Shear-driven Annular Flow-boiling in Millimeter-scale channels: Direct Numerical Simulations for Convective Component of the Overall Heat Transfer Coefficient

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Abstract: Many contemporary high heat-flux cooling applications are facilitated by controlled operation of millimeter-scale flow-boilers that operate in the steady annular or steady-in-the-mean (with superposed large amplitude standing waves in the liquid film) annular regimes – with micrometer-scale liquid film thicknesses. That is, a thin evaporating liquid film flow covers the heated boiling-surface – with or without superposed micron/sub-micron-scale nucleate boiling regime. Therefore, to begin with, to characterize convective boiling component of experimentally measured values of heat transfer coefficient (HTC), it becomes important to fully characterize the underlying steady annular flows under the assumption of suppressed nucleation.

For such steady cases, and liquid thickness values in the range of tens to hundreds of micrometers that are much smaller than the mm-scale hydraulic diameter of the ducts, this paper presents a direct numerical simulations (DNS) approach for laminar liquid and laminar vapor flows. Representative detailed steady solutions for annular flow-boiling of FC-72 in a horizontal channel (heated from below) are presented, the flow-physics is studied, and HTC values are correlated. Furthermore, a one-dimensional correlations-based design tool is developed and discussed, along with its future extensions for covering laminar liquid and turbulent vapor are annular flow realizations that may also occur in the aforementioned operations of flow-boiling.

Keywords: Shear-driven annular flow-boiling, convective and nucleate boiling component of heat transfer coefficient, millimeter scale flow-boilers, steady annular evaporating flows, high heat-flux cooling.
1. Introduction

This paper presents fundamental modeling and first-principles based computational results for obtaining the *convective component* of heat transfer coefficient (HTC) values associated with annular regime flow-boiling operations [1]. An assumption of suppressed nucleation becomes necessary for defining *convective* HTC for steady and shear/pressure-driven annular flow-boiling cases in horizontal millimeter-scale (mm-scale) channels – i.e., rectangular cross-section ducts with large aspect ratios – that are of primary interest to this paper. Both steady and subsequent steady-in-the-mean pulsatile innovative operations [1] of innovative flow-boilers (see Figure 1) ensure annularity over the entire length of the test-section – therefore defining and obtaining convective boiling HTC ($h_{|cb}$) estimates by CFD have an important role to play in characterizing and understanding experimentally obtained total HTC (denoted $h_{|total}$ or $h_{|Expt}$) values which may be due to both nucleate and convective boiling contributions (respectively denoted as $h_{|nb}$ and $h_{|cb}$). These two types of local HTC contributions are defined later on in this paper. However, the definition of $h_{|total}$ ($\equiv h_{|Expt}$) is straightforward as it is given by the relationship: 

$$q_{w|total}(x) \equiv q_{w|Expt}(x) \equiv h_{|total} \ast \Delta T$$

where, $\Delta T$ is the characteristic excess temperature defined as “$T_w(x) - T_{sat}(p_{in})$,” with $T_w(x)$ being the local wall temperature and $T_{sat}(p_{in})$ being the characteristic saturation temperature associated with channel inlet pressure $p_{in}$ at $x^p = 0$ location in Figure 1. The liquid film-thickness values in innovative operations (Figure 1) are in tens to hundreds of microns and much less than the mm-scale hydraulic diameters of the duct. The liquid flows may or may not have significant contribution from nucleation and if nucleation is present, the bubble departure diameters (under conditions where viscous or inertial forces typically dominate transverse gravity-induced buoyancy forces) are necessarily limited by liquid film-thicknesses and, therefore, correspond to micron/sub-micron departure sizes. For the steady CFD
simulations presented here, both temperature and heat-flux controlled steady heating of the bottom horizontal plate are considered. This paper focuses on the algorithm and results for laminar liquid and laminar vapor conditions. Future extensions of the paper’s methodologies – that cover annular flows that involve laminar liquid and turbulent vapor flows – are also discussed in section 5.

Besides innovative operations, annular flow-boiling regimes (with or without nucleation) also occur in most traditional flow-boiling operations (see Figure 2) involving liquid only inlet (at saturation or slightly subcooled temperatures) and vapor only exit. The broader context [2, 3] of traditional flow-boiling operations in Figure 2 deal with issues such as: different “methods of heating,” multiple flow regimes resulting from competing effects of nucleation and convection, effects of gravity, effects of hydraulic diameter of the duct, surface-liquid-vapor interactions (associated with wettability, intermolecular forces, nano- and/or micro-structures present on the surface, etc.) affecting nucleation, and mechanisms of critical heat-flux (CHF). These are typically investigated and explored by a mix of experimental and modeling approaches – with a predominant focus on experiments with uniform heat-flux “method of heating” (see [4-7]) and development of correlations for HTC and pressure-drop (see [8-11]).

For innovative operations (Figure 1) of interest, there are three broad categories of annular flow-boiling realizations with regard to presence or absence of contributions from nucleate boiling on overall HTC:

(i) Nucleate boiling is suppressed for the entire length \(0 \leq x \leq L\) of annular realization. This is likely if the boiling surface is machined smooth (e.g., with roughness less than 0.1 µm and over 30 µm apart, number of available nucleation sites become small), exhibits low liquid film-thickness values (i.e., high mass-flux \(G\) and high quality \(X\)), and heating levels – in terms of average heat-flux or imposed temperature-difference values – are low to moderate.
(ii) Contributions from nucleate boiling mechanisms \( (h_{x|nb}) \) is dominant in the overall HTC \( (h_{x|total}) \), as discussed in [12], throughout \( (0 \leq x \leq L) \) the length of the channel.

(iii) Nucleation is present in a certain upstream part (say, \( 0 \leq x \leq x^* \)) of the annular flow-boiling channel (in Figure 1) and is effectively suppressed in the corresponding downstream part (say \( x^* \leq x \leq L \)). This third category of realizations, which have been observed in [13], and the characterization of suppression of nucleation, require availability, definitions and comparison of \( h_{x|total} \) with \( h_{x|cb} \) values over \( 0 \leq x \leq L \).

It is expected, from one-dimensional analysis of a pure fluid’s flow-boiling in Figure 1, that a suppressed nucleation assumption-based CFD simulation capable of yielding convective component of HTC \( h_{x|cb} \), film thickness \( \Delta(x) \) etc., requires – at the bare minimum – the following boundary conditions: (a) inlet liquid and vapor flow rates \( (\dot{M}_{L-in} \text{ and } \dot{M}_{V-in}) \) respectively; (b) inlet liquid film-thickness \( \Delta_{in} \) (which may have higher order dependence on prior-to-inlet heating approach as well as flow-guiding gap configurations of the splitter plate shown in Figure 1); (c) channel geometry (gap height “h” in Figure 1); and (d) knowledge of heating condition of the bottom boiling-surface – i.e., either heat-flux \( q^"(x) \) or characteristic excess temperature \( \Delta T \). It is expected that results of such CFD simulations will actually agree with experimental values – provided the steady annular flow is realized under category (i) above, i.e., nucleate boiling is physically suppressed over \( 0 \leq x \leq L \).

However, in the presence of significant nucleate boiling, the definition of \( h_{x|cb} \) that is proposed here involves an ability to implement CFD simulations under the knowledge of aforementioned boundary conditions with the following assumptions: (a) an assumed flow-regime (annular, plug-slug, etc.) – annular regime in this case; (b) a hypothetically assumed absence of nucleation; (c) an assumption that part of the heating condition of the boiling surface, the wall temperature \( T_w(x) \)
of the bottom wall, is experimentally known and is to be the thermal wall boundary condition; and
(d) the other heating condition of the boiling-surface, wall heat-flux $q_{w|\text{Expt}}(x)$, is also
experimentally known but the suppressed nucleation assumption-based actual CFD predictions
(convective boiling component) of wall heat-flux $q_w^\prime(x) \equiv q_{w|\text{CFD}}(x) \equiv q_{w|cb}^\prime(x)$ and liquid film-
thickness $\Delta_{\text{CFD}}(x) (\equiv \Delta_{cb}(x))$ will not, in general, agree with experimentally measured values of
$q_{w|\text{Expt}}(x) \equiv q_{w|\text{Total}}(x) (\neq q_{w|cb}^\prime(x))$ and $\Delta_{\text{Expt}}(x) \equiv \Delta_{\text{actual}}(x) (\neq \Delta_{cb}(x))$. This last
assumption recognizes that, in the presence of significant nucleate boiling, both wall heat-flux
rates $q_w^\prime(x)$ and wall temperatures $T_w(x)$ characterize the actual thermal boundary conditions of
the surface and also $\Delta_{\text{actual}}(x) \geq \Delta_{cb}(x)$. Therefore, it is expected that, although experimentally
measured values of $T_w(x)$ is all that is needed for computing $h_{x|cb}$ and $q_{w|cb}^\prime(x)$, concurrent and
independent knowledge of $q_{w|\text{Expt}}(x) (\equiv q_{w|\text{Total}}(x))$ is needed for defining the nucleate boiling
contribution ($h_{x|\text{nb}}$) in the overall HTC. The definition of $h_{x|cb}$, obtainable by CFD, then becomes:
$q_{w|cb}^\prime(x) \equiv h_{x|cb} \ast \Delta T$. The above rigorous definitions are further standardized here by defining
the nucleate boiling HTC ($h_{x|nb}$) contribution through the relationship: $h_{x|\text{total}} \equiv h_{x|cb} + h_{x|nb}$.
This definition of $h_{x|nb}$ is preferred over other power-law forms in literature [14, 15] where no
clear definition of $h_{x|cb}$ is used. The CFD-based definition of $h_{x|cb}$, as given above, is a major
contribution of this paper as it removes a significant amount of ad hoc and ambiguous definitions
present in the literature. Going back to the three possible experimental realizations of annular flow,
one then sees that:

(i) For cases where nucleation is suppressed over the entire length of the channel, $h_{x|\text{total}} \approx
h_{x|cb}$. 

(ii) For cases where nucleate boiling has a dominant or significant contribution, $h_{x|\text{total}} \approx h_{x|\text{nb}}$

or $h_{x|\text{total}} \equiv h_{x|\text{cb}} + h_{x|\text{nb}}$.

(iii) For cases where nucleation is suppressed ($h_{x|\text{total}} \approx h_{x|\text{cb}}$) only in the downstream region (say, $x^* \leq x \leq L$), and $h_{x|\text{total}} = h_{x|\text{cb}} + h_{x|\text{nb}}$ over the upstream locations (say $0 \leq x \leq x^*$).

Although direct or indirect numerical simulations [16-21] in support of scientific investigation of nucleate pool boiling [2, 3] have been advancing for quite some time, there are limited literature and analytical techniques on annular flow-boiling CFD for $h_{x|\text{cb}}$ (except some that also include other flow-regimes, integral methods, and/or correlations-based estimates [22-26]). The available results/tools cannot reliably support the design of mm-scale innovative boiler operations [1, 27] of interest here. Furthermore, available numerical or experimental studies [28-31] of external gravity-driven falling film evaporation are not applicable to shear-driven evaporative flow-boiling under consideration.

Adapting and utilizing the ability of the reported steady/unsteady simulation techniques for steady internal condensing flows [32-34], this paper shows that it is now possible to use computational methods to obtain solutions and develop correlations for steady annular convective flow-boiling situations of interest to this paper. Furthermore, the paper shows that its direct numerical simulations (DNS) – a first-principles based (i.e. all governing equations are well established with regard to their scope and limitations) subset of computational fluid dynamics (CFD) – for laminar liquid and laminar vapor conditions can be used to develop correlations (with possible empirical corrections coming from synthesis with experimental results) for use in simplified predictive tools in support of deciding experimental runs involving steady non-pulsatile annular flow-boiling operations (which form part of innovative operations [1, 35]).
The nearly exact 2-D steady annular (suppressed nucleation cases) laminar/laminar simulation approach and results, as presented in this paper, address some critical issues with regard to flow-physics understanding as well as development and usage of HTC correlations. The paper proposes a sample convective HTC correlation in a well-defined range of non-dimensional numbers and discusses the validity (while improving the understanding and associated techniques) of popular one-dimensional correlations-based prediction tools. The paper is also able to adapt results obtained in some earlier papers [33, 34] to reasonably predict instability that characterizes transition from non-annular to annular flow-regimes. Relying on analogy with stability analyses for annular flow-condensation [33, 34] and associated discussions in this paper, instability signatures present in the nearly exact steady annular flow-boiling solutions are identified. These can be used to estimate the lower threshold of vapor quality $X_{cr|NA-A}$ – below which (i.e. $X < X_{cr|NA-A}$) non-annular (typically plug-slug regime) flow-regimes are typically observed in (and modeled through) experiments [1, 8, 35] involving horizontal ducts with moderate total mass-flux values ($G \leq 100 \text{ kg/m}^2\text{s}$) of refrigerants or water, micrometer-scale liquid film thickness, and pressure-drop minimizing range of mm-scale hydraulic diameters ($2 \text{ mm} \leq D_h \leq 8 \text{ mm}$).

With planned extensions of this approach to laminar liquid and turbulent vapor conditions, the paper also enables future and forthcoming “experiments-simulations” synthesis towards developing models/correlations – which are of importance to the design of mm-scale innovative flow-boilers (Figure 1). Such a modeling approach (also see section 5 on forthcoming results) can deal with: (i) development of a criterion for onset of suppressed-nucleation annular flow-boiling as the liquid film becomes thinner and exhibits suppressed nucleation $h_{x|total} \approx h_{x|cb}$ values at downstream locations within an annular regime while it continues to experience both annular and nucleate boiling contributions at upstream locations (also see [8, 36]), (ii) semi-empirical modeling
for the annular flow-boiling realm in the presence of nucleation (situations where $h_{x|\text{total}} \neq h_{x|\text{cb}}$), (iii) effects of transverse and axial components of gravity on non-annular to annular flow-regime transitions, (iv) the thin-film and dry-out instability related characterization of CHF mechanism (out of at least three-to-four different mechanisms of CHF discussed in the literature [3, 36]) that is typically relevant to the innovative annular flow-boiling approach (see [1] and Figure 1), (v) experimental and computational support for non-annular to annular flow-regime transition criteria as discussed in the earlier paragraph’s approach, which is based on recently reported instability analyses tools [33, 34] capable of estimating/identifying such conditions, (vi) development of pressure-drop correlations, etc.

Furthermore, the proposed “experiments-simulations” synthesis approach will enable development of empirical models that can be superposed on steady HTC ($h_{x|\text{tot-st}}$) correlations of the type being enabled here – and this will also assist in yielding HTC correlations for the technologically important high heat-flux pulsatile operations’ HTC ($h_{x|\text{tot-pulsatile}}$) values in the form of: $h_{x|\text{tot-pulsatile}} = h_{x|\text{tot-st}} + \Delta h_{\text{Enh}}$, where $\Delta h_{\text{Enh}}$ is the enhancement value (see [1]). The $\Delta h_{\text{Enh}}$ values in these operations depend on superposition of large-amplitude standing waves on annular thin film-boiling (over hydrophilic or super-hydrophilic boiling surfaces) cases associated with steady non-pulsatile realizations.

2. Problem Statement and Governing Equations
The computational algorithm and solutions presented here are for steady annular/stratified flow-boiling (under suppressed nucleation conditions) inside a channel, as shown in Figure 3. These boiling flows are achieved by heating the bottom wall and keeping the top wall at close to, or slightly above, local vapor saturation temperatures. In Figure 3, gravity-driven cases correspond to $\alpha > 0$ and shear-driven cases correspond to zero-gravity ($\bar{g} = 0$) or horizontal ($\alpha = 0$) cases.
The suppressed nucleation annular length $L$ (the distance between inlet and outlet) in Figure 3a typically corresponds to $L \leq x^P_A$, the maximum length possible for the annular regime as shown in Figure 2. However, the actual annular length for the flow in Figure 2 can be defined to be the sum “$L + L_{nb}$” of both suppressed and nucleate boiling lengths, i.e. “$x^P_A - L_{do}$,” the distance between the modeled “point of transition” between non-annular and annular regimes and exit of the device. In Figure 2, $L_{do}$ is the distance between the exit and the hypothetical “point of dry-out” – which is positive as it lies downstream of the exit. For $L_{do} < 0$, i.e. the dry-out point inside the test-section, the total annular length in Figure 2 will become “$L + L_{nb} - |L_{do}|$.”

The two-dimensional computational approach employed to investigate annular flow-boiling inside channels and tubes is based on the full governing equations described here and in [37]. Analogous flow-condensation approaches are given in [33, 34].

