used in risk assessment.
2. Understand simple source models for liquids and gases: holes, pipe flow, pools.
3. Understand more complex source models: holes in tanks.
4. Understand how to use simple source models to build models for more complex situations.
Source Models

• **What:** Describe how material escapes from a process
• **Why:** Required to determine potential consequences of an accident

\[ \text{Risk} = f(\quad, \quad) \]
What do Source Models Provide?

• Release rate, mass/time
• Total amount released
• State of material: liquid, solid, gas, combination
Why do we need Source Models?

Source models are used to estimate the consequences.
Consequence Models

Figure 4-1
Release Mechanisms - 1

Wind
Direction

Vapor

Small Hole in Vapor
Space of a Pressurized Tank

Immediately
Resulting
Vapor Cloud

Liquefied Gas under Pres.

Catastrophic Failure of
Pressurized Tank
Release Mechanisms - 2

Intermediate Hole in Vapor Space of a Pressurized Tank

Escape of Liquefied Gas from a Pressurized Tank
Release Mechanisms - 3

- Evaporation
- Liquid in Bund
- Spillage of Refrigerated Liquid into Dike
- Liquid Jet
- Evaporating Cloud
- SS Spill
- Boiling Pool
- Spillage of Refrigerated Liquid onto Water
Release Mechanisms - 4

In infinitely more possibilities!

All of these complicated cases can be built-up from primitive cases, i.e. hole in vessel, flow thru pipe, etc.
Release Mechanism Parameters

Nature of release depends on lots of parameters:

1. Temperature and pressure of released material.
2. Composition of released material.
3. Ambient temperature and pressure.
4. Ambient wind, humidity
5. Geometry of release (hole, rupture, catastrophic failure)
7. Velocity of release.
8. Many others!
Source Model: Liquid thru a hole

Physical Facts:
1. Pressure is
2. Pressure energy is
3. Losses due to
Mechanical Energy Balance for Incompressible flow

\[
\frac{\Delta P}{\rho} + \frac{\Delta \bar{u}^2}{2g_c} + \frac{g}{g_c} \Delta z + F = -\frac{W_s}{\dot{m}}
\]

Eq. 4-28

\(P = \) Pressure
\(\rho = \) Density
\(\bar{u} = \) Velocity
\(g_c = \) Gravitational Constant
\(g = \) Acceleration due to gravity
\(z = \) Height above datum
\(F = \) Friction
\(W_s = \) Shaft work
\(\dot{m} = \) Mass flow
\[
\frac{\Delta P}{\rho} + \frac{\Delta \bar{u}^2}{2g_c} + \frac{g}{g_c} \Delta z + F = -\frac{W_s}{\dot{m}}
\]

\[
\frac{\Delta P}{\rho} =
\]

\[
\frac{\Delta \bar{u}^2}{2g_c} =
\]

\[
\frac{g}{g_c} \Delta z =
\]

\[
F =
\]

\[
-\frac{W_s}{\dot{m}} = \text{Mechanical Energy from pumps / turbines}
\]
Make Assumptions for Hole:

Horizontal:

No Pumps / turbines:

\( F \neq 0 \)

Solve ME balance for \( u \)

Apply: \( Q_m = \rho u A \)

\[
Q_m = \left( \frac{\text{kg}}{\text{m}^3} \right) \left( \frac{\text{m}}{\text{s}} \right) \left( \text{m}^2 \right) = \text{kg/s}
\]
Orifice Discharge Equation

\[ Q_m = C_o A \sqrt{2 \rho g_c \Delta P} \quad \text{Eq. 4-7} \]

\( C_o = \text{Discharge coefficient accounts for friction} \)

\[ = 1 \quad \rightarrow \]

\[ = 0.61 \text{ for turbulent flow of liquids.} \]

