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Chapter 1. Introduction

Computational physics based simulations are needed to assist in the design and operation of condensers that need stable, repeatable, and predictable realizations of annular condensing flows. This is particularly important because innovative operations of efficient high heat-flux condensers (and boilers) have been shown ([1]) to involve conditions where the flow regime is annular or wavy annular over the entire length of the device. Besides potential applications in certain zero gravity space-based thermal management systems, the gravity-insensitive shear driven annular condensing flows (with or without pulsations induced heat-flux enhancements [1]) can also be employed in avionics cooling, electronics/data-center cooling, etc. applications that require efficient boiling/condensing flows through narrow passages. Though the simulation capabilities presented here through simple extensions allow modeling and simulations for: annular gas-liquid flows without phase-change, and flow boiling (under conditions of suppressed nucleation) inside innovative boilers ([1]); the dissertation limits itself to annular flow condensation.

The typical traditional operation of ground-based larger hydraulic diameter (say > 2 mm) gravity driven condensers (or gravity assisted asymmetrically stratified flow in horizontal “tube” condensers [2]) are known to be effective due to the presence of the longer annular regimes (Figure 1.1a) relative to the purely shear driven horizontal cases (Figure 1.1b). Therefore, gravity driven traditional operations have a better inherent ability to use the higher thermal and hydrodynamic efficiency of the annular regimes over the less desirable non-annular regimes (plug/slug, bubbly, etc.). However, recent development of innovative shear driven condenser operations ([1, 3]) have allowed the length of the annular regime $x_A$ in Figure 1.1b to be extended to cover the entire length of the device in Figure 1.1c (i.e., $x_A \geq L$). In addition, newly proposed pulsatile flow operations ([1]) also allow very large amplitude wavy annular flows that achieve very high heat-flux values relative to the non-pulsatile cases studied here. Future extensions/implementations of the algorithm reported here can also deal with pulsatile large amplitude wavy annular flows.
Figure 1.1: Schematic of (a) inclined gravity driven channel flow, (b) horizontal shear/pressure driven channel flow, and (c) an innovative all annular non-pulsatile condenser operation ([1]). These devices have pure vapor flow at the inlet and has condensation only at the bottom surface.

Accuracy in scientific computational simulations have been elusive for annular (or stratified) shear driven internal condensing flows. Relative to earlier reported gravity driven flow results ([4]), the computational challenges have been quite significant for the more sensitive shear driven cases. The sensitivity is because the liquid condensate’s motion as well as interface location is strongly coupled to the vapor’s motion and is therefore susceptible to inadvertent and inevitable presence of computational or physical noise. The research goal of this work has been to develop capabilities for reliable capturing of dynamic wave-propagation (both in amplitude and phase) and develop an understanding of various
physical mechanisms (particularly instability mechanisms) associated with the shear driven cases.

The motivation for developing the algorithm presented in this work are two folds. One is to focus on reliably obtaining key flow physics information (such as heat-transfer rates, length of the annular regime, etc.) that are of contemporary interest. The other is to setup the algorithm in a way that numerical analysts can further the cause of diagnosing and addressing the well-known difficulties associated with satisfying interfacial mass-flux equalities. These difficulties arise whenever solutions of a certain class (of the type considered here) of phase-change problems are attempted with the help of commercially available single domain (i.e. combined consideration of the two phases) computational approaches based on level-set ([5]), VOF ([6, 7]), front-tracking ([8], [9], etc. techniques. These difficulties are often said to arise from one of two sources:

(i) “Numerical diffusion” associated with the solution technique for the interface tracking equation – even if the interface is modeled as “sharp.” This is said to be associated with spatio-temporal grid size restrictions - either in meeting Courant-Friedrichs-Lewey, CFL, type requirements in capturing the amplitudes of waves on the interface or more stringent grid refinement needs in capturing the phase of the waves ([10]) for solving the interface evolution equation. “Numerical diffusion” is said to arise when this spatio-temporal grid size starts becoming comparable to the grid-size needed for solving the underlying fluid flow equations or it starts becoming comparable to inherent small scales associated with the interface topology (e.g. interfacial curves/surfaces bend or fold to become very close to one another) itself.

(ii) “Numerical diffusion” that arises when the interface is modeled as one of “finite thickness” and CFD is used to solve the flow field in this finite thickness interfacial domain. In this case, additional convergence errors may arise because: (i) the spatio-temporal grid size associated with solving the interface equation becomes close to the “finite thickness” of the interface as iterations attempt to reduce the
thickness ([11]) to levels where it becomes effectively “sharp” relative to the length scales of interest, and/or (ii) the initially erroneous definition of the velocity field appearing in the interface advection equation remains erroneous or non-convergent during the “interface thickness” reduction process.

The above problems have been resolved for some 2-D/3-D gas-liquid flow simulations dealing with zero ([8, 12-14]) or non-zero ([9, 11]) interfacial mass-flux values—as in problems dealing with unsteady bubble dynamics, where the time varying solution are able to accurately locate the vapor-liquid interface. These are bubble-dynamics solutions in the non-oscillatory bubble regime or oscillatory volume (not shape) regime of bubble behavior ([15]). These solutions are not looking to resolve bubbles in oscillatory volume and shape regimes – the kind addressed by experiments and analysis in [16]. It is the algorithms capable of handling bubbles in oscillatory volume and shape regimes that may be able to handle the wave dynamics related to stability analysis for steady annular flow solution.

There is a lack of reliable techniques that have overcome the above described interfacial mass-flux equalities issues in capturing wave-dynamics (both in amplitude and phase) for annular flow condensation or flow boiling – a resolution that is necessary for stability analysis of shear driven internal condensing flows.

In order to meet the objectives of accurate solution and numerical scheme diagnostics, this work has chosen to implement a method of characteristics based scheme which maintains following differences from the commercially available level-set ([5]) and/or VOF ([6, 7]) based techniques:

(i) The “interface