The liquid and vapor phases in the flow are denoted with subscripts $I = 1$ and $I = 2$ (alternatively, as $I = \text{‘L’}$ and ‘V’) respectively. Both phases are modeled as incompressible (i.e. vapor Mach numbers are low). The fluid properties (density $\rho$, viscosity $\mu$, specific heat $C_p$, and thermal conductivity $k$) are denoted with subscript $I$. The properties are to take their representative constant values for each phase ($I = 1$ or 2).

Let the temperature, pressure, and velocity fields over the two phases be respectively denoted as $T_i, p_i,$ and $\vec{v}_i = u_i \hat{i} + v_i \hat{j}$. Also, let $T_{sat}(p^i_2)$ be the saturation temperature of the vapor as a function of local pressure $p^i_2$ at the interface, $\Delta$ be the film thickness, $\dot{m}_p$ be the local interfacial phase-change mass flux (kg/m$^2$-s), and $T_w(x) (> T_{sat}(p^i_2))$ be a known temperature variation of the heated bottom surface (with its length-averaged mean value being $\overline{T}_w$, where $\overline{T}_w \equiv \frac{1}{L} \int_0^L T_w(x^P) dx^P$). Let $g_x$ and $g_y$ be the components of gravity along the x and y axes, $p_0$ be the
steady inlet pressure at point A in Figure 3b, \( \Delta T(\equiv T_w(x) - T_{sat}(p_0)) \) be a representative characteristic temperature difference between the bottom plate and the liquid, \( h_{fg} \) be the heat of vaporization at temperature \( T_{sat}(p_2) \) which is typically close to \( T_{sat}(p_0) \), and \( U \) be the average inlet vapor speed determined by vapor density \( \rho_2 \) and the inlet mass flow rate per unit width \( \dot{M}_\text{in}'(\equiv \rho_2 \cdot U \cdot h) \), where \( \dot{M}_\text{in}' \) is related to total mass flow rate \( \dot{M}_\text{in} \) and total mass-flux \( G \) for a flow through a rectangular cross-section channel of height \( h \) and width \( w \), provided \( \frac{h}{w} \ll 1 \). This relationship is given by: \( \dot{M}_\text{in}' = \dot{M}_\text{in} \cdot w = G \cdot h \cdot w \). Let \( (x^p, y^p) \) represent physical distances of a point with respect to the axes in Figure 3 (for which \( x^p = 0 \) is at the inlet and \( y^p = 0 \) is at the heated bottom wall surface). Next a new list of fundamental non-dimensional variables, \((x, y, \delta, u_I, v_I, \dot{m}, \theta_I, \pi_I)\) are introduced through the following definitions:

\[
[x^p, y^p, \Delta, u_I^p, v_I^p] \equiv [h \cdot x, h \cdot y, h \cdot \delta, U \cdot u_I, U \cdot v_I]
\]

\[
[\dot{m}^p, T_1, p_1] \equiv [\rho_1 \cdot U \cdot \dot{m}, T_{sat}(p_0) + \Delta T \cdot \theta_I \cdot p_0 + \rho_1 \cdot U^2 \cdot \pi_I]
\]

The above annular flow-boiling specification is appropriate for prescribed or known wall temperatures \( T_w(x) \). This is typically characterized as \( \Delta T(x) \equiv T_w(x) - T_{sat}(p_0) \equiv \Delta T \cdot \theta_w(x) \). Note that, a “fixed” \( \theta_w(x) \neq 1 \) characterizes the same fixed non-uniform “method of heating” for different values of the mean temperature difference \( \Delta T \). In this case, boiling surface heat-flux associated with convective boiling \( q_{\text{wlc}}(x) \) and local convective boiling HTC \( h_{\text{xlcb}}(\equiv q_{\text{wlc}}(x)/\Delta T) \) are values to be found as part of the CFD solution.

For prescribed heat-flux “method of heating”, \( q_w(x)(\equiv \bar{q}_w \cdot \Psi_q(x)) \) values are known. This is equivalent to knowing the mean-heat flux \( \bar{q}_w \equiv \frac{1}{L} \int_0^L q_w(x^p)dx^p \) value and associated “method of heating” characterization function \( \Psi_q(x) \). Again note that a “fixed” \( \Psi_q(x) \neq 1 \)
characterizes the same fixed non-uniform “method of heating” for different values of the mean heat-flux $\dot{q}_w$. In this case, $T_w(x)$, $\overline{T}_w$ and $\Delta T$ – with an assumption that nucleation is suppressed – are some of the quantities that are obtained as part of the CFD solution.

The representative constant values of the fluid properties are obtained from Engineering Equation Solver (EES) software [38] and other data handbooks. However, there are some inherent uncertainties associated with experimental data reported in the handbook values. Therefore, key results presented in non-dimensional terms should be assumed to have some additional uncertainties associated with fluid properties (appearing in non-dimensional parameters) over and above computational error uncertainties (associated with level of convergence, discretization/truncation errors, etc.).

2.1. Interior Equations

The simulations emphasized here assume laminar vapor and laminar liquid flows. For most shear-driven flows of interest to annular flow-boiling in mm-scale ducts, the laminar liquid flow assumption holds up to the end of the computational domain (i.e. the distance in Figure 1 between the inlet, $x^p = 0$, and the exit, $x^p = L$ – where $L$ is typically less than the length $x_A^p$ of the annular regime, also see corresponding locations in Figure 2). It is expected that the comparisons of results obtained from these simulations with corresponding experimental results for suppressed nucleation cases will be quite good even if the core vapor flow far from the near interface laminar zone (connected with the laminar liquid flow) is turbulent up to a certain level. This level is to be determined by local values of vapor-phase Reynolds number that does not correspond to such thinness of the near interface laminar zone that the *interfacial stresses* for the liquid film flow start becoming significantly different from their values obtained from laminar/laminar simulations. This agreement is expected, and later on substantiated by streamline patterns given in this paper,
because dominant values of near-interface vapor flow variables, e.g., $x$ and $y$-components of the interfacial vapor velocity, will remain very small and locally laminar (see later predictions of stream-line patterns) as the liquid flow remains thin and dominated by viscous forces. Any additional randomness introduced through interfacial waviness arising from random noise sources on the flow domain boundaries and far field vapor core turbulence may, at most, contribute to “laminar interfacial turbulence” but this will not have sufficient impact on the significantly stronger instability mechanisms (see analogous discussion in [29, 30] for condensing flows) that yield an estimate – based on non-linear stability analyses for laminar/laminar flows - for the length $\hat{x}_A \equiv \frac{x_A^p}{h}$ of the annular regime.

Under laminar/laminar assumption, the non-dimensional differential forms of mass, momentum ($x$ and $y$-components), and energy equations for the two-dimensional flow in the interior of either of the incompressible phases ($I = 1$ or $2$) are the well-known equations:

$$
\frac{\partial u_I}{\partial x} + \frac{\partial v_I}{\partial y} = 0
$$

$$
\frac{\partial u_I}{\partial t} + u_I \frac{\partial u_I}{\partial x} + v_I \frac{\partial u_I}{\partial y} = -\left(\frac{\partial \pi_I}{\partial x}\right) + \frac{1}{\text{Re}_I} \left(\frac{\partial^2 u_I}{\partial x^2} + \frac{\partial^2 u_I}{\partial y^2}\right) + \text{Fr}_{x^-}^2 + \frac{1}{\text{Re}_I} \left(\frac{\partial^2 v_I}{\partial x^2} + \frac{\partial^2 v_I}{\partial y^2}\right)
$$

$$
\frac{\partial v_I}{\partial t} + u_I \frac{\partial v_I}{\partial x} + v_I \frac{\partial v_I}{\partial y} = -\left(\frac{\partial \pi_I}{\partial y}\right) + \frac{1}{\text{Re}_I} \left(\frac{\partial^2 v_I}{\partial x^2} + \frac{\partial^2 v_I}{\partial y^2}\right) + \text{Fr}_{y^-}^2 + \frac{1}{\text{Re}_I} \left(\frac{\partial^2 v_I}{\partial x^2} + \frac{\partial^2 v_I}{\partial y^2}\right)
$$

$$
\frac{\partial \theta_I}{\partial t} + u_I \frac{\partial \theta_I}{\partial x} + v_I \frac{\partial \theta_I}{\partial y} \approx \frac{1}{\text{Re}_I, \text{Pr}_I} \left(\frac{\partial^2 \theta_I}{\partial x^2} + \frac{\partial^2 \theta_I}{\partial y^2}\right)
$$

where $\text{Re}_I \equiv \frac{\rho_1 U h}{\mu_1}$, $\text{Pr}_I \equiv \frac{\mu_1 c_{p1}}{k_1}$, $\text{Fr}_{x^-}^2 \equiv \frac{g_x h}{U^2}$ and $\text{Fr}_{y^-}^2 \equiv \frac{g_y h}{U^2}$.

### 2.2. Interface Conditions

Superscript “i” is used for the values of flow variables at the interface. The interface, is explicitly located by the expression $\Phi \equiv y^p - \Delta (x^p) = 0$. The nearly exact interface conditions (see [33,
39, 40] etc.) need to be better qualified, extended to cover sub-micron liquid film thickness values of current interest – as well as for planned future simulations-assisted investigations. These conditions and issues are re-stated here in Appendix A1. The “Newtonian” fluid models for stresses \( \mathbf{T}_1 \) and \( \mathbf{T}_2 \) defined in Appendix A1 also define the values of the liquid and vapor phases’ traction vectors \( \mathbf{T}^{pl}_1 \) and \( \mathbf{T}^{pl}_2 \) at any point on the interface (\( \Phi = 0 \)). At any point on the interface (see Figure 3a), the unit normal (directed from the liquid to the vapor phase) is denoted by \( \mathbf{n} \) and unit tangent vector by \( \mathbf{t} \). Note that traction vectors (see Appendix A1 or [40]) \( \mathbf{T}^{pl}_2 \equiv \mathbf{T}^{pl}_2 \mathbf{n} \equiv \tau^{pl}_{2x} \mathbf{i} + \tau^{pl}_{2y} \mathbf{j} \) and \( \mathbf{T}^{pl}_1 \equiv \mathbf{T}^{pl}_1 \mathbf{n} \equiv \tau^{pl}_{1x} \mathbf{i} + \tau^{pl}_{1y} \mathbf{j} \). The non-dimensional values of the stress vector components are, respectively, defined as \( \mathbf{T}^{pl}_2 \equiv (h/\mu_2 U) \mathbf{T}^{pl}_2 \equiv \tau^{pl}_{2x} \mathbf{i} + \tau^{pl}_{2y} \mathbf{j} \) and \( \mathbf{T}^{pl}_1 \equiv (h/\mu_1 U) \mathbf{T}^{pl}_1 \equiv \tau^{pl}_{1x} \mathbf{i} + \tau^{pl}_{1y} \mathbf{j} \). Non-dimensional Cartesian co-ordinate forms of the interface conditions, for the flow in Figure 3a, are given below:

- The continuity of tangential component of velocities is a requirement (see Eq. (A1.2)). This requirement non-dimensionalizes to:

\[
\begin{align*}
  u^2_i &= u^1_i - \delta_x (v^2_i - v^1_i) \\
\end{align*}
\]  

where, \( \delta_x \equiv \partial \delta / \partial x \).

- The normal component of momentum balance at the interface, after ignoring the normal component of viscous stresses in comparison to interfacial pressures, is modeled by Eq. (A1.3) in Appendix A1. This relationship non-dimensionalizes to:

\[
\begin{align*}
  \pi^1_i &= \frac{\rho_2}{\rho_1} \pi^2_i - \frac{1}{We} \left( \frac{\delta_{xx}}{(1 + \delta_x^2)^{3/2}} \right) + \dot{m}^2 \left( \frac{\rho_1}{\rho_2} - 1 \right) \\
\end{align*}
\]  

where, \( We \equiv \rho_1 U^2 h/\sigma \) and surface tension \( \sigma \) for the pure vapor depends on local representative interfacial temperature \( T^i \) (i.e. \( \sigma = \sigma (T^i) \)).
• The tangential component of momentum balance at the interface (see Eq. (A1.4)) non-dimensionalizes to:

\[ \frac{\partial u_1}{\partial y} \bigg|^{\text{i}} = \frac{\mu_2}{\mu_1} \frac{\partial u_2}{\partial y} \bigg|^{\text{i}} + [t] \]  

(5)

where, the term \([t]\) in Eq. (5) is defined as:

\[
[t] = \left\{ \frac{\mu_2}{\mu_1} \frac{\partial v_2}{\partial x} \bigg|^{\text{i}} - \frac{\partial v_1}{\partial x} \bigg|^{\text{i}} \right\} + \frac{2\delta_x}{[1 + \delta_x^2]} \left\{ \frac{\partial u_1}{\partial x} \bigg|^{\text{i}} - \frac{\partial v_1}{\partial y} \bigg|^{\text{i}} \right\} 
- \frac{2\delta_x}{[1 + \delta_x^2]} \frac{\mu_2}{\mu_1} \left\{ \frac{\partial u_2}{\partial x} \bigg|^{\text{i}} - \frac{\partial v_2}{\partial y} \bigg|^{\text{i}} \right\}
\]

(6)

Following discussions given for Eq. (A1.4), the right side of Eq. (6) has ignored the Marangoni term (whose effects, for the class of problems studied here, have earlier been verified to be negligible).

• The non-dimensional form of non-zero physical values of interfacial mass fluxes \(\bar{m}_{\text{L,K}}^{\text{p}}\) and \(\bar{m}_{\text{V,K}}^{\text{p}}\) (defined in Eq. (A1.5)) arise from kinematic constraints associated with the liquid and vapor velocity values at the interface. In the non-dimensional form these are given by:

\[
\bar{m}_{\text{L,K}}^{\text{p}} = \left[ -u_1^{\text{i}} \left( \frac{\partial \delta}{\partial x} \right) + \left( v_1^{\text{i}} - \frac{\partial \delta}{\partial t} \right) \right] \sqrt{1 + \left( \frac{\partial \delta}{\partial x} \right)^2} \text{ and}
\]

\[
\bar{m}_{\text{V,K}}^{\text{p}} = \frac{\rho_2}{\rho_1} \left[ -u_2^{\text{i}} \left( \frac{\partial \delta}{\partial x} \right) + \left( v_2^{\text{i}} - \frac{\partial \delta}{\partial t} \right) \right] \sqrt{1 + \left( \frac{\partial \delta}{\partial x} \right)^2}
\]

(7)

• The non-dimensional form of non-zero physical values of interfacial mass flux \(\bar{m}_{\text{Energy}}^{\text{p}}\) (as given by Eq. (A1.6)) represents the constraint imposed by the dominant net thermal energy transfer rates across the interface and is given by:

\[
\bar{m}_{\text{Energy}}^{\text{p}} \approx \frac{Ja}{Re_1 Pr_1} \left\{ - \frac{\partial \theta_1}{\partial n} \bigg|^{\text{i}} + \frac{k_2}{k_1} \frac{\partial \theta_2}{\partial n} \bigg|^{\text{i}} \right\}
\]

(8)

where, \(Ja \equiv C_p \Delta T/h_{fg}\) and \(h_{fg} \equiv h_{fg}(T_{\text{sat}}(p_2^{\text{i}})) \cong h_{fg}(T_{\text{sat}}(p_0^{\text{i}})).\) Recall that liquid Reynolds
number \( \text{Re}_1 \) and Prandtl number \( \text{Pr}_1 \) are given by their definitions that immediately follow Eq. (2).

For the case of prescribed heat-flux “method of heating” \( (q^*_w(x) \equiv \bar{q}^*_w \cdot \Psi_q(x)) \) – with average value heat-flux of \( \bar{q}^*_w \) over \( 0 \leq x \leq L \) – Eq. (A1.6) in Appendix A1 can be used to rewrite Eq. (8) in its alternative non-dimensional form:

\[
\dot{m}_{\text{Energy}} = \frac{\bar{q}^*_w}{\rho_2 U_h f g} \cdot \frac{\rho_2}{\rho_1} \cdot \tilde{\Psi}_q(x) \equiv \text{Bl} \cdot \frac{\rho_2}{\rho_1} \cdot \tilde{\Psi}_q(x) \tag{9}
\]

where, \( \text{Bl} = \bar{q}^*_w / (\rho_2 U_h f g) \) & \( \tilde{\Psi}_q(x) \equiv q^*_w(x) / \bar{q}^*_w \). Here interfacial heat-flux \( q^*_w(x) \) is in the normal \( \vec{n} \) direction at any point (associated with distance \( x \) and associated position vector \( \vec{x} \) on the interface) and equals \( \dot{m}_{\text{Energy}}^p \cdot h_{fg} \) where \( \dot{m}_{\text{Energy}}^p \) is given by Eq. (A1.6). However for thin film flows of interest to this paper, the relationship \( q^*_w(x) \approx q^*_w(x) \) and \( \tilde{\Psi}_q(x) \approx \Psi_q(x) \) approximately hold.