Rules for Discharge Coefficient:
Orifice Discharge Coefficient

See Perry’s for more details!
Example

1-inch diameter hole

100 psig upstream pressure

Water

\[
A = \frac{\pi D^2}{4} = \frac{(3.14) \left[ (1 \text{ in}) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) \right]^2}{4} = 5.45 \times 10^{-3} \text{ ft}^2
\]

\[C_o = 0.61\] for highly turbulent flow

\[\Delta P = 100 \text{ psig} - 0 \text{ psig} = 100 \text{ psi} = 100 \text{ lb}_f / \text{in}^2\]
Substitute in Orifice Equation

\[ Q_m = C_o A \sqrt{2 \rho g_c \Delta P} \]

\[ Q_m = \left( 0.61 \right) \left( 5.45 \times 10^{-3} \text{ ft}^2 \right) \]

\[
\times \sqrt{2 \left( 62.4 \frac{\text{lb}_m}{\text{ft}^3} \right) \left( 32.17 \frac{\text{ft}-\text{lb}_m}{\text{lb}_f \cdot \text{s}^2} \right) \left( 100 \frac{\text{lb}_f}{\text{in}^2} \right) \left( 144 \frac{\text{in}^2}{\text{ft}^2} \right)}
\]

\[ Q_m = 25.3 \text{ lb}_m / \text{s} \]

This is 3.03 gallons/sec.

The discharge velocity is 74 ft/sec!
Pressure at hole due to hydrostatic head plus ambient pressure.

Flow is maximum at \( t = 0 \) and decreases with time.
\[ Q_m = \rho AC_o \sqrt{2 \left( \frac{g_c P_g}{\rho} + gh_L \right)} \]  

Eq. 4-12

\[ Q_m = \text{Mass flow rate} \]
\[ \rho = \text{Liquid density} \]
\[ A = \text{Hole area} \]
\[ C_o = \text{Discharge coefficient} \]
\[ g_c = \text{Gravitational constant} \]
\[ P_g = \text{Gauge pressure in vapor space} \]
\[ g = \text{Acc. due to gravity} \]
\[ h_L = \text{Liquid height above hole.} \]
Hole in a Tank

Mass balance: Accumulation = -Output
Hole in a Tank

Can solve above equations to determine:

1. Total draining time.
2. Liquid level as a function of time.
3. Discharge rate as a function of time.

See textbook for details, pp 126 - 130.
Liquid Flow Thru Pipes

Physical Facts:

1. Pressure is $P_1 > P_2$
2. Velocity is
3. Losses due to
Mechanical Energy Balance for Pipe Flow

\[
\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2 g_c} + \frac{g}{g_c} \Delta z + F = -\frac{W_s}{m}
\]

\[\frac{\Delta P}{\rho} = \text{Pressure Energy}\]
\[\frac{\Delta u^2}{2 g_c} = \text{Kinetic Energy (KE)}\]
\[\frac{g}{g_c} \Delta z = \text{Potential Energy (PE)}\]

\[F = \text{Frictional Losses}\]

\[-W_s / m = \text{Shaft Work from Mechanical Linkage}\]
Frictional Losses for Pipe Flow

\[ F = K_f \left( \frac{u^2}{2g_c} \right) \]

where \( K_f \) is the excess head loss

\[ \left( \frac{u^2}{2g_c} \right) \]

is the Fanning friction factor

For pipe lengths: \( K_f = \frac{4fL}{d} \)

where \( f \) is the Fanning friction factor
(see text for computing)

\( L \) is the pipe length
\( d \) is the pipe diameter
Friction term, $F$, given by:

$$F = \frac{2fL\nu^2}{g_c d}$$

$L$ = Pipe Length, $g_c$ = grav. constant
$\nu$ = Liquid ave. velocity, $d$ = Pipe diam.

$f$

$= f$(Reynolds no., pipe roughness)

Equations (4-31 to 4-37) and Figure (4-7) provided in textbook for $f$.

Differs from Moody friction factor!
Frictional Losses for Pipe Flow -2

For pipe fittings:

\[ K_f = \frac{K_1}{Re} + K_\infty \left(1 + \frac{1}{ID_{\text{inches}}} \right) \]

where \( K_1 \) and \( K_\infty \) are constants (see Table 4-2)

- \( \text{Re} \) is the Reynolds number
- \( ID_{\text{inches}} \) is the fitting diameter in inches

\( K_1 \) important at low \( \text{Re} \) while \( K_\infty \) important at high \( \text{Re} \).
Example – Horizontal Pipe, no fittings

\[ KE \approx 0 \rightarrow u = \text{constant} \rightarrow \Delta u^2 = 0 \]

\[ \Delta z \approx 0 \text{ since horizontal} \]

\[ W_s \approx 0 \text{ since no pumps or turbines} \]

\[ \frac{\Delta P}{\rho} = -F = -\frac{2fLu^2}{g_c d} \]
Example:

What is pressure drop across 150 ft of 1-inch Sch. 40, new commercial steel pipe if flow = 30 gpm? Viscosity = 1.0 cp (water), cp = centipoise

Procedure:

1. Convert to appropriate units
2. Select equation and simplify to this situation
3. Determine Reynolds number and then $f$
4. Calculate answer and check if it makes sense.
1. Convert to Appropriate Units

I.D. = 1.049" = 0.0874 ft = 26.6 mm

\[ A = \frac{\pi D^2}{4} = \frac{(3.14)(0.0874 \text{ ft})^2}{4} = 0.0060 \]

\[ Q_v = (30 \text{ gal/min}) \left( \frac{0.1337 \text{ ft}^3}{1 \text{ gal}} \right) = 4.011 \]

\[ u = \frac{Q_v}{A} = \frac{4.011 \text{ ft}^3 / \text{min}}{0.0060 \text{ ft}^2} = 668 \text{ ft/min} = 11.1 \]

Note: Typical pipe liquid velocity about 10 ft/sec.
2. Select Equation:

Mechanical Energy Balance:

No Pumps:
Horizontal:
Velocity constant:

\[
\frac{\Delta P}{\rho} = -F = -\frac{2fLU^2}{g_c d}
\]
3. Determine Reynolds No. and then Friction Factor

\[ Re = \frac{Du \rho}{\mu} \]

\[ D = \text{diam.}, \ u = \text{velocity}, \]

\[ \rho = \text{density}, \ \mu = \text{viscosity} \]

1 cp = \(6.72 \times 10^{-4}\) lb \(_m\) / ft-sec

\[ Re = \frac{(0.0874 \text{ ft})(11.1 \text{ ft/sec})(62.4 \text{ lb}_m / \text{ft}^3)}{6.72 \times 10^{-4} \text{ lb}_m / \text{ft-sec}} \]

\[ Re = 9.01 \times 10^4 \] (no units!)
3. Determine Reynolds No. and then Friction Factor

From Table 4-1,
\[ \varepsilon = 0.046 \text{ mm (pipe roughness)} \]

Then
\[ \left( \frac{\varepsilon}{d} \right) = \frac{0.046 \text{ mm}}{26.6 \text{ mm}} = 0.00173 \]

From Figure 4-7 (or equations in text),
\[ f = 0.00616 \]
4. Calculate Answer:

\[ \Delta P = \frac{-F}{\rho} = -\frac{2fLu^2}{g_c d} \]

\[ \Delta P = -\frac{(2)(0.00616)(150 \text{ ft})(11.1 \text{ ft/s})^2(62.4 \frac{\text{lb}_m}{\text{ft}^3})}{(32.17 \frac{\text{ft lb}_m}{\text{lb}_f \text{s}^2})(0.0874 \text{ ft})} \]

\[ \Delta P = -5052 \text{ lb}_f / \text{ft}^2 = -35.1 \text{ lb}_f / \text{in}^2 \quad (\text{psi}) \]
General Pipe Flow Problem

For the general case, with fittings, changes in elevation, pumps, etc., problem is by trial and error.

Procedure:

1. Guess velocity
2. Compute Reynolds Number
3. Compute fitting head losses
4. Compute friction factor, \( f \)
5. Calculate velocity
6. Continue until guessed velocity = calculated velocity.

Can all be done easily by spreadsheet!
Gas Flow thru a Hole

$P_0 > \text{Outside P}$

Physical Facts:
1. Pressure is
2. 
3. 

Isentropic process --> use Equation (4-48)
Flow rate a function only of supply or upstream pressure and is independent of downstream pressure.

Sonic Velocity reached in hole
For ideal gases:

\[ a = \sqrt{\gamma g_c R_g T / M} \]

Equation 4-53

For air at 20°C sonic velocity = 344 m/s = 1129 ft/s

This represents the maximum speed that information can be transmitted through the gas.
Choked Flow Equation - Equation (4-50)

\[(Q_m)_{choked} = C_o A P_o \sqrt{\frac{\gamma g_c M}{R_g T_o} \left( \frac{2}{\gamma + 1} \right)^{(\gamma + 1)/(\gamma - 1)}}\]

\(Q_m = \text{Mass Flow}\)
\(C_o = \text{Discharge coef.} \rightarrow 1.0 \text{ for choked gas flow}\)
\(A = \text{Area}\)
\(P_o = \text{Upstream pressure (absolute)}\)
\(M = \text{Molecular weight}\)
\(T_o = \text{Temperature (absolute)}\)
\(g_c = \text{grav. constant}\)
\(R_g = \text{Ideal gas constant}\)
An absolute pressure ratio of greater than 1.67 to 2 will insure choked flow.