- The interfacial mass balance (in Eq. (A1.9) or, when necessary, by Eq. (A1.10)) requires that the net mass-flux (in kg/m\(^2\)-s) at a point on the interface, must be the same for all the different physical processes that impose a constraint on its local value. The non-dimensional form of this requirement becomes:

\[
\dot{m}_{\text{LK}} = \dot{m}_{\text{VK}} = \dot{m}_{\text{Energy}} \equiv \dot{m} \tag{10}
\]

It should be noted that negligible interfacial thermal resistance and equilibrium thermodynamics is assumed to hold on either side of the interface. This is reasonable, except for some situations discussed in Appendix A1. This is because the liquid film-thickness values considered here are typically greater than a few micrometers and much less than, or at most, same order (factor of 1/4 to 1/2) as the mm-scale channel height \( h \). This modeling assumption typically holds for almost all “\( x \)” values of interest \( (0 \leq x \leq L) \) over which the CFD solution
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is sought.

- The non-dimensional thermodynamic restriction on interfacial temperatures (as given by the approximation in Eq. (A1.7), becomes:

\[ \theta_1 \approx \theta_2 \approx \theta_s(\pi_2) \] (11)

Within the vapor phase, for the refrigerants and mm-scale ducts considered here, the inlet pressure \( p_0 \ll p_{cr} \), where \( p_{cr} \) is the critical pressure [2] of the vapor. As a result, the changes in absolute pressure relative to the inlet pressure are big enough to affect vapor motion but, at the same time, they are usually too small to significantly affect saturation temperatures (except in micron-scale ducts and at high mass flux \( G \) values). Therefore, computations also show that, we have \( \theta_s(\pi_2) \approx \theta_s(0) \).

2.3. Boundary Conditions for Combined Consideration of the Vapor and Liquid Domains

The problem is computationally solved subject to the boundary conditions shown on a representative, not-to-scale, film profile in the vapor-liquid domain of Figure 3b.

**Top wall:** The upper wall physical temperature \( T_2(x^p, h) > T_{sat}(p_0) \) is at a superheated value (typically 5-10°C above saturation temperature) and this, along with the assumption that \( p_0 \ll p_{cr} \), makes the vapor solutions almost indistinguishable from those that assume vapor phase temperature to be a uniform \( T_{sat}(p_0) \).

**Bottom wall:** Besides the no-slip condition at the boiling surface, a steady boiling surface temperature \( T_1(x^p, 0) = T_w(x^p)(> T_{sat}(p_0)) \) – or a steady wall heat-flux \( q_w(x) \) – define its thermal boundary condition. Also, as experimentally established [41], a specific choice for earlier defined non-dimensional temperature function:
\[ \theta_w(x) \equiv \theta_1(x,0) = \frac{T_1(x,0) - T_{\text{sat}}(p_0)}{T_w - T_{\text{sat}}(p_0)} \]  

for wall temperature \( T_w(x) \) – or a specific \( \Psi_q(x) \) in case of wall heat flux prescription \( q_w(x) \equiv \bar{q}_w \cdot \Psi_q(x) \) – define a specific “method of heating.”

**Inlet conditions and significance of its resolution:** At the inlet \( (x^P = 0) \), presence of evaporative annular flow-boiling (Figure 3a) is assumed, and one requires among other variables, a prescription of a finite non-zero film thickness, \( \Delta(0) = \Delta_0 \). Because of the finiteness of \( \Delta_0 \) (unlike \( \Delta_0 \approx 0 \) in the onset of condensation condition discussed in [33, 40]), this value has to be “special” as all inlet variable profiles – such as inlet liquid velocity, pressure and temperature profiles \( (u_1(0,y), v_1(0,y), p_1(0,y), T_1(0,y)) \) over \( 0 \leq y \leq \Delta_0 \); and inlet vapor velocity, pressure and temperature profiles \( (u_2(0,y), v_2(0,y), p_2(0,y), T_2(0,y)) \) over \( \Delta_0 \leq y \leq h \); inlet values of interfacial stress vectors \( (\tau_{1^i}^p(x^P = 0, y^P = \Delta_0), \tau_{2^i}^p(x^P = 0, y^P = \Delta_0)) \); and interfacial mass-flux \( (\dot{m}^P(x^P = 0, y^P = \Delta_0)) \) – have to be “mutually consistent” (satisfy all the interfacial conditions) for the proposed laminar/laminar DNS. Such restrictive compatibility requirements among so many variables make full 2-D annular flow boiling DNS a challenge – particularly when one compares it with simpler and popular correlations-based one-dimensional (1-D) simulations/models for annular boiling (to be described in section 3.3) which only requires prescriptions of total mass flow rate per unit width and inlet vapor quality (which is often close to its thermodynamic estimate) at \( x^P = 0 \). That is, for correlations-based simpler calculations, only total mass flow rate per unit width \( \dot{M}_p(x^P = 0) \equiv \dot{M}_1^p(0) + \dot{M}_2^p(0) \equiv \int_0^{\Delta_0} \rho_1 u_1^p(0, y^P) dy^P + \int_{\Delta_0}^h \rho_2 u_2^p(y^P) dy^P \) and inlet quality \( X_{in} \equiv X(0) = \dot{M}_2^p(0)/\dot{M}_p(x^P = 0) \) are needed at \( x^P = 0 \). Therefore, it is expected that, perhaps, detailed inlet conditions information for the 2-D steady simulation are quite important only for implementation of DNS and associated interest in 2-D nature of flow...
variables’ spatial variations – not so much for 1-D functions (such as x-variations of $h_{x\mid cb}$, $X$ etc.) of interest. Despite this complexity, the usefulness of DNS lies in the processed 1-D values (and their correlations) that it yields x-variations of local convective boiling HTC $h_{x\mid cb}$ (or its non-dimensional form, $Nu_{x\mid cb}$) and the corresponding quality $X$. The paper shows that the 1-D calculations based on utilizing CFD-enabled HTC correlation proposals are indeed relatively insensitive to such 2-D details with regard to inlet conditions needed for DNS.

To benefit from detailed CFD solution and to address the needs of this rather restrictive specification of inlet conditions, the following enabling approach is recommended. The proposed enabling approach to deal with this situation is to assume a “prior” adiabatic laminar/laminar flow (see $-x^p* < x^p < -x^{p+i}$ in Figure 4, where “i” in $x^{p+i}$ takes integer values 1, 2, 3, etc. associated with different “prior” heating methods HM-i shown in Figure 5) and, also, that a “prior” and spatially non-uniform “method of heating” be prescribed (see Figure 5) for this part of $x^p < 0$. Whether it is a wall temperature $T_w(x)$ or a wall heat-flux $q_w(x)$ (Figure 5) prescription, at a certain $x^p = -x^p*$, it is assumed that liquid and vapor enter the channel as adiabatic isothermal laminar/laminar flows (i.e., both phases are at same uniform temperature and experience no active heating over the adiabatic zone, viz. $-x^p* < x^p < -x^{p+i}$). For this adiabatic zone, “mutually consistent” analytical prescriptions for all required inlet-conditions are available at $x^p = -x^p*$ (see Appendix A2).

At the location $x^p = -x^p*$ in Figure 5, the fluid temperatures and wall temperatures all equal $T_{sat}(p^*)$, where $p^*$ is the absolute pressure assumed for the top wall location at $x^p = -x^p*$. At $x^p = -x^p*$, the consistent values of liquid and vapor phases’ velocity, pressure and temperature profiles; interfacial stress vectors; and interfacial mass-flux are as given in Appendix A2. For any assumed non-uniform “prior” heating method (denoted as “HM-i”, i = 1, 2 & 3 over $-x^{p+i} <$
\( x^P < 0 \) as in the caption of Figure 5), mass flow rate \( \dot{M}_{in}' \), and suitably assigned inlet conditions such as quality values at \( x^P = -x^{P*} \) associated with liquid and vapor flow rates – \( \dot{M}_L'(x^P = -x^{P*}) \) and \( \dot{M}_V'(x^P = -x^{P*}) \) respectively, the CFD solution over \( x^P > -x^{P*} \) automatically yields correct and consistent (with analytical solution in Appendix A2) inlet conditions up to \( x^P = -(x^P)^{**i} \) followed by the transition heating zone of \( -(x^P)^{**i} < x^P < 0 \) on to the uniform heating method zone of interest, which begins at \( x^P = 0 \). The actual physical value of the steady pressure \( p_{in}(= p_0) \) at \( x^P = 0 \) and \( y^P = h \) is not directly used in CFD but it indirectly appears through fluid properties and important thermodynamic properties such as \( h_{fg}(p_2^i) \approx h_{fg}(p_0) \) and \( T_{sat}(p_2^i) \approx T_{sat}(p_0) \).

**Steady Exit conditions:** For the steady problem, the flow is parabolic and no exit condition is needed. Pressure is not directly prescribed across the exit boundary for the computational simulations. Its arbitrary “reference” value \( p_{exit} \) is specified, to begin with, in the vapor domain – at the corner point of the intersection of the exit and the top wall (point B in Figure 3b). This value is then re-adjusted to ensure a reference pressure value of \( p_{ref|A} = 0 \) for the reference location point A (at \( x = 0 \) in Figure 3b).

**Initial Conditions:** The steady problem considered here needs no initial condition prescription as there are no time considerations. It does, however, require some reasonable but arbitrary initial guessed values for the first iteration, as described in step (i) of the algorithm in section 3 below.

### 3. Computational Approach, Algorithm and Grid-size Restrictions

#### 3.1 Computational Approach and Algorithm
The 2-D steady computational algorithm will be described for obtaining steady solutions of the steady boundary value problem shown in Figure 3b. The solution can be obtained by the steady approach described below.
The simulation uses an approach of separately solving, on COMSOL, the (steady) liquid and vapor domain governing equations over their respective domains – domains that result from the assumed “sharp” interface model in Figure 3b. The steady algorithm – after making choices for the gap height $h$, the pure fluid, inlet pressure $p_0$, and cooling conditions – obtains fluid properties, sets $\theta_1^i \equiv \theta_s(0)$, and begins with assuming reasonable first-guess values of interface location function $\Delta(x)$ (or non-dimensional $\delta(x)$) along with key interfacial flow variable functions $u_1^i(x)$ and $\dot{m}(x)$ (where $\dot{m}$ is for obtaining $v_1^i(x)$ values). The steady single domain direct numerical solution (DNS) approach for each of the two phases retains all the steady terms in the governing equations (including interface conditions) of section 2 – except that, to model steady flows, all partial time derivatives are set to zero.

The approach used here for annular suppressed nucleation steady flow-boiling is essentially the same as the steady algorithm for annular flow-condensation described in [33, 34]. With respect to Figure 3, the algorithm consists of the following steps:

(i) Utilizing the liquid side interfacial flow variables first guesses of $u_1^i, v_1^i, \theta_1^i$ and the first guess of steady film thickness $\delta(x)$, the liquid domain in Figure 3b is treated as a separate “fixed” domain and the governing interior equations of mass, momentum and energy are solved on COMSOL. The exit boundary at $x = L_{\text{comp}}$ is first treated a location where an arbitrary first guess uniform pressure value which is known and which equals the previous iteration or first guess value of corner pressure (i.e., $p_1(L_{\text{comp}}, y) \equiv p_1(L_{\text{comp}}, y = \Delta(L_{\text{comp}})) \equiv p_1^i$). The boundary conditions for the interface is one which has the prescribed aforementioned first guess values of velocity $(u_1^i, v_1^i)$ and temperature $(\theta_1^i)$. The two-phase flow simulations’ inlet of interest at $x = 0$ is extended upstream to a de facto inlet at $x^p = -x_p^*$, as shown in Figure 4, which is associated with known adiabatic flow.
conditions (Appendix A2). The bottom wall thermal boundary conditions for the $x < 0$ zone are as prescribed in Figure 5. The uniform temperature and velocity profiles $u_1(x_p = -x_p^*, y)$ and $v_1(x_p = -x_p^*, y)$ – all are available from analytically known adiabatic flow results (for any appropriate liquid mass flow rate $\dot{M}_{L}'$ and associated film thickness $\Delta_0a = \Delta(x_p = -x_p^*)$). These are obtained as per procedures and results given in Appendix A2.

The COMSOL solution at this step is used to yield reasonable first guess values of interior liquid domain flow variables, viz. $u_1(x,y), v_1(x,y), \pi_1(x,y)$ and $\theta_1(x,y)$.

(ii) Next, continuity of tangential velocity (Eq. (3)) and $\dot{m}_{LK} = \dot{m}_{VK}$ (part of Eq. (10)) with terms as in Eq. (7) are used to obtain $u_2^i(x)$ and $v_2^i(x)$ values. The non-dimensional temperature, $\theta_2^i(x)$ is obtained from Eq. (11). The mathematical operations for obtaining these functions are performed within a MATLAB program and results are transferred to the CFD formulation on COMSOL.

(iii) Utilizing the currently available location $\delta(x)$ and vapor side interfacial flow variables $u_2^i(x), v_2^i(x)$, and $\theta_2^i(x)$ obtained through the previous step; the temporarily (for this iteration) “fixed” vapor domain in Figure 3b is used to solve the interior governing equations of mass, momentum and energy on COMSOL. Here, the interface is one of the boundaries which has prescribed velocity components and temperature conditions from step (ii) above. The exit at $x = L_{\text{comp}}$ is treated as prescribed outflow boundary condition with initially zero value for reference pressure at point B of Figure 3b. The upstream extended inlet at $x_p = -x_p^*$ (with bottom wall thermal boundary conditions for the liquid as in Figure 5) – also has known velocity profiles for $u_2$ or $v_2$ at $x_p = -x_p^*$. These are associated with the adiabatic flow results for vapor mass flow rate $\dot{M}_V = \dot{M}_{in} - \dot{M}_L$, and film thickness $\Delta_0a = \Delta(x_p = -x_p^*)$ – and all these values are as per results given in
Appendix A2. The computationally predicted velocity profiles of $u_2(x,y)$, $v_2(x,y)$ are retained and pressure profile $\pi_2(x,y)$ is re-adjusted so as to make the reference pressure zero at point A ($x^p = -x^p^*$) instead of at point B in Figure 3b. Next, COMSOL is used to obtain the x and y components of the interfacial stress vector $\mathbf{\tau}_2^{\text{pl}}$ or its non-dimensional value $\mathbf{\tau}_2^i$.

(iv) Using the normal and tangential components of interfacial momentum balance conditions (Eqs. (4) and (5)) along with the x and y-components of the computed values of $\mathbf{\tau}_2^i$ in step (iii) above, MATLAB is used to obtain the x and y components of the liquid side’s interfacial stress vector $\mathbf{\tau}_1^{\text{pl}}$ and its non-dimensional value $\mathbf{\tau}_1^i$.

(v) Using the stress components of at the interface of the liquid domain as boundary condition to replace the velocity components ($u_1^i, v_1^i$), while retaining the remaining prescriptions associated with step (i); the liquid domain problem is re-solved on COMSOL for $-x^p^* < x^p < L_{\text{comp}}$ – with bottom wall thermal boundary condition as in Figure 5. Key variables from the resulting solution are saved. These are interior liquid domain values of the variables $u_1, v_1, \pi_1$ and $\theta_1$ – as well as their interfacial values associated with the one-dimensional interfacial functions $u_1^i$ and $v_1^i$.

(vi) At this point all the interfacial conditions in section 2.2, except the remaining equality of Eq. (10), namely: $m_{\text{L,K}} = m_{\text{Energy}}$, has been satisfied. As discussed in Ranjeeth et al. [33, 34], this equality leads to an interface tracking equation whose steady form is:

$$\frac{d\delta(x)}{dx} = \frac{\bar{v}}{\bar{u}} \cdot x^p \geq -x^p^*$$

where, $\delta(x^p = -x^p^*) = \delta_0a \equiv \Delta_0a/h$ is known from Appendix A2’s Eqs. (A2.10) - (A2.11). For prescribed temperature boundary conditions, the definitions of $\bar{u}(x)$ and $\bar{v}(x)$...
in Eq (13) arise from use of Eq. (8) for $\dot{m}_{\text{Energy}}$. This yields:

$$\bar{u} \equiv u_1^i + \left[Ja/(Re_1.Pr_1)\right] \frac{\partial \theta}{\partial x} \bigg|_i$$

and, $$\bar{v} \equiv v_1^i + \left[Ja/(Re_1.Pr_1)\right] \frac{\partial \theta}{\partial x} \bigg|_i$$

(14)

For prescribed heat-flux boundary-conditions, the definitions of $\bar{u}(x)$ and $\bar{v}(x)$ in Eq. (13) arise from use of Eq. (9) for $\dot{m}_{\text{Energy}}$. This leads to:

$$\bar{u} \equiv u_1^i \ \text{and}, \ \bar{v} \equiv v_1^i + B\frac{\rho_2}{\rho_1}\Psi q(x)$$

(15)

Next, on MATLAB, Eqs. (13) and (14) are solved by a simple numerical integration scheme (trapezoidal Simpson rule or higher order, as needed) to yield a new estimate of the interface location $\delta(x)$ for a certain simple equi-spaced fixed-grid (of width $\Delta x_{f-g}$) discretization of the $x$-axis, where $x = x_i = i.(\Delta x_{f-g})$ and integer $i = 0, 1, 2, \ldots$. At this point in the algorithm, the location is updated after Eq. (13) is solved, and the changed location is used to change the domain (by a simple mapping technique) of all the previously computed interior liquid domain variables $u_1, v_1, \pi_1,$ etc. available over $-x^{p^*} < x^p < L_{\text{comp}}$. The domain change is in the $y$-direction – from the earlier $y$-domain to this step’s new $y$-domain of $0 \leq y \leq \delta(x)|_{\text{new-}(vi)}$. If prescribed heat-flux $q_w(x)$ is the thermal boundary condition for the heated bottom wall, similar numerical solutions of Eqs. (13) and (15) are used.

(vii) With the updated liquid domain solution and interface location from step (vi) above, steps (ii) through (vi) are repeated until converged solutions are obtained. Besides COMSOL’s convergence tests for numerical solutions of interior equations for each of the two-phases, it is checked that all interior, interface, and boundary conditions are satisfied.

In the implementation of the above algorithm, a COMSOL-specific point with regard to post
solution evaluation of interfacial stress vector $\tau^\text{pl}_2$ in step (iii) above should be noted. The x and y-components, $\tau^p_{2x}$ and $\tau^p_{2y}$ are directly and concurrently evaluated in COMSOL at any interior “$x = x_i$” where $-x^p_* < x^p < L_{\text{comp}}$. It appears that COMSOL’s default procedure is to obtain these values by a higher order central differencing type approach (that also utilizes solution variables’ values at $x_{i-1}, x_{i+1}$ etc.) at an interior $x = x_i$ location. However, at the left and right boundary points of $x_i = x^p = -x^p_*$ and $x_i = x^p = L_{\text{comp}}$, the stress vector should be obtained by a one-sided differencing approach as upstream or downstream values outside the computational domain are not known. This default procedure on COMSOL can introduce significant errors at the left and right boundary points if the issue is not properly addressed.

The above issue of evaluation of $\tau^\text{pl}_2$ was addressed here by using available values of $\tau^p_2$ (from analytical adiabatic solution in Appendix A2) for $x^p = -x^p_*$. For $x^p = L_{\text{comp}}$, the values close to $x^p \approx L_{\text{comp}}$ (for computations over $x^p \leq L < L_{\text{comp}}$) were used from certain stored estimates for $x^p \approx L_{\text{comp}}$. These stored estimates were obtained from an earlier longer domain computations involving $L_{\text{comp earlier}} > L_{\text{comp}}$. For this reason, the solution reported here is only valid for $-x^p_* < x^p < L_{\text{comp}}$ domain. Note that the flow boiling solution of interest is typically only for uniform thermal boundary condition zone, i.e., $x^p > 0$ in Figure 5 and that too, typically, one is interested in values that discard any entrance zone effect (as discussed later) associated with prior ($x^p < 0$) heating in Figure 5.

### 3.2 Grid-size Restrictions, Grid-size Independent Solutions, and Convergence

From the Discrete Fourier Transforms (DFT) of key x-dependent functions of $\delta(x), \bar{u}(x), \bar{v}(x), \tau^l_{2x}, \tau^l_{2y}$ etc.; their dominant spatial frequencies are ascertained. Then the smallest spatial length $\lambda_x$ that needs to be resolved is ascertained. Then the spatial discretization $\Delta x_{f-g}$ in
step (vi) is so chosen that it not only satisfies all interfacial conditions but that it can also resolve the flow-physics constraints on the resolvable length scales of interest (including the largest non-dimensional length $L = L_{\text{comp}}$). That is, the Nyquist criteria [42] is satisfied by imposing a more conservative restriction of $\lambda_x / 6 < \Delta x_{f-g} < L / 2$ (rather than Nyquist criteria: $\lambda_x / 2 < \Delta x_{f-g} < L$).

It should be noted that, after ensuring mesh-type independence (quadrilateral v/s triangular meshes) for steady solutions, only triangular meshes were chosen for superior performance in steady CFD simulations used for the 2-D liquid and the vapor domains (for discretization of interface conditions used as interface boundary conditions in COMSOL solvers, the choice was $\Delta x_{f-g} < \Delta x_{f-g}^*$. Here $\Delta x_{f-g}^*$ values were such that, post-convergence, both the more conservative Nyquist criteria and discretized interface conditions were satisfied. Part of the vapor domain in Figure 3b shows the choice of triangular elements. In both the phases, the actual mesh is non-uniform as COMSOL’s mesh generation function makes them more refined near the interface and the walls. This mesh-generation function is considered “fixed” for the reported simulations and mesh-size calculations. The square root of the representative average areas of the triangles for the liquid and vapor domains are obtained and respectively denoted as representative mesh sizes $\Delta s_L$ and $\Delta s_V$. Though these mesh-size values lie between the ones obtained from COMSOL’s minimum and maximum areas, the mesh-generation “function” (with boundary-layer type refinement features near the wall and the interface) was found to correlate with the “minimum” area. Hence, for simplicity, reported representative mesh-sizes (for liquid ($\Delta s_L$) and vapor ($\Delta s_V$) domains) in Table 1 correspond to non-dimensionalized square root of the minimum triangle areas – as obtainable on COMSOL.

Besides the fluid-physics based constraints on $\Delta x < \Delta x_{f-g}^*$, there are additional constraints that arise for obtaining grid-size independent convergent solutions from the algorithm. For grid-size
independence of a representative convergent solution variable (see definition of this in [43]), one requires: \( \Delta x < \Delta x_{f-g}^\ast \), \( \Delta s_L < \Delta s_L^\ast \), and \( \Delta s_V < \Delta s_V^\ast \). The questions remain as to how the numbers \([\Delta x_{f-g}^\ast, \Delta s_L^\ast, \Delta s_V^\ast]\) are defined/obtained; how they depend on each other and other relevant parameters; and how they depend on the convergence criteria. Another relevant question is what the “orders of convergence” are if grid refinement towards convergence is undertaken from coarser grids satisfying: \( \Delta x > \Delta x_{f-g}^\ast \), \( \Delta s_L > \Delta s_L^\ast \), and \( \Delta s_V > \Delta s_V^\ast \). These issues are discussed here and in [43].

To begin with a trial-and-error approach is chosen to obtain \([\Delta x_{f-g}^\ast, \Delta s_L^\ast, \Delta s_V^\ast]\) values that lead to: satisfaction of interface conditions, satisfaction of convergence criteria on COMSOL with regard to satisfying the interior governing equations (Eq.(2)) for both the phases, and grid-size independent convergence criteria. For a chosen \( \Delta x_{f-g}^\ast \) values that lead to: satisfaction of interface conditions, satisfaction of convergence criteria on COMSOL with regard to satisfying the interior governing equations (Eq.(2)) for both the phases, and grid-size independent convergence criteria. For a chosen \( \Delta x_{f-g}^\ast \approx 0.64 \), Table 1 shows three representative liquid and vapor mesh sizes for which some numerical solutions of representative flow-variables are obtained for each of the two-phases (e.g. y-variations in the x-component of liquid and vapor velocity profiles at \( x^p = 0.02 \) m, as shown in Figure 6). It is then required that some representative single numerical measures of interest, e.g. \( VL \equiv \int_0^h u_1^p \cdot dy^p \) and \(VV \equiv \int_0^h u_2^p \cdot dy^p\), of these different solutions be obtained for a range of different mesh-sizes (as in Table 1) and their averages be computed. If the relative error of values of all the variables for the different mesh-sizes lie within a certain pre-defined allowed range of scatter around their averages (e.g. within relative error \( e_{\text{max}} \), which is 0.001 or 0.1% for the data shown in Figure 6), then the solutions have reached plateau towards their convergent values – and are termed convergent and grid size-independent over the range of mesh-sizes. For \( e_{\text{max}} \) and the length of the domain \( L \) associated with Figure 6, one can obtain conservative non-dimensional values of \([\Delta x_{f-g}^\ast, \Delta s_L^\ast, \Delta s_V^\ast]\) by simply
declaring the guessed value of \(\Delta x_{f-g}\)\(_{\text{guess}}\) to be equal to \(\Delta x_{f-g}^* \approx 0.64\) and the largest of the three different mesh-sizes (for each of the two phases) in Table-1 to be \(\Delta s_L^* (e_{\text{max}}, L, \Delta x_{f-g}^*\)\(_{\text{guess}}\) \approx 3.5 \times 10^{-7}\) and \(\Delta s_V^* (e_{\text{max}}, L, \Delta x_{f-g}^*\)\(_{\text{guess}}\) \approx 5.32 \times 10^{-6}\). The actual upper bounds representing \([\Delta x_{f-g}^*, \Delta s_L^*, \Delta s_V^*]\) are typically bigger than the above reported conservative values and they can be found by the procedure described in the next paragraph. Supporting figures that explain the details for this procedure are similar to what is reported in [43] and are omitted for this paper.

Numerically obtained values of a representative domain variable \(V_L\) (or \(V_V\)) are plotted against several mesh-size \(\Delta s_L\) (or \(\Delta s_V\)) values on a log-log plot for the guessed value of \(\Delta x_{f-g}^*\)\(_{\text{guess}}\). The resulting curve is closely approximated by two banded straight lines – one horizontal (band-width being \(e_{\text{max}}\) discussed above) and one inclined. The horizontal line’s upper bound – the approximate point of intersection between the horizontal and inclined banded lines – yields \(\Delta s_L^* (e_{\text{max}}, L, \Delta x_{f-g}^*\)\(_{\text{guess}}\). For the vapor variable \(V_V\), one obtains, by the same procedure, the estimated value of \(\Delta s_V^* (e_{\text{max}}, L, \Delta x_{f-g}^*\)\(_{\text{guess}}\). By repeating the above procedure for increasing values of \(\Delta x_{f-g}^*\)\(_{\text{guess}}\), while holding fixed the chosen values of \(e_{\text{max}}\) and \(L\), one can arrive at an upper threshold of \(\Delta x_{f-g}^*\)\(_{\text{guess}}\) beyond which it is impossible to satisfy the \(e_{\text{max}}\) error criteria needed for identifying the plateau convergent region in the aforementioned log-log plots. This value of \(\Delta x_{f-g}^*\) and associated values of the above described estimates of \(\Delta s_L^* (e_{\text{max}}, L, \Delta x_{f-g}^*\)\(_{\text{guess}}\) and \(\Delta s_V^* (e_{\text{max}}, L, \Delta x_{f-g}^*\)\(_{\text{guess}}\) yield the desired values of \([\Delta x_{f-g}^*, \Delta s_L^*, \Delta s_V^*]\). It should be noted that the slopes of the three inclined lines for each of the three log-log plots associated with \([\Delta x_{f-g}^*, \Delta s_L, \Delta s_V] > [\Delta x_{f-g}^*, \Delta s_L^*, \Delta s_V^*]\) yield the respective “order of convergence” values (see [43]).

It is found that, typically, \(\Delta x_{f-g}^* \approx 0.64\) in the above example) needed for resolving fluid physics
and accurate satisfaction of all interface conditions is much coarser than the mesh-sizes for the liquid and vapor domains (i.e., $\Delta x_{f-g}^* \gg \Delta s_{L}^* \approx 3.5 \times 10^{-7}$ and $\Delta s_{V}^* \approx 5.32 \times 10^{-6}$ in the above example). This relative coarseness of $\Delta x_{f-g}^*$ allows CFD predicted x-variations (on $\sim \Delta s_{L}^*$ or $\sim \Delta s_{V}^*$ scales) associated with interfacial functions, such as: $\delta(x), \bar{u}(x), \bar{v}(x), \tau_{2x}, \tau_{2y}$ etc., to be “smoothed” and then re-mapped onto the desired $x_i = i (\Delta x_{f-g})$ grid.

For mesh-independent solution in Figure 6 and Table 1, Tables 2a-b show satisfaction of all the interface conditions (in physical variables) at discretized x-locations. Since satisfaction of interfacial conditions is a built-in feature of the algorithm in section 3.1, Table 2 merely shows correctness in coding and implementing the algorithm.

3.3 Solution Representation Formats and Relationship of Solutions to Popular HTC (or Nusselt-number) versus Quality $X(x)$ Correlations for Convective Boiling

The numerical solution of the steady boiling flow problem depicted in Figure 3 is obtained as per the above described procedures. In this paper, specific sample solutions are presented in physical variables to give a clear idea of the fluid used and the physical dimensions involved. However, generalized and correlated results representing solution variables of interest (also see [32] for analogous condensing flow results) can be presented (and one such result is given here) in non-dimensional forms – with clearly marked boundaries of the non-dimensional parameter-space covered by the set of solutions used. A similar procedure is typically used for developing correlations from available experimental data.

The physical variables appearing in the interior equations and wall boundary conditions (in section 2) are non-dimensionalized as per the relationships specified in Eq. (1), definition of $Ja$ in Eq. (8) and definition of $Bl$ in Eq. (9). These lead to non-dimensional numbers or functions that are specified in (or for) Eqs. (2), (4), (8), (9) and (12). Besides these non-dimensional numbers and
functions, inlet boundary conditions require introduction of non-dimensional forms of \( \dot{M}_{L-in}' \) and \( \dot{M}_{V-in}' \) which relate to non-dimensional values of mass-flux \( G \) and inlet quality \( X_{in} \) (already non-dimensional) through the relationships: \( \dot{M}_{L-in}' = G(1 - X_{in}) \) and \( \dot{M}_{V-in}' = G \times X_{in} \). Further, mass-flux \( G \) is non-dimensionalized as \( Re_{T-V}(\equiv \frac{G h}{\mu^2}) \) and it should be noted that \( Re_1 \) appearing in Eqs. (2) and (4) relates to \( Re_{T-V} \) through: \( Re_1 \equiv \frac{\rho_2 U}{\mu_2} = Re_{T-V} \times \frac{\rho_1}{\rho_2} \). After including the non-dimensional parameters specifying the inlet condition (liquid and vapor flow rates) and those that appear in the non-dimensional forms of interior, interface, and wall conditions (for known wall-temperatures, \( T_\text{w}(x) \), specifying the “method of heating”); it is clear that the complete set of non-dimensional numbers, functions, and distance values that affect the non-dimensional values of 1-D physical variables (such as HTC, pressure-drop etc.) of interest are given by the list in Eq. (16) below.

\[
\{ Re_{T-V}, Ja, Fr_x^{-2}, Fr_y^{-2}, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}, Pr_1, We, \theta_\text{w}(x), X_{in}; x \} \tag{16}
\]

In Eq. (16), \( Re_{T-V} \equiv \frac{\rho_2 U}{\mu_2}, \) \( Ja \equiv \frac{C_p \Delta T}{h_{fg}}, \) \( Pr_1 \equiv \frac{\mu_1 C_p}{k_1}, \) \( Fr_x^{-2} \equiv \frac{g_x h}{U^2}, \) \( Fr_y^{-2} \equiv \frac{g_y h}{U^2}, \) and \( We \equiv \frac{\rho_1 U^2 h}{\sigma}. \) In Eq. (16), while \( Ja \) is sufficient to specify uniform wall temperature \( T_\text{w}(x) = \overline{T}_\text{w} (\theta_\text{w}(x) = 1) \) “method of heating.” However, for non-uniform wall temperatures cases, a “method of heating” becomes specific only when the function \( \theta_\text{w}(x) \) (i.e. \( \theta_\text{w}(x) \neq 1 \)) is specified – and, therefore, its presence is retained in Eq. (16). For mm-scale innovative flow-boilers considered here, the liquid film is \( \mu \text{m-scale} \) thick and one can verifiably ignore the liquid flows’ convection term appearing in the differential form of its energy equation (i.e. Eq. (2) for \( I = 1 \)). This means \( Pr_1 \) appears only as “\( Ja/Pr_1 \)” in Eq. (8) and one can replace \( Ja \) and \( Pr_1 \) in Eq. (16) by \( Ja/Pr_1 \). Also, for horizontal channels considered here, \( Fr_x^{-2} = 0, \) \( Fr_y^{-2} \) is not important except in the determination of the annular length \( x_A^R \) in Figure 2, and surface-tension
through Weber number (We) is not important for the steady zero curvature (in the z-direction) solutions of the channel problem – however it is likely that We is important for small diameter tubes. Because of interest in the steady annular part of thin-film flows in Figure 1 and the above discussions, the parametric set in Eq. (16) is adequately approximated as:

$$\left\{ \text{Re}_T-V, \text{Ja}, \text{Fr}_x, \text{Fr}_y, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}, \text{Pr}_1, \text{We}, \theta_w(x), \text{in; } x \right\} \cong \left\{ \text{Re}_T-V, \frac{\text{Ja}}{\text{Pr}_1}, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}, \theta_w(x), \text{in; } x \right\}$$

(17)

Note that, the liquid and vapor-phase Reynolds number that are best suited for assessing laminar or turbulent nature of these separated flows are not Re$_T-V$ but, respectively, Re$_L-\Delta$ ($\equiv G(1-X)\Delta/\mu_1$) and Re$_V-n$ ($\equiv GX_h/\mu_2$). Typically, Re$_L-\Delta < 1000$ [44] corresponds to laminar thin-film flows and Re$_V-n < 2000$ corresponds to laminar vapor flows over the entire vapor; whereas Re$_V-n < 10000$ (or much higher, depending on flow specifics) corresponds to vapor-flow which is laminar in the vicinity of the thin liquid film’s laminar flow (Re$_L-\Delta < 1000$).