.... Choked flow is the usual case.
Gas Flow thru Pipes

Physical Facts:
1. Pressure is
2. As P decreases
3. T can

\[ P_2 > P_1 \]
Gas Flow thru Pipes - Sonic Conditions

Two Cases:

1. Adiabatic: Gas velocity is sonic at end of pipe
   \[ Q = \] (long pipelines approach this)

2. Isothermal: (long pipelines approach this)
   \[ T = \]

Gas velocity = \( \frac{a}{\sqrt{\gamma}} \)
at end of pipe
Several Modeling Approaches (see text)

Adiabatic choked flow

---- Real Case here???

Isothermal choked flow

Adiabatic choked mass flow

Isothermal choked mass flow
Adiabatic Choked Flow thru Pipe

Rigorous solution requires a trial and error solution of equation (4-67) coupled with equations (4-63) to (4-66).

\[
\begin{align*}
\frac{T_{\text{choked}}}{T_1} &= \frac{2Y_1}{\gamma + 1}, \quad (4-63) \\
\frac{P_{\text{choked}}}{P_1} &= Ma_1 \sqrt{\frac{2Y_1}{\gamma + 1}}, \quad (4-64) \\
\frac{\rho_{\text{choked}}}{\rho_1} &= Ma_1 \sqrt{\frac{\gamma + 1}{2Y_1}}, \quad (4-65) \\
G_{\text{choked}} &= \rho \bar{u} = Ma_1 P_1 \sqrt{\frac{\gamma g_c M}{R_g T_1}} = P_{\text{choked}} \sqrt{\frac{\gamma g_c M}{R_g T_{\text{choked}}}}, \quad (4-66) \\
\frac{\gamma + 1}{2} \ln \left[ \frac{2Y_1}{(\gamma + 1)Ma_1^2} \right] - \left( \frac{1}{Ma_1^2} - 1 \right) + \gamma \left( \frac{4fL}{d} \right) &= 0. \quad (4-67)
\end{align*}
\]
\[ G = \frac{\dot{m}}{A} = Y_g \sqrt{\frac{2 g_c \rho_1 (P_1 - P_2)}{\sum K_f}} \]

Equation (4-68)

\( Y_g = \) expansion factor (Figure 4-14 or Table 4-4)

\( P_1 - P_2 = \) sonic pressure drop (Figure 4-13 or Table 4-4)

Direct solution possible with this approach.

See Example 4-5.
Adiabatic Choked Flow thru Pipe

Given: Type, length and diameter of pipe

Pressure drop across pipe

Molecular weight, heat capacity ratio of gas

Temperature

\[ Q = 0 \]
Simplified Approach: Adiabatic Choked Flow thru Pipe

1. Determine friction factor, \( f \). Usually assume fully developed turbulent flow. \( f = f(d, \varepsilon) \) – eqn. 4-34.

2. Determine \( \Sigma K_f \) from pipe length and fittings.

3. Determine sonic pressure drop from Figure 4-13 or Table 4-4. Use sonic pressure in step 5.

4. Determine expansion factor, \( Y_g \) from Figure 4-14 or Table 4-4.

5. Substitute into Equation 4-68 to get mass flux, \( G \)

6. Mass flow = \( GA \).
Equations provided in Table 4-4
Adiabatic Expansion Factor – Figure 4-14

Equations provided in Table 4-4
Isothermal Pressure Drop Ratio – Figure 4-17

Equations provided in Table 4-4
Isothermal Expansion Factor – Figure 4-18

Equations provided in Table 4-4
Asymptotic Solution: Isothermal and Adiabatic

\[ m = A \sqrt{\frac{\rho_1 P_1 g_c}{\sum K}} \]