Similar to the above, an inspection of: all the non-dimensional governing equations, interface conditions, and boundary conditions for the steady heat-flux specified “method of heating” (recall $q'_w(x) \equiv \bar{q}_w \cdot \Psi_q(x)$ and see Eq. (9)) leads to the set specified in Eq. (18) below.

$$\left\{ x; \text{Re}_T-V, \text{Bl}, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}, \text{Pr}_1, \Psi_q(x), \text{in; } x \right\}$$

(18)

In Eq. (18), Bl $= \bar{q}_w/(\rho_2 U h_{fg})$ is a boiling number that represents a characteristic non-dimensional heat-flux. The non-dimensional parameters in Eq. (18) also consists of one non-dimensional function for specifying non-uniform heat-flux $\Psi_q(x)$, and one non-dimensional distance variable $x$. Analogous to Eq. (17), the set in Eq. (18) determines the non-dimensional values of 1-D physical variables of interest (HTC, pressure-drop etc.).

It should be noted that the solution obtained for any particular non-dimensional form of a flow
variable; such as interfacial shear, interfacial speed, wall heat-flux, etc. may depend, in a non-linear way, on the parameters in Eqs. (17) and (18). Other equivalent combinations involving other non-dimensional number labels are also possible. This is why it is not uncommon to see different non-dimensional numbers appearing in engineering correlations [2, 3] for heat-flux or critical vapor quality associated with the length of the annular regime \( x_A \equiv x_A^p / h \) (for condensing flows, see similar discussions in [32]). Non-dimensional form of physical wall heat-flux, for any given wall temperature variation \( T_w(x) \), is typically reported after the defining introduction of local heat transfer coefficient, \( h_x \), i.e., \( h_{x|cb} (\equiv h_{x|CFD}) \) given by:

\[
q''_{w|cb} (x) \equiv -k_1 \left( \frac{\partial T_1}{\partial y_p} \right)_{y_p=0} = h_{x|cb} [T_w(x) - T_{sat}(p_o)]
\]  

(19)

and, subsequently, a convective boiling Nusslet Number (non-dimensional form of \( h_{x|cb} \)) definition is introduced as:

\[
Nu_{x|cb} \equiv h_{x|cb} \frac{h}{k_1}
\]  

(20)

From here and henceforth, the discussions in sections 3.3 and 4 will exclusively focus on convective boiling and therefore, unless otherwise specified by use of sub-scripted variables, the shorthand notations for \( q''_{w|cb}, h_{x|cb}, \) and \( Nu_{x|cb} \) will be \( q''_w, h_x, \) and \( Nu_x \), respectively.

Often, in Eq (20), instead of gap height \( h \), some other characteristic length (such as hydraulic diameter, etc.) may be used.

A fixed non-dimensional function \( \theta_w(x) \) defined in Eq. (12), or a fixed \( \Psi_q (x) \) defined for Eq. (9), represent a fixed “method of heating.” It is expected that \( Nu_x(x) \) as well as non-dimensional annular length \( x_A \), will generally depend on the arguments listed in Eq. (17) (or those in Eq. (18)) – depending on whether temperature or heat-flux prescriptions are known or are to be used.
(sometimes as guessed values in an iterative calculation procedure). Many equivalent non-dimensional replacements of these parameters, which may appear to look different, are feasible and may replace the indicated set of non-dimensional numbers. Particularly, for reasons to be discussed below, it is popular to empirically correlate $\text{Nu}_x(x)$ in Eq. (20) as a function of quality $X(x)$, replacing the distance variable $x$ by quality variable $X$ – and, many practices also drop, often without proper discussions, the dependencies on non-dimensional functions $\theta_w(x)$ or $\Psi_q(x)$ in the arguments list given in Eqs. (17) and (18). In other words:

$$\text{Nu}_x = \text{Nu}_x \left(x, x_{in}, \text{Re}_{T-V}, \frac{Ja}{Pr_1}, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}, \theta_w(x)\right)$$

$$\approx \text{Nu}_x \left(X, x_{in}, \text{Re}_{T-V}, \frac{Ja}{Pr_1}, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}\right) \tag{21}$$

$$\text{Nu}_x = \text{Nu}_x \left(x, x_{in}, \text{Re}_{T-V}, \text{Bl}, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}, \text{Pr}_1, \Psi_q(x)\right)$$

$$\approx \text{Nu}_x \left(X, x_{in}, \text{Re}_{T-V}, \text{Bl}, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}, \text{Pr}_1\right) \tag{22}$$

Reasons for proposing correlations for $\text{Nu}_x(X)$ in the structure indicated above – whether it is empirically or computationally obtained – is that one assumes that quality ($X$) based $\text{Nu}_x(X)$ correlations are likely to have a much weaker dependence on any spatial variations in $T_w(x)$ or $q''_w(x)$ if their respective mean values of $\overline{T}_w$ or $\overline{q''}_w$ are used in the correlations. This expectation/assumption can be assessed if the correlations for $\text{Nu}_x(X)$ – if they are accurate – are used along with proper one-dimensional Energy Balances to obtain the spatial variations for quality $X(x_p)$ for different non-uniform heating methods and, subsequently, comparing these $X(x_p)$ predictions with those obtained from 2-D steady CFD approach of this paper. This is possible because one-dimensional Energy Balance for a control-volume between a location “$x_p$”
and “x^p + Δx^p” in Figure 3a yields:

\[
\frac{\dot{M}_{in}'}{dx^p} \frac{dX(x^p)}{dx^p} = \frac{q_w^*(x^p)}{h_{fg}} = \frac{h_x[(T_w(x) - T_{sat}(p_0))]}{h_{fg}}
\]

(23)

Utilizing, \(\dot{M}_{in}^' \equiv \rho_2 Uh, x^p \equiv \hat{x}, h, Re_{T-V} \equiv \rho_2 Uh/\mu_2, Bl \equiv \bar{q}_w''/(\rho_2 Uh_{fg})\), the non-dimensional form of Eq. (22) for specified \(T_w(x)\) becomes:

\[
\frac{dX(\hat{x})}{d\hat{x}} = Nu_x \frac{Ja}{Pr_1} . \frac{1}{Re_{T-V}} \cdot \frac{\mu_1}{\mu_2} \cdot \theta_w(x)
\]

(24)

For specified heat-flux \(q_w''(x)\), Eq. (22) non-dimensionalizes to:

\[
\frac{dX(\hat{x})}{d\hat{x}} = Bl \cdot \Psi_{q}(x)
\]

(25)

Because of direct appearance of \(\theta_w(x)\) and \(\Psi_{q}(x)\) on the right sides of Eqs. (24) and (25), an integration of these equations will yield the values of quality \(X\)’s spatial variation with \(x\) – provided a known \(Nu_x\) in Eq. (20) is used to integrate Eq. (23) or known constant \(Bl\) is used to integrate Eq. (25) (yielding a linear function of \(x\)). Once \(X(x)\) is known by the above 1-D approach, \(h_x\) and \(q_w''(x)\) can also be obtained as functions of \(x\) for the known \(T_w(x)\) cases – this is through use of the defining relations available from Eq. (19) and Eq. (20). The efficacy of use of popularly assumed simplifications proposed for the right sides of Eqs. (21) and (22) in Eqs. (24) and (25) need to be assessed with the help of this paper’s DNS/CFD approach.

4. Results and Discussions

4.1 Basic Flow Features of Suppressed Nucleation Annular Boiling

The steady flow simulations yield elucidating information on two-dimensional spatial variations of key 2-D flow variables of interest (\(l = 1\) or 2), viz. velocity components \((u, v_1)\), temperatures \((T_l)\), pressures \((p_1)\), streamlines etc. They also yield one-dimensional spatial variations of key flow variables of interest such as: film thickness \(\Delta(x)\), x-component of interfacial velocity \(u_1^1\),
characteristic speed $\bar{u}(x)$ associated with interfacial wave-propagation resulting from initial disturbances of infinitesimal amplitude, interfacial shear $\tau^{\text{int}}(x) \equiv \tau_2^{\text{pi}} = (\tau_2^{1x} + \Delta'(x)\tau_2^{1y})/\sqrt{1 + \Delta_x^2}$, interfacial mass flux $m^p(x^p)$, wall heat-flux $q_w^w(x^p)$, local values of HTC $h_x \equiv q_w^w(x^p)/[T_w(x) - T_{\text{sat}}(p_0)]$, Nusselt number $\text{Nu}_x(\equiv h_x.h/k_1)$, and quality $X(x^p)$. The results also yield interfacial mechanical energy transfer terms $\dot{W}^{\text{int}}_{\text{mech}}$ (see details in [34] and [43]), and identify the most significant of different $\dot{W}^{\text{int}}_{\text{mech}}$ terms, along with the relationship of significant terms to one another in the interior of the flow field.

For a representative horizontal ($\alpha = 0$) flow situation in Figure 3a (also see Figure 4), and under a steady “method of heating” of the type defined in Figure 5 (with $-x^p^* = -0.05$ m and $-x^p^{**} = -0.03$ m; $\Delta T = 10^\circ$C), the steady solution has been obtained by the algorithm proposed in section 3 and the plots for: film thickness $\Delta(x)$ versus x, and cross-sectional profiles of: $v_1^p(x^*,y), T_1(x^*,y)$ and $p_1(x^*,y)$ (for $I = 1 \& 2$) versus y, for a representative $x = x^*$, are respectively shown in Figures 7 a-d. Note that the cross-sectional profile of $u_1^p$ the x-component of liquid and vapor velocities remain the same as shown in Figure 6. In Figure 7, $x^p \geq 0$ with $x^p = 0$ as indicated/defined in Figures 4-5.

It is important to note that, relative to $h = 2$ mm, liquid film thickness ($\Delta$) in Figure 7a is very small (order of $(\Delta/h)$ is $10^{-1}$). Also, the already small order of magnitude ($\sim 10^{-2}$ m/s) of x-component of liquid velocity $u_1(x,y)$ relative to maximum vapor speed of $\sim 1$ m/s (see Figure 6), is much larger than the order of magnitude ($\sim 10^{-5}$ m/s) of y-component of liquid velocity $v_1(x,y)$ – which has magnitudes, shown in Figure 7b, that are not even noticeable relative to the magnitude of $v_2(x,y)$ which is of order $\sim 10^{-3}$ m/s. Evaporation at the interface is associated with large density reduction – so there is a large increase in y-component of fluid velocity (between $v_2(x,y)$ values of vapor and $v_1(x,y)$ values of liquid) near the interface. The cross-sectional temperature $T_1(x^*,y)$
variations are shown in Figure 7c. The pressure variation $p_I(x^*,y)$ in the vapor ($I = 2$) and liquid ($I = 1$) phases – as shown in Figure 7d – is primarily hydrostatic (for $g_y = -g$). These results are consistent with streamline patterns shown in Figure 8. The starting points of the streamlines were chosen at $x^p = -x^{p*}$ (for both liquid and vapor mass phases) in a fashion so as to yield equal mass flow rates between each streamline – and hence the streamlines are spatially unequally distributed at $x^p = -x^{p*}$. It is significant to note that Figure 8 clearly demonstrates how evaporated vapor of laminar liquid flow pushes the incoming vapor away from the interface region – into the core region. As a result, for thin film flows of interest here, even in the presence of non-deterministic interfacial waves, the liquid flow remains laminar – dominated by viscous forces (with negligible inertia) – and at most exhibits \textit{wavy laminar interfacial turbulence}. Figure 8 also shows that even if vapor-phase Reynolds number is sufficiently high (i.e., $2000 \leq \text{Re}_V (\equiv G X h / \mu_2) \leq 10000$, or much higher depending on the flow specifics), classical turbulence will be restricted to the vapor core and would only affect pressure drop predictions (associated with changes in vapor phase velocity away from the interface) – not the liquid flow’s predicted dynamics, thickness, and associated heat transfer rates.

For the flow case’s results given in Figures 7-8, the $x$-variations in key variables of interest, viz: characteristic velocity, $\bar{u}^p(x^p)$, and interfacial velocity, $u_{1i}^{pl}(x^p)$; interfacial shear stress, $\tau^{\text{int}}(x^p)$; interfacial mass-flux, $\dot{m}^p(x^p)$; wall heat-flux, $q_w''(x^p)$, along with heat transfer coefficient, $h_x$; and Nusselt Number, $Nu_x$ along with quality, $X(x^p)$ are respectively shown in Figures 9 a – f.

4.2 Equivalence of wall heat-flux $q_w''(x)$ and wall-temperature $T_w(x)$ as “method of heating” specifications

For the flow-case used for predicting the results reported in Figure 7, $T_w(x) = T_{sat} + 10^\circ C$ for $x^p \geq 0$ was used – and this is shown as a part of the solid $T_w(x)$ curve in Figure 10. For this uniform
wall temperature imposition over $x^p \geq 0$, the corresponding predicted wall heat-flux values (as a part of results obtained from the 2-D steady solution approach of section-3), are also shown (by another solid curve) in Figure 10.

Treating the wall heat-flux values given by the solid curve in Figure 10 as a prescribed “method of heating” boundary condition (of a different non-uniform $\Psi_q(x) \neq 1$ type), the algorithm described in section 3.1 is again implemented for this heat-flux boundary condition (shown by $q_w(x^p)$ curve, made up of solid square points, in Figure 10), and the corresponding wall temperature $T_w(x^p)$ predictions are obtained as part of the solution (and shown by the dotted $T_w(x)$ curve in Figure 10). It is seen that the dotted curve $T_w(x^p)$ predictions in Figure 10 are nearly the same as the solid-curve $T_w(x^p)$ values initially used as input to the temperature boundary condition problem – the average difference between them being 1.5% (within numerical errors).

The representative pair of results above was expected, but the numerical confirmation prove the equivalence of the two types of thermal boundary conditions for suppressed nucleation convective boiling cases – this has been accomplished, perhaps for the first time, in the present context of two-phase flows.

4.3 Improved understanding of correlations-based modeling and implementation of 1-D solution techniques (for different heating approaches over $x^p \geq 0$ in Figure 5).

4.3.1 Quality based Nusselt number Correlations and Spatial Variations in HTC values

For a given pure fluid and fixed values of mass-flux $G$, channel height $h$, inlet quality $X(0) \equiv X_{in}$, and a fixed “method of heating” over $x^p \geq 0$ – such as uniform temperature “method of heating,” i.e. $\theta_w(x) = 1$, in Figure 5; consider three different “prior methods of heating” HM-i (i = 1, 2, 3.. in Figure 5) for $x^p < 0$. For the full direct numerical solution (DNS) technique described in section 3 for laminar/laminar steady flows, by methodically adjusting $X(-x^p)$ values for a given mass-
flux $G$, channel height $h$, and a given fluid; it is relatively easy – as described in the next paragraph – to arrive at the same $X(0)$ value and the same $\theta_w(x) = 1$ for different “prior methods of heating” over $x^p < 0$.

The methodical iterative adjustments of $X(-x^p)$ values are achieved by starting with two initial guesses (say $X(-x^p)|_{\text{guess}-1}$ and $X(-x^p)|_{\text{guess}-2}$) for $X(-x^p)$ – such that the DNS solution technique of section 3 yields predictions of quality $X(0)$ denoted as $X_{\text{pred}-1}(0)$ and $X_{\text{pred}-2}(0)$ which form an interval within which lies the sought-for $X(0)|_{\text{sought}}$ value, i.e. $X_{\text{pred}-1}(0) \leq X(0)|_{\text{sought}} \leq X_{\text{pred}-2}(0)$. With subsequent improvements on these two initial guesses of $X(-x^p)|_{\text{guess}-1}$ and $X(-x^p)|_{\text{guess}-2}$ – achieved by a combination of bisection and interpolation methods (the kind used in Newton Raphson techniques for finding zeroes [45] of a scalar valued function) – it is easy to obtain a pair of nearly identical solutions over $x^p \geq 0$ for which $X_{\text{pred}-1}(0)|_{\text{final}} \approx X(0)|_{\text{sought}} \approx X_{\text{pred}-2}(0)|_{\text{final}}$.

Figure 11a shows the three different “prior methods of heating” (i.e. for $x^p < 0$) for which $G = 13.98 \text{ kg/m}^2\text{s}$, $h = 2 \text{ mm}$, quality $X(0) \approx 0.714$, and method of heating $\theta_w(x) = 1$ are achieved over $x^p \geq 0$. For these three cases, $X(x^p)$ and $h_x(x^p)$ predictions over $x^p \geq 0$ are shown in Figure 11b and $h_x(X)$ predictions over $x^p \geq 0$ are shown in Figure 11c.

It can be seen from the plots that the effect of the “prior method of heating” is almost non-existent for quality but have some effect on the heat transfer coefficient values. Note that $h_x(x^p)$ curves are within 0.23% of each other in Figure 11b and $h_x(X)$ curves are within 0.52% of each other in Figure 10c. These are small changes and hence the differences can be largely neglected if the “prior methods of heating” immediately before the beginning of the same uniform heating ($x^p \geq 0$) are different (perhaps not too different!).

Also, in Figure 11b, the drop in $h_x$ immediately after the beginning of uniform heating (over $0 \leq$
x^p \leq x_e^p ) is essentially a “thermal entry zone” effect and after that prior non-uniform heating methods become unimportant and the flow behaves as if it has been exposed to uniform heating alone. Slightly different curves in the entrance zone of Figure 11b correspond to different non-uniform prior heating methods within the same “family.” It can be seen that a more gradual “prior heating” leading to the uniform temperature heating results in lower drop in \( h_x \) values over the “entrance” zone of 0 \( \leq x^p \leq x_e^p \). For these reasons, while obtaining correlations, only data points after the minimum \( h_x \) values (i.e., \( x^p \geq x_e^p \) – see Figures 11 b-c) are emphasized.

The above results support the simplified engineering practice of proposing \( \text{Nu}_x(X) \) correlations with parametric dependence as in Eq. (21) or Eq. (22) and then, as needed, predicting \( X(x) \) variations as solutions of Eq. (24) or Eq. (25).

The next question that arises is whether such engineering practices are in fact reasonable enough (i.e. within acceptable accuracies) to yield good HTC values even for non-uniform “methods of heating” – which is, prediction of \( q_w^c(x) \) for the case of non-uniform prescribed values of wall temperature \( T_w(x) \) (i.e., \( \theta_w(x) \neq 1 \) for \( x^p \geq 0 \)) and prediction of \( T_w(x) \) for the case of non-uniform prescribed heat-flux \( q_w^c(x) \) (i.e., \( \Psi_q(x) \neq 1 \) for \( x^p \geq 0 \)). These questions are addressed in the next sub-section.

**4.3.2 DNS-based assessment of correlations-based 1-D modeling and prediction abilities for non-uniform heating (over \( x^p \geq 0 \))**

For the uniform \( T_w(x^p) \) prescription shown as solid-curve in Figures 10 and 12a (for \( x^p \geq 0 \)), the relevant \( \text{Nu}_x(X) \) correlation could be of the structure given by the right side of Eq. (17), for all \( x > x_e \). Next, along with the uniform, two non-uniform \( T_w(x) \) variations (for \( x^p > 0 \)) are also shown in Figure 12a. The non-uniform prescriptions have the same \( \overline{T}_w \) as the uniform case’s \( T_w(x) \) for 0 \( < x^p < L_p \). From Eq. (12), it is easy to see that this non-uniform prescription corresponds to
\[ \theta_w(x) \equiv \frac{T_w^\delta}{\Delta T} \left( 2 \frac{x^p}{L_p} - 1 \right) + 1 \] (26)

The non-uniform cases considered in Figure 12a are respectively shown by dotted curves with \( T_w^\delta = 1^{\circ}C \) and \( T_w^\delta = 2^{\circ}C \). The corresponding quality variations obtained by 2-D computational tools of this paper are part of the results shown in Figure 12b.

Because of one-to-one correspondence between the physical distance \( x \) and quality \( X \) in Figure 12b, it is clear that \( h_x(X) \) solid curve shown in Figure 12c, which is associated with uniform temperature heating (\( \theta_w(x) = 1 \)) in Figure 12a, can be correlated in the form of right side of Eq. (21). The resulting correlation, by definition, is close to its actual theoretical values shown in Figure 12c. In engineering practices, these \( h_x(X) \) values are typically recommended for use in Eq. (24) – for both \( \theta_w(x) = 1 \) and \( \theta_w(x) \neq 1 \) – as empirical correlations are seldom developed by considering different types of non-uniform heating. The question is: how reasonable is this practice?

For \( X(0) = 0.669 \), the 1-D ODE solver for Eq. (24) yields three \( X(x^p) \) predictions in Figure 12b: one for \( \theta_w(x) = 1 \), and two for \( \theta_w(x) \neq 1 \) specified by Eq. (25) under \( T_w^\delta = 1^{\circ}C \) and \( T_w^\delta = 2^{\circ}C \).

In addition, in Figure 12b, the two \( X(x^p) \) curves for \( \theta_w(x) \neq 1 \), i.e. Eq.(25) with \( T_w^\delta = 1^{\circ}C \) and \( T_w^\delta = 2^{\circ}C \), are also obtained and plotted by the 2-D CFD technique of this paper. Next three different \( h_x(X) \) curves were obtained in Figure 12c by simply replacing the distance variable \( x^p \), by the corresponding qualities (i.e. \( x^p \rightarrow X(x^p) \)), in the DNS obtained \( h_x(x^p) \) variations. The dotted \( h_x(X) \) curves, shown in Figure 12c, were obtained by using \( h_x(x^p) \) values obtained by 2-D steady-solver associated with the non-uniform heating (i.e., \( \theta_w(x) \neq 1 \) for \( T_w^\delta = 1^{\circ}C \) and \( T_w^\delta = 2^{\circ}C \) in Eq. (25)). The \( X(x^p) \) curves for non-uniform heating methods – in Figure 12b – have mean differences of 0.21% and 0.52% (for the cases \( T_w^\delta = 1^{\circ}C \) and \( T_w^\delta = 2^{\circ}C \)) with respect to the uniform
heating method results. In Figure 12c, the mean difference between the 2-D prediction curves for uniform heating $h_x(X)$ values and values obtained for the two non-uniform heating approaches (for $T_w^\delta = 1^\circ C$ and $T_w^\delta = 2^\circ C$) are 0.37% and 1.29%, respectively. The difference between 1-D and 2-D prediction increases with increase in the value of $T_w^\delta$. Further, some of the differences in quality versus heat transfer coefficient values in Figure 12c can also be attributed to greater numerical error in negotiating the “entrance zone” effect – as discussed in the previous sub-section. However, the proximity and monotonicity of these 1-D and 2-D prediction values of $X(x^p)$ in Figure 12b for different $\theta_w(x) = 1$ and $\theta_w(x) \neq 1$ values justify engineering practice that replaces the physical distance $x$ present in the correlation structure of Eq. (17), by the simplified one in Eq. (21). Furthermore the results in Figures 12a-c indicate that one can further simplify Eqs (24) and (25) as well, by setting $\theta_w(x) \equiv 1$ and $\Psi_q(x) \equiv 1$ – provided $\overline{T_w}$ (or $\overline{q_w}$) are used in evaluation of $Nu_x$ correlation in the right side of Eq. (21) (or Eq. (22)) and $||\theta_w(x) - 1|| < 0.1$. Here, for $0 < x^p < L_p$, the “$T_w(x)$” to “$\overline{T_w}$” distance function is defined to be $||\theta_w(x) - 1|| \equiv \int_0^1 \left[\{\theta_w(x)-1\}^2+\left\{\theta_w'(x)\right\}^2\right] dx$ where $x = x^p / L_p$. A similar justification of replacement of the structure in Eq. (18) by the one in Eq. (22) – along with discussions of some limitations in doing so – is being given in [46].

4.4 Effects of Nucleation on HTC values and Transition from Non-Annular to Annular Flow Regimes

As discussed in section 1, both presence and absence of nucleate boiling is possible within the annular regime [36]. These possibilities are also shown in Figure 13a in the context of this paper’s focus on realizing non-pulsatile flows of the type shown in Figure 13b. For a case where nucleation is considered to be physically suppressed, the length $L_{A-N}$ defining the “Annular-with-nucleation” zone in Figure 13a is small relative to the suppressed nucleation annular length $L_{A-SN}$ flow.
Otherwise, when nucleation is dominant, the length \( L_{A-SN} \) goes to zero and \( L_{A-N} \) covers the entire test-section in Figure 13a. Furthermore, it is assumed that for most heating conditions of interest to innovative boiling (Figure 1), the recirculation flow rate \( \dot{M}_{v-in-recirc} \) in Figure 13b can be increased/adjusted to achieve the indicated annular flow-regime (with or without nucleation). Presence of nucleation, over a portion of the flow in Figure 13a, may not only affect the heat-transfer rates but may also affect the transition criteria to from non-annular to annular flow-regimes indicated in Figure 13a.

While convective boiling component of HTC \( h_{x|cb} \) depends only on the parameter set shown in Eq. (21) (or Eq. (22) depending on the heat-flux characterization of the “method of heating”), nucleate boiling component HTC \( h_{x|nb} \) depends, along with the parameters set in Eq. (21) (or Eq. (22)), on additional parameters such as: surface texture (or surface roughness), wettability, heating level etc. This work provides a unique opportunity to use CFD-based \( h_{x|cb} \) correlation in conjunction with experimental results on \( h_{x|total} \) – towards addressing aforementioned issues on when to expect any particular type of annular flow realization (with regard to presence or absence of nucleation) and a way forward towards developing correlations \( h_{x|total}, h_{x|cb}, h_{x|nb} \) and criteria for suppression of \( h_{x|nb} \).

Presence or absence of nucleation also affects the flow-regime transition from non-annular to annular regimes, as indicated in Figure 13a. These transition zones may be characterized by critical value of quality \( X \) (denoted \( X_{cr} \)) – provided all other non-dimensional numbers characterizing an annular flow-boiling situation is known. The transition criteria of interest are: non-annular to annular regime transition in the presence of nucleation (denoted \( X_{cr|NA-A-WN} \)); non-annular to annular regime transition that occurs when all of the annular regime realized has no nucleation (denoted \( X_{cr|NA-A-NN} \)); and “annular-with-nucleation” to “suppressed nucleation annular” regime
(denoted $X_{cr|A-SN}$) shown in Figure 13a.

This paper limits itself to considering non-annular to annular transition $X_{cr|NA-A-NN}$ in the absence of nucleation (i.e., by assuming $L_{A-N} = 0$ or $L_{A-N} \ll L_{A-SN}$ in Figure 13a).

This characterization for $L_{A-N} = 0$ flows in Figure 13a is possible with the help of the current 2-D solver. This is because the instability mechanisms associated with such transition boundaries in Figure 13a can be related to transition criteria identified in Figures 14a-c (and discussed, by analogous considerations of issues in [33, 34, 43]) – which are to be more extensively discussed in [46].

For flow-boiling, as seen from Figure 14a, mechanical energy from outside (see terms and their definitions in [43]) is primarily transferred into the liquid control volume (of width $\Delta x$ shown in Figure 3a) through three terms viz.: pressure work terms ($PW_{L-conv}$ & $PW_{L-int}$) and interfacial viscous work term ($VW_{L-int}$) – with the sum of these terms being positive (Figure 14b). These are dissipated by equal and opposite outgoing viscous dissipation term ($VD_L$) shown in Figure 14a. As shown in Figure 14b, the sum of these three terms represents energy input per unit length for a liquid control-volume of width $\Delta x$ and is more or less counter balanced by the viscous dissipation term $VD_L$.

For suppressed nucleation annular flows, the transition zone (related to quality $X_{cr|NA-A-NN}$) between non-annular to annular flow-regimes typically falls between peak of characteristic speed ($\bar{u}_p$) and extremum value of significant mechanical energy transfer terms (see [43]) shown in Figure 14b. The maxima in the characteristic speed, as shown in Figure 14c, corresponds to the lower values of distance (from inlet) and lower quality for which transition may occur – whereas the extremum in mechanical energy transfer terms, typically, correspond to the higher values of distance (from inlet) and higher quality for which transition may occur.
Further, in the presence of transverse gravity, the critical transition quality is expected [43] to be closer to the lower limit – i.e., the peak of characteristic speed in Figure 14c – and in the absence of transverse gravity \((g_x = g_y = 0)\), the transition quality \(X_{\text{cr}|\text{NA-A-NN}}\) is expected [43] to be closer to the minima of the viscous dissipation term.

In Figure 14, for the run parameters mentioned, the “prior” inlet quality \(X(−x^p) = 0.45\) was used.

It is clear that, for negligible presence of nucleation in annular boiling, \(X_{\text{cr}|\text{NA-A-NN}}\) may not depend on “confinement number,” nucleation site density, and wettability issues – and, therefore, is likely to be of the form:

\[
X_{\text{cr}|\text{NA-A-NN}} = X_{\text{cr}|\text{NA-A-NN}} \left( \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1} \right) \text{ or } X_{\text{cr}|\text{NA-A-NN}} \left( \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1} \right)
\]

(27)

The \(X_{\text{cr}|\text{NA-A-NN}}\) type correlations – based on the above identification principle – have been reported for annular flow-condensation in [32-34] and are being reported in [46] for the present annular flow-boiling cases of interest.

For preliminary guidance in selecting flow-control strategies in experiments for the innovative operations of the type reported in [1] – one needs to satisfy the twin requirements of \(X_{\text{in}}\) being sufficiently greater than \(X_{\text{cr}|\text{NA-A-NN}}\) and \(\Delta_0 \cong 0\) (100 \(\mu m\)). The above described \(X_{\text{cr}|\text{NA-A-NN}}\) correlation (to be given in [46]) and \(\Delta_{0a}\) correlations (given in Eqs. (A2.13) & (A2.15)) suffice for preliminary design of innovative boilers shown in Figure 1.

4.5 A sample convective boiling HTC correlation of the type \(\text{Nu}_x = \text{Nu}_x(X, J_a/\text{Pr}_1, \text{Re}_{T-V}, \rho_2/\rho_1, \mu_2/\mu_1)\) for annular flow-boiling

A majority of engineering heat transfer correlations for \(\text{Nu}_x\) described in literature that include different flow-regimes (plug-slug, annular etc.) in convective flow-boiling, such as the ones in [8-11] (which are primarily considered in this study), use quality \(X\) and mass-flux \(G\) (or its non-
dimensional equivalent $\text{Re}_{T-V}$ in Eqs. (17) and (18)) as key flow-variables.

For choices of physical variables and ranges associated with the flow conditions listed in Table 3, a $\text{Nu}_{x|\text{cb}}$ correlation – for just the convective component of overall HTC and for uniform wall-temperature method of heating or actual suppressed annular flow-boiling ($X_{\text{in}} > X_{\text{cr}\mid A-SN}$) – is sought in the form given by the right side of Eq. (21).

Based on the physical parameter ranges mentioned in Table 3 several simulation cases were run and the data was used to correlate the dependence of convective boiling Nusselt. The resulting correlation is of the form:

$$\text{Nu}_{x|\text{cb}} = 1.72 \cdot X_{\text{in}}^{1.61} X_{\text{in}}^{0.128} \text{Re}_{T-V}^{0.0284} \left( \frac{\text{Ja}}{\text{Pr}_1} \right)^{-0.0583} \left( \frac{\rho_2}{\rho_1} \right)^{-0.399} \left( \frac{\mu_2}{\mu_1} \right)^{0.454}$$

(28)

where, $0.5 \leq X_{\text{in}} \leq 0.86$, $616.5 \leq \text{Re}_{T-V} \leq 9880.7$, $0.0048 \leq \text{Ja}/\text{Pr}_1 \leq 0.0424$, $0.00466 \leq \rho_2/\rho_1 \leq 0.0097$, $0.0216 \leq \mu_2/\mu_1 \leq 0.0295$. The linear regression fit associated with Eq. (28) yielded a mean error of 4.95%. More accurate correlations for Eq. (27) are being given in [46].