Equation 4-84

For a circular pipe, with friction due to pipe length:

\[ m = \frac{\pi}{8} \sqrt{\frac{\rho_1 P_1 D^5 g_c}{fL}} \]

For an ideal gas,

\[ m = \frac{\pi}{8} \sqrt{\frac{P_1^2 M D^5 g_c}{RT_1 fL}} \]
Example 4-5 in Text:

Several methods to calculate:
1. Choked flow thru hole
2. Adiabatic flow (trial and error, direct)
3. Isothermal flow (trial and error, direct)
Example 4-5: Direct Method

1. Friction factor, $f = 0.00564$ (assume fully developed turbulent flow – equation 4-34).

2. $K_f = 4fL/d = 8.56$ due to pipe length only.

3. From Figure 4-13: \[ \frac{P_1 - P_2}{P_1} = 0.770 \Rightarrow P_2 = 49.4 \text{ psia} \]

   Since actual downstream $P$ is less than this, flow is sonic. Use this pressure in Eqn. 4-68.

4. From Figure 4-14, $Y_g = 0.69$.

5. From Equation 4-68, $m = 1.78 \text{ lb}_m / \text{sec}$
Example 4-5 in Text:

Nitrogen

200 psig

33 feet

1-inch sch. 40

1 atm

0 psig

Modeling Approaches:

Choked flow thru hole: 4.16 lb/sec

Adiabatic choked flow thru pipe:

Direct method (approx.): 1.78 lb/sec

Trial and error (exact): 1.81 lb/sec

Isothermal choked flow thru pipe: 1.76 lb/sec

Recommendation: Use adiabatic choked flow, or choked flow thru a hole
**Asymptotic Solution** – valid for velocity head losses > 100, less than 2.2% error

\[ \dot{m} = A \sqrt{\frac{\rho_1 P_1 g_c}{\sum K}} \]

\[ \dot{m} = (6.00 \times 10^{-3} \text{ ft}^2) \]

\[ \times \sqrt{\left( \frac{1.037 \text{ lb}_m}{\text{ft}^3} \right) \left( 214.7 \frac{\text{lb}_f}{\text{in}^2} \right) \left( 144 \frac{\text{in}^2}{\text{ft}^2} \right) \left( 32.17 \frac{\text{ft-lb}_m}{\text{lb}_f \cdot \text{s}^2} \right) } \]

\[ \dot{m} = 2.08 \text{ lb}_m / \text{sec} \]

Compared to a rigorous solution of 1.81 \text{ lb}_m / \text{sec}

% error = 14.9%
Flashing Liquids

P₁ → P₂
Energy for flashing comes from sensible energy in liquid

\[ f_v = \frac{C_p \left( T_o - T_{BP} \right)}{\Delta H_{vap}} = \text{Mass Fraction Vap.} \]

\[ T_o = \text{Storage} / \text{Ambient Temperature} \]

\[ T_{BP} = \text{Normal Boiling Point Temperature} \]
Other Source Models (see textbook)

Flashing liquid flowing thru hole: assume liquid flashes outside of the hole.

Flashing liquid flowing thru pipe:

• See equation 4-92 for liquids stored at $P$ higher than saturation vapor pressure.

• See equation 4-105 for liquids stored at saturation vapor pressure.

Boiling Pool see eqns. (4-106) and (4-107)
Source Models do not need to be exact!

If uncertain about model, physical property, geometry, etc., select the one to obtain maximum discharge. See Table 4-6.

Maximum discharge ---> Maximum Consequence

Problem: can lead to a very large result.

We should always try to do best we can using good engineering judgement!
Realistic Release Incidents: Table 4-6

Process Pipes: Rupture of largest diameter as follows:

- For \( d < 2 \) in., assume full bore rupture
- For 2-4 in. assume rupture equal to 2-inch pipe
- For \( d > 4 \) in, assume rupture area = 20% of pipe area

Vessels: Assume rupture based on largest diameter connecting pipe and then use criteria above.

Relief Device: Use calculated total release rate at set pressure. Assume everything is airborne.
Assume release of the largest quantity of substance handled on site in a single process vessel at any time. Assume entire quantity is released in 10-minutes.

Assume release on ground.

Assume F-stability, 1.5 m/s wind speed (Chapter 5)

Assume highest daily max. T and average humidity.

See Table 4-6
THE END

“Think Safety!”