A comprehensive examination of validity of DNS obtained correlations, such as the one in Eq. (28) – which are being extended to cover more useful and practical ranges, involving turbulent vapor and laminar liquid, and their prediction for extended correlations of $h_{x|\text{cb}}$ are not part of this paper but are briefly discussed next.

5. Forthcoming Results Enabled by this Work

Laminar liquid and turbulent vapor cases of annular boiling: The 2-D CFD laminar liquid/laminar vapor approach described here holds good for reasonably high levels of vapor turbulence (up to approximately $\text{Re}_V \leq 10000$), as per discussions about streamlines in section 4.1. However, to achieve higher Reynolds number values (and consequently higher mass-flux $G$ values) typically associated with high heat-flux flow-boiling experiments, algorithm for simulations involving
laminar liquid/turbulent vapor, that may or may not involve significant interactions between liquid and vapor, so as to affect the interfacial stresses (determined, typically, by the laminar liquid flows), have to be developed and carried out. For this, a new CFD approach – that employs Reynolds Average Navier Stokes (RANS) and/or Large Eddy turbulent vapor simulations and replaces laminar vapor modeling in the current approach – will be developed.

Convective boiling HTC ($h_{x\mid cb}$) CFD algorithm for known heat-flux “method of heating”: The aforementioned suppressed nucleation CFD can be carried out under the assumption of known $q_w^\ast(x) \equiv q_w^\ast \cdot \Psi_q(x)$. The resulting wall temperature prediction $T_{w\mid CFD}(x)$ will be much larger – for cases where nucleate boiling is important – than experimentally measured wall temperature values of $T_{w\mid Expt}(x)$. This can be used to estimate $h_{x\mid cb}$ through the relationship: $q_w^\ast(x) \equiv h_{x\mid cb} \ast (T_{w\mid CFD}(x) - T_{sat}(p_0))$. Correlations obtained from such $h_{x\mid cb}$ or $Nu_{x\mid cb}$ values, in the format of Eq. (22), are to be proposed in fashion analogous to the correlation proposed here in Eq. (28).

Effects of interfacial mass-flux: The existing simulation approaches for adiabatic flows (Appendix A2 of this paper), annular condensing flows [33, 34], and annular flow boiling simulation results – when considered together – enable a systematic study of the effects of zero, negative, and positive values of interfacial mass-flux on known correlations for: film thickness, interfacial-shear, pressure-drop, relationship of void-fraction with quality and other parameters. Such comparisons are to be reported in [46].

6. Conclusions

In summary, this paper accomplishes the following:

- It reports the details of an accurate CFD-based solution approach for steady annular flow-boiling. A laminar liquid/laminar vapour approach is described here and a similar laminar liquid/turbulent vapour approach is enabled by the approach implemented here. In case of
significant presence of micron/sub-micron nucleate boiling, the approach provides reliable estimates of convective boiling contribution to the total HTC. In case of suppressed nucleation annular flow-boiling, the approach accurately yields the entire flow-physics.

- The paper addresses some critical questions on how to obtain convective boiling component of HTC correlations by detailed DNS/CFD and proposes a correlations-based one-dimensional prediction approach for engineering design. This approach has been and is being used by the authors in the design of innovative flow-boilers [12, 46].

- The solution technique, established here for the first time in the context of annular-boiling, establishes the expected equivalence of heat-flux and temperature-controlled “methods of heating” – as far as convective boiling characterization is concerned.

- The paper outlines an approach for identifying a criterion for “onset” of suppressed nucleation annular boiling and, also, proposes theoretical/computational criteria for estimating transition of suppressed nucleation annular flows to relevant neighbouring non-annular (plug – slug, etc.) regimes – as the inlet quality decreases.

Acknowledgements
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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bl</td>
<td>Boiling Number ((\frac{q_w}{\rho_2 U h_{fg}}))</td>
</tr>
<tr>
<td>(C_p)</td>
<td>Specific Heat, J/kg-K</td>
</tr>
<tr>
<td>Fr_x</td>
<td>Froude Number in x-direction, ((U/(g_x h)^{1/2}))</td>
</tr>
<tr>
<td>Fr_y</td>
<td>Froude Number in y-direction, ((U/(g_y h)^{1/2}))</td>
</tr>
<tr>
<td>G</td>
<td>Mass-flux for steady annular boiling, kg/m²s</td>
</tr>
</tbody>
</table>
\( g_x \)  Gravity component in x-direction, m/s\(^2\) 
\( g_y \)  Gravity component in y-direction, m/s\(^2\) 
\( h \)  Height of the channel, m 
\( h_{fg} \)  Heat of vaporization, J/kg 
\( J_a \)  Liquid Jakob Number \((C_{p1}\Delta T/h_{fg}(p_0))\) 
\( k \)  Conductivity, W/m-K 
\( L \)  Length of the channel or test-section, m 
\( \dot{m}^p \)  Local interfacial mass flux, kg/m\(^2\)-s 
\( p_0 \)  Steady inlet pressure (also \( p_{in} \)), kPa 
\( Pr_1 \)  Liquid Prandtl Number \((\mu_1 C_{p1}/k_1)\) 
\( \overline{q_{w}} \)  Mean wall heat flux, W/m\(^2\) 
\( Re_{T-V} \)  Reynolds number representing non-dimensional \( G \) \((Gh/\mu_2)\) 
\( \overline{T_w} \)  Mean Boiling surface temperature, °C 
\( T_{sat}(p_0) \)  Saturation Temperature at pressure \( p_0 \), °C 
\( u_{Ix}, v_I \)  Non-dimensional velocities in x and y-directions 
\( U \)  Average inlet vapor velocity in the x-direction (obtained from \( G = \rho_2 U \)), m/s 
\( w \)  Cross-sectional width of the channel, m 
\( We \)  Liquid Weber Number \((\rho_1 U^2 h/\sigma)\) 
\( x, y \)  Non-dimensional distances along with and perpendicular to the boiling surface 
\( x_A \)  Non-dimensional length of the annular regime 
\( \Delta s^p \)  Mesh size, m 

Greek Symbols 
\( \delta \)  Non-dimensional liquid film thickness 
\( \Delta \)  Physical value of liquid film thickness, m 
\( \theta \)  Non-dimensional temperature 
\( \mu \)  Viscosity, kg/m-s
\( \rho \) Density, kg/m\(^3\)

**Subscripts**

1 or \( L \) Represents liquid phase of the flow variable

2 or \( V \) Represents vapor phase of the flow variable

\( e \) Represents “entrance effect” due to heating conditions prior to inlet

**Superscripts**

\( p \) Physical value of a variable, e.g. \( x^p \) – associated with non-dimensional \( x \) values

\( i \) Value of the flow variable at the interface
APPENDICES
APPENDIX A1

The surface velocity $\vec{v}_s$ of a point on the interface ($\Phi = 0$) at time $t$ is associated with this point’s movement to a new mapped position on the interface at time $t + \Delta t$. All such mappings must be such that the normal component of this $\vec{v}_s$ is given by:

$$\vec{v}_s \cdot \hat{n} = - \frac{\partial \Phi}{\partial t} / |\nabla \Phi|$$

(A1.1)

The tangential component of the vapor and the liquid velocities at the interface must be continuous, i.e.

$$\vec{v}^{pl}_1 \cdot \hat{t} = \vec{v}^{pl}_2 \cdot \hat{t}$$

(A1.2)

Allowing for variations in the surface tension, $\sigma$, over the interface such that the vector $\vec{v}_s \sigma$ is primarily in the tangent plane, the normal component of momentum balance at a point on the interface is given in [32, 33] and simplifies to:

$$p^1_i = p^2_i + \dot{m}^p \left[ \frac{1}{\rho_2} - \frac{1}{\rho_1} \right] + \vec{v}_s \sigma \cdot \hat{n}$$

(A1.3)

The tangential component of momentum balance at any point on the interface, which allows for surface variations in the surface tension $\sigma$, reduce to:

$$S^{1}_i \dot{\hat{n}} \cdot \hat{t} = S^{2}_i \dot{\hat{n}} \cdot \hat{t} + \vec{v}_s \sigma \cdot \hat{t}$$

(A1.4)

For the phase-change flow problems considered here, interfacial temperature variations are negligible and there are no interfacial impurities. Hence the Marangoni term $\vec{v}_s \sigma \cdot \hat{t}$ contributions can be ignored relative to the interfacial shear driven motion.

The mass-flux $\dot{m}^p$ is denoted, separately as $\dot{m}^{pV}_{VK}$ and $\dot{m}^{pL}_{LK}$, to indicate independent kinematic restrictions imposed by interfacial values of vapor and liquid velocities. Thus, the definitions are:
\[
\dot{m}^{\text{VK}}_p \equiv \rho_2 (\vec{v}_2^p - \vec{v}_s). \hat{n}, \quad \text{and} \\
\dot{m}^{\text{LK}}_p \equiv \rho_1 (\vec{v}_1^p - \vec{v}_s). \hat{n}
\]  

(A1.5)

The energy balance at a point on the interface, with energy fluxes being relative to moving interface, also imposes a restriction on the interfacial mass flux \(\dot{m}_{\text{Energy}}^p\). Its approximation as discussed in [43] is:

\[
\dot{m}_{\text{Energy}}^p \approx \frac{1}{h_{fg}} \left[ -k_1 \left| \frac{\partial T_1}{\partial n} \right|^i + k_2 \left| \frac{\partial T_2}{\partial n} \right|^i \right]
\]  

(A1.6)

The assumption of equilibrium thermodynamics at the interface allows one to use thermodynamics tables [38] to estimate “\(h_{fg}\)” as \(h_{fg} \approx h_{fg}(T_s(p_2^i)) \approx h_{fg}(T_s(p_0))\).

However, when the liquid film in Figure 3 becomes sufficiently thin with \(|\Delta(x)| < \Delta_{cr}\), where \(\Delta_{cr}\) could be as little as 10-15 nm or much larger, depending on the dynamics of the approach as well as the physical material constituting the fluid and the wettability of the boiling surface, disjoining pressure effects may be observed (see explanation in [43]).

Whenever the film thickness is sufficiently large (say >10μm) over most of the boiling flow regime, equilibrium thermodynamic assumption as well as negligible interfacial thermal resistance assumption typically hold (see [43]). For problems considered here, over \(x^p > 0\), equilibrium thermodynamic assumptions are good. This is because interfacial mass transfer rates, \(\dot{m}^p\), is sufficiently small in non-dimensional terms i.e. \(\dot{m} \equiv \dot{m}^p / \rho_1 U \ll 1\). Under these equilibrium conditions, for \(x^p > 0\), \(T^i_1\) and \(T^i_2\) respectively denote the liquid and vapor temperatures at the interface, the following scientific model of the equilibrium thermodynamics holds at the interface:

\[
T^i_1 \approx T^i_2 \equiv T_{\text{sat}}(p^i_2)
\]  

(A1.7)
However for some “thin film” situations not considered here, Eq. (A1.7) assumption of negligible thermal resistance, i.e. $\Delta T^i/\Delta T \ll 1$ (where $\Delta T^i \equiv |T_1^i - T_2^i|$) assumption does not hold and $T_1^i \neq T_2^i$ can be modeled by one of the two approaches given in [43].

For such conditions, where liquid film is “thin” over most of the significant parts (as in pulsatile high heat flux cases in [1]) of the length of the channel in Figure 3, one allows $T_1^i \neq T_2^i$ and introduces other modelling equations [43] and another restriction on the interfacial mass flux that requires $\dot{m}^p = \dot{m}_{\text{kinetic}}^p$, and $\dot{m}_{\text{kinetic}}^p$ is obtained through phase-change models based on Kinetic theory of gases [2] and is defined as:

$$\dot{m}_{\text{kinetic}}^p \approx \frac{2\sigma_c}{2 - \sigma_c} \left[ \frac{p_{\text{sat}}(T_2^i)}{(2\pi R T_2^i)^{1/2}} - \frac{p_{\text{sat}}(T_1^i)}{(2\pi R T_1^i)^{1/2}} \right]$$

(A1.8)

where $\sigma_c$ is an “accommodation” coefficient [2, 3] and $R \equiv R_u / \bar{M}$ is a gas constant related to the universal gas constant, $R_u$, and fluid’s molecular weight $\bar{M}$.

As discussed in [43], mass balance at any point on the interface requires a single-valued interfacial mass flux. That is, when $\Delta T^i/\Delta T \ll 1$, one only needs to satisfy

$$\dot{m}_{\text{LK}}^p = \dot{m}_{\text{VK}}^p = \dot{m}_{\text{Energy}}^p \equiv \dot{m}^p$$

(A1.9)

If $\Delta T^i/\Delta T$ is not insignificantly small, the model in Eq. (A1.9) is replaced by the new interfacial mass balance requirement:

$$\dot{m}_{\text{LK}}^p = \dot{m}_{\text{VK}}^p = \dot{m}_{\text{Energy}}^p = \dot{m}_{\text{kinetic}}^p \equiv \dot{m}^p$$

(A1.10)

APPENDIX A2

For the adiabatic zone $-x^{p*} < x^p < -x^{p**}$ in Figure 5, the annular liquid and vapor flows are at
uniform temperatures $T_1(x, y) = T_2(x, y) = T_{\text{sat}}(p_{0a})$ and interfacial mass flux values are $\dot{m}^p = 0$. Here $p_{0a}$ is the adiabatic zone pressure at $x^p = -x^p^*$ and $y = h$, (in Figure 5). This annular adiabatic laminar/laminar flow zone shown in Figure A2.1, easily yields an analytical solution of the type:

$$
\mathbf{g} = g_x \mathbf{i} - g_y \mathbf{j}
$$

$$
\mathbf{v}_1 = u_1(y) \mathbf{i}
$$

$$
\mathbf{v}_2 = u_2(y) \mathbf{i}
$$

and

$$
\Delta(x) = \Delta_{0a}
$$

(A2.1)

The x and y components of liquid ($I = 1$) and vapor ($I = 2$) momentum balance in Eq. (2) of section 2, when written in physical variables, yield solutions of the following structure:

$$
p_1(x, y) = -\rho_1 g_y y + \lambda_1 x + \lambda_{11}
$$

$$
p_2(x, y) = -\rho_2 g_y y + \lambda_2 x + \lambda_{22}
$$

$$
u_1(y) = \frac{k_1}{2\mu_1} y^2 + k_{11} y + k_{12}
$$

(A2.2)

$$
u_2(y) = \frac{k_2}{2\mu_2} y^2 + k_{21} y + k_{22}
$$

$$
\Delta(x) = \Delta_{0a}
$$

At the interface $y = \Delta_{0a} = \text{constant}$, (the adiabatic zone film thickness) the continuity of tangential velocities (Eq. (3) with $\delta_x = \frac{1}{h} \Delta'(x)$), tangential component of interfacial momentum balance (Eq. (5) with $[t] = 0$), and normal component of interfacial momentum balance (Eq. (4) with $\dot{m} = 0 = \delta_{xx}$) come together to yield the following for the horizontal ($g_x = 0$) channel:

$$
k_1 = k_2 = \lambda_2 = \lambda_1
$$

$$
k_{11} = \frac{\mu_2}{\mu_1} k_{21}
$$

(A2.3)

The non-slip conditions at $y = 0$ and $y = h$ yield
\[ k_{12} = 0 \]
\[ k_{22} = \frac{k_2}{2\mu_2} h^2 + k_{21} h \]  

(A2.4)

Using, \( p_2(-x^p, h) = p_{0a} \), the following is obtained for Eq. (A2.1):  
\[ \lambda_{11} = \lambda_{22} + (\rho_1 - \rho_2) g y h \]
\[ \lambda_{22} = p_{0a} + \rho_2 g_y \Delta_{0a} \]  

(A2.5)

Next it can be found that the vapor flow rate per unit width, \( \dot{M}_V' \), in Figure A2.1 is given by:
\[ \dot{M}_V' \equiv \int_{\Delta_{0a}}^{h} \rho_2 u_2^p(y^p) dy^p = \rho_2 k_2 \phi(\Delta_{0a}, h) \]  

(A2.6)

where,  
\[ \phi(\Delta_{0a}, h) = \left[ \frac{1}{2\mu_2} \left( \frac{\Delta_{0a}^3}{3} \right) + \Psi(\Delta_0, h) \left( \frac{\Delta_{0a}^3}{2} \right) - (h - \Delta_{0a}) \left( \frac{h^2 + \Psi(\Delta_{0a}, h)}{2} \right) \right] \]  
and
\[ \Psi(\Delta_{0a}, h) = \frac{1}{2\mu_2} \left[ \frac{-\Delta_{0a}^3 (1 - \frac{\mu_2}{\mu_1}) + h^2}{\Delta_{0a}^3 (1 - \frac{\mu_2}{\mu_1}) - h} \right]. \]

Further it can be found that,
\[ \dot{M}_L' \equiv \int_{0}^{\Delta_{0}} \rho_1 u_1^p(0, y^p) dy^p = \frac{\dot{M}_V'}{\rho_2 \phi(\Delta_0, h)} \left( \frac{\Delta_{0a}^3}{6\mu_1} + \frac{\mu_2 \Delta_{0a}^2}{\mu_1} \Psi(\Delta_{0a}, h) \right) \]  

(A2.7)

Using the notation,
\[ \dot{M}_{in}' = \dot{M}_L' + \dot{M}_V' \]  

(A2.8)

\[ X_{in} = \dot{M}_V'/\dot{M}_{in}' \]  

(A2.9)

it is easily shown that \( \delta_{0a} \equiv \Delta_{0a}/h \) is the zero of the following non-dimensional equation:
\[
\frac{1 - X \rho_2}{X \rho_1} - \frac{1}{\Phi(\Psi, \delta_{0a})} \left[ \frac{\mu_2}{\mu_1} \cdot \frac{1}{6} \delta_{0a}^3 + \frac{\mu_2}{\mu_1} \cdot \frac{\delta_{0a}^2}{2} \cdot \Phi \left( \delta_{0a}, \frac{\mu_2}{\mu_1} \right) \right] = 0 \quad (A2.10)
\]

where, \( \Psi \equiv \frac{1}{2} \cdot \frac{-\delta_{0a} (1 - \frac{\mu_2}{\mu_1}) + 1}{[\delta_{0a} (1 - \frac{\mu_2}{\mu_1}) - 1]} \) and, \( \Phi \equiv \left[ \frac{1}{6} (1 - \delta_{0a}^3) + \Psi \cdot \frac{(1 - \delta_{0a})}{2} \right] \left( \frac{1 - \delta_{0a}}{2} + \Psi \cdot (1 - \delta_{0a}) \right) \]

Clearly, the constant film thickness \( \delta_{0a} \), a zero of Eq. (A2.10), is of the type:

\[
\delta_{0a} = \delta_{0a} \left( \frac{1 - X \rho_2}{X \rho_1}, \frac{\mu_2}{\mu_1} \right) \quad (A2.11)
\]

Considering flow of refrigerants at an inlet pressure of \( p_{0a} = 1 - 2 \) bars, and annular zone qualities of \( 0.1 \leq X \leq 0.9 \), it is found that for

\[
0.003 \leq \frac{\rho_2}{\rho_1} \leq 0.016 \quad (A2.12)
\]

\[
0.02 \leq \frac{\mu_2}{\mu_1} \leq 0.036
\]

computationally obtained roots of Eq. (A2.10) for the parameters are correlated with mean error of 5.16% and maximum absolute error of 12.81% by the relationship:

\[
\delta_{0a} = 0.4227 \left( \frac{\mu_2}{\mu_1} \right)^{-0.2496} \left( \frac{1 - X \rho_2}{X \rho_1} \right)^{0.3524} \quad (A2.13)
\]

For parameters, covering both refrigerants and water at \( p_{0a} = 1 - 2 \) bars and \( 0.1 \leq X \leq 0.9 \):

\[
0.0006 \leq \frac{\rho_2}{\rho_1} \leq 0.016
\]

\[
0.02 \leq \frac{\mu_2}{\mu_1} \leq 0.055 \quad (A2.14)
\]

the correlation:
\[\ln \delta_{0a} = -0.8147 - 0.1337 \ln \frac{\mu_2}{\mu_1} + 0.29726 \ln \left( \frac{1 - X}{X} \cdot \frac{\rho_2}{\rho_1} \right) \]

\[-0.0188 \left[ \ln \left( \frac{1 - X}{X} \cdot \frac{\rho_2}{\rho_1} \right) \right]^2 + 0.0371 \ln \left( \frac{\mu_2}{\mu_1} \right) \cdot \ln \left( \frac{1 - X}{X} \cdot \frac{\rho_2}{\rho_1} \right) \]  

when compared with computed results has mean error of 0.35% and maximum absolute error of 1.87%.

The above results imply void fraction models of:

\[\varepsilon \equiv \frac{h - \Delta_{0a}}{h} = 1 - \delta_{0a} = 1 - \left[ 0.4227 \frac{\mu_2}{\mu_1}^{-0.2496} \left( \frac{1 - X}{X} \cdot \frac{\rho_2}{\rho_1} \right)^{0.3524} \right] \]  

and

\[\varepsilon = 1 - \delta_{0a} = 1 - \exp \left( -0.8147 - 0.1337 \ln \frac{\mu_2}{\mu_1} + 0.29726 \ln \left( \frac{1 - X}{X} \cdot \frac{\rho_2}{\rho_1} \right) \right) \]

\[-0.0188 \left[ \ln \left( \frac{1 - X}{X} \cdot \frac{\rho_2}{\rho_1} \right) \right]^2 + 0.0371 \ln \left( \frac{\mu_2}{\mu_1} \right) \cdot \ln \left( \frac{1 - X}{X} \cdot \frac{\rho_2}{\rho_1} \right) \]  

A graphical comparison of Eq. (A2.16) and Eq. (A2.17) with Zivi correlation [47] given in Eq. (A2.18) is shown in Figure A2.2. Comparisons with other popular correlation are to be reported in [46].

\[\varepsilon = \frac{1}{1 + \frac{1 - X}{X} \left( \frac{\rho_2}{\rho_1} \right)^{2/3}} \]  

With known, the velocity profiles in Eq. (A2.1) are obtained through:

\[k_1 = k_2 = \lambda_2 = \lambda_1 = \frac{\dot{M}_V}{\rho_2 \phi(\Delta_{0a}, h)} \]  

(A2.19)
\[ k_{21} = k_2 \Psi (\Delta_{0a} h) \]

\[ k_{22} = -\left( \frac{k_2}{2 \mu_2} h^2 + k_{21} h \right) \]

\[ k_{11} = \frac{\mu_2}{\mu_1} k_{21} \]

The results in Eq. (A2.19) also give pressure fields (with \( p_{0a} \)) and interfacial stress vector (for \( \vec{f} = \vec{j} \) in Figure A2.1) fields \( \tau^i_1 (\equiv \tau^i_{2x} \hat{i} + \tau^i_{2y} \hat{j}) \) through the relations:

\[
\begin{align*}
  p_1(x, y) &= -\rho_1 g y + k_2 x + (\rho_1 - \rho_2) g y \Delta_{0a} + p_{0a} + \rho_2 g y \Delta_{0a} \\
  p_2(x, y) &= \rho_2 g y (\Delta_{0a} - y) + k_2 x + p_{0a} \\
  \tau^i_{2x} &= -(k_2 \Delta_{0a} + \mu_2 k_{21}) \\
  \tau^i_{2y} &= p^i_2 = p_{0a} + \rho_2 g y (h - \Delta_{0a})
\end{align*}
\]
REFERENCES

27. Kivisalu, M.T. Experimental investigation of certain internal condensing and boiling flows: Their sensitivity to pressure fluctuations and heat transfer enhancements. Ph. D, Michigan Technological University. 2015.


**Figure Caption List**

Figure 1: Innovative boiler’s non-pulsatile operation [1].

Figure 2: A schematic of a traditional flow-boiling operation in a channel with bottom wall heating.

Figure 3: (a) Schematic of a representative suppressed nucleation case of annular flow-boiling in a channel. (b) Schematic of a representative instantaneous interface location showing the interfacial variables used as boundary conditions for the liquid and the vapor domains. The computational domain’s exit at \( x = L_{\text{comp}} \) in (b) is often slightly larger than the exit at \( x = L \) in (a).

Figure 4: Representative film thickness profile for annular/stratified flow-boiling – including “prior method of heating.”

Figure 5: Representative wall temperature \( (T_w(x)) \) (and heat-flux) values are prescribed as indicated for \( x^P \geq 0 \) and \( -x^P_* < x^P < 0 \).

Figure 6: Cross-sectional profile plots at \( x^P = 0.02 \) m are shown for \( x \)-component of velocity \( u^P_I \) with mesh comparison for a representative liquid and vapor domain solution. The “order of convergence” study, not reported here, yields results similar to what has been reported in [33, 43]. (Run parameters: Fluid is FC72, \( U = 1 \) m/s, \( p_0 = 105.1 \) kPa, \( \Delta T = 10^\circ \)C, \( h = 2 \) mm, \( G \equiv \rho_2 U = 13.98 \) kg/m\(^2\)s).

Figure 7: (a) Plot of a steady film thickness profile for a horizontal case in the presence of transverse gravity. Cross-sectional profile plots at \( x^P = 0.02 \) m are shown for: (b) \( y \)-component of velocity \( v^P_I \), (c) Temperature \( T_i \), and (d) Pressure \( p_i \). (Run parameters: Same as in Figure 6)

Figure 8: Streamline patterns for \( \vec{v}_I \), for \( I = 1 \) and 2, are shown above for the flow in Figure 7. The background shade representing magnitude of \( \vec{v}_I \), for \( I = 1 \) and 2, is the vertical axis on the right of the plot. (Run parameters: same as in Figure 6).
Figure 9: The x-variation of 1-D flow variables for the flow in Figure 7: (a) Characteristic speed $\bar{u}(x_p)$ and interfacial velocity $u_1^{pl}(x_p)$. (b) Interfacial shear stress, $\tau^{int}(x_p)$. (c) Interfacial mass-flux, $\dot{m}(x_p)$. (d) Wall heat-flux $\dot{q}_w(x_p)$ and heat transfer coefficient $h(x_p)$. (e) Nusselt Number, $Nu_x(x_p)$ and quality, $X(x_p)$. (f) Heat transfer coefficient as a function of quality $X$, $h_x(X)$. (Run case: same as in Figure 6.)

Figure 10: The $T_w(x)$ prescription is shown by the solid curve. The solid $T_w(x)$ prescription yields a 2-D solution whose $\dot{q}_{w}(x)$ predictions are shown as solid lines. The dotted line $\dot{q}_{w}(x)$ is an identical depiction of the solid line $\dot{q}_{w}(x)$ prescription – indicating that it is now used as a boundary condition to obtain 2-D solutions - with its $T_w(x)$ predictions shown as dotted line.

Figure 11: (a) Three different “prior methods of heating” are shown. Nearly identical $X(0)$ values are achieved over $x_p < 0$. (b) Variation of Heat transfer coefficient $h_x(x_p)$ and Quality $X(x_p)$ with distance over $x_p \geq 0$ are shown for the three different “prior methods of heating”. (c) Variation of Heat transfer coefficient $h_x(X)$ with the respective qualities for the three different “prior methods of heating”. (Run parameters: Same as in Figure 6)

Figure 12: (a) The uniform heating method ($\Delta T = 10^\circ C$ and $T_w^\delta = 0^\circ C$) and two proposed non-uniform heating – with $T_w^\delta = 1^\circ C$ and $T_w^\delta = 2^\circ C$ – are indicated. (b) For the three heating methods in (a), the three different $X(x_p)$ predictions from this paper’s 2-D steady solver along with a comparison with their 1-D prediction values are shown. (c) For the three heating methods in (a), the three different $h_x(X)$ predictions from this paper’s 2-D steady solver are shown. (Run parameters: Same as in Figure 6)

Figure 13: Possible flow-regime scenarios in an innovative flow-boiler [1] for different vapor recirculation rates $\dot{M}_{v-recirc}$ or $X_{in}$. (a) Shows presence of non-annular, annular with nucleation, and annular regime with suppressed nucleation. (b) Shows presence of annular regime with and without nucleation from the inlet. Furthermore inlet liquid thickness $\Delta_{0a} \approx \Omega (100\mu m)$. 


Figure 14: (a) Relative magnitude of energy transfer terms. (b) Sum of significant energy transfer terms and Viscous Dissipation term, $V_{DL}$. (c) Characteristic speed $\bar{u}(x^p)$ and Viscous Dissipation term, $V_{DL}$. (Run parameters: Fluid is FC-72, $U = 1 \text{ m/s}$, $p_0 = 105.1 \text{ kPa}$, $\Delta T = 15^\circ\text{C}$, channel height = 2 mm, $G \equiv \rho_2 U = 13.98 \text{ kg/m}^2\text{s}$).

Figure A2.1: Schematic of the adiabatic laminar/laminar flow zone corresponding to uniform liquid film thickness of $\Delta_0$

Figure A2.2: Comparison of correlations with Zivi Correlation (Parameters: $(\rho_2/\rho_1) = 0.0095$, $(\mu_2/\mu_1) = 0.024$)
Figures

Figure 1
Figure 2
Figure 3a
Figure 3b
Figure 5

Wall Temperature, $T_w(x^p)$
[or Heat Flux, $q_w(x^p)$]

$T_{sat}(p^*)$
[or $q_w = 0$]

Uniform $T_w(x^p)$ [or $q_w(x^p)$] for $x^p \geq 0$

HM-i ($i = 1, 2, 3$)

$\Delta T$ [or $q''_w$]

CFD enabling "prior heating method"

Actual "heating method" of interest

Distance along the length of the channel, $x^p$
Figure 6

Distance from heated surface, $y^p$ (m)

Velocity, $u^p$ (m/s) at $x^p = 0.02$ m ($l = 1$ or 2)

Vapor profile

Liquid profile

$e_{\text{max}} \leq 0.1\%$

Mesh-1

Mesh-2

Mesh-3
Figure 7a

Film Thickness, $\Delta (m)$

Distance along the length of the channel, $x^p (m)$
Figure 7b
Figure 7d

Distance from heated surface, $y^p$ (m)

Pressure, $p_1$ (Pa) @ $x^p = 0.02$ m ($I = 1$ or $2$)

- Liquid - $p_1$
- Vapor - $p_2$
Figure 9a

Interfacial Velocity, $u_l^p$ (m/s)

Characteristic Speed, $\bar{u}^p$ (m/s)

Distance along the length of the channel, $x^p$ (m)
Figure 9b
Figure 9c

Interfacial Mass Transfer, \( \dot{n}_i^P \) (kg/m²s)

Distance along the length of the channel, \( x^P \) (m)
Figure 9d

The graph shows the relationship between the distance along the length of the channel, $x_p$ (m), and the heat flux per unit area, $q_w$ (W/m$^2$), as well as the heat transfer coefficient, $h_x$ (W/m$^2$K). The graph indicates the variation of these parameters as the distance changes. The heating method associated entrance length, $x_e$, is also indicated on the graph.
Figure 9e

The graph shows the variation of Nusselt Number ($N_u_x$) and Quality ($X$) with Distance along the length of the channel ($x_p$) in meters. The Nusselt Number increases as the distance increases, while the Quality also increases slightly with distance.
Figure 9f

Heat Transfer Coefficient, $h_X$ (W/m²K)

Quality, $X$

$X_e - X(0)$

81
Figure 11a

Wall Temperature, $T_w(x)$ (K)

Distance along the length of the channel, $x^p$ (m)
Figure 11b

Heat Transfer Coefficient, $h_x$ (W/m$^2$K)

Quality, $x$

Distance along the length of the channel, $x_e$ (m)

Legend:
- HM-1
- HM-2
- HM-3
Figure 12a
Figure 12c

Heat Transfer Coefficient, $h_x$ (W/m$^2$K) vs. Quality, X

- Uniform
- Non-Uniform $T_w^\delta=1{^\circ C}$
- Non-Uniform $T_w^\delta=2{^\circ C}$
Figure 13a
Figure 13b

\[ \dot{M}_{v\text{-recirc}} \]

\[ \Delta_0 \]

\[ X(0) \]
Figure 14a
Figure 14c

Characteristics Speed, $\tilde{u}^P$ (m/s) vs. Distance along the length of the channel, $x^P$ (m)

Viscous Dissipation, $V_{D_c}$ (W/m)

- Solid line: $\tilde{u}^P$
- Dotted line: $V_{D_c}$
Figure A2.1

\[ \mathbf{g} = g_x \hat{i} + g_y \hat{j} \]

with

\[ g_x = 0 \]
Table Caption List

Table 1: Table shows representative non-dimensional mesh sizes for different meshes.

Table 2: (a)-(b): Tables show representative satisfaction of interface conditions (in physical variables) for different locations along the length of the channel.

Table 3: Choices and ranges of physical flow variables for fluid flow conditions considered in this study.
### Tables

#### Table 1

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Representative Liquid Domain Mesh sizes</th>
<th>Representative Vapor Domain Mesh sizes</th>
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<td>$\Delta s_{L2} = 3.51 \times 10^{-7}$</td>
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Table 2 (a)

<table>
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<th>Location</th>
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<th>Continuity of Tangential Velocities</th>
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<td>( \dot{m}_{LK} )</td>
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### Table 2 (b)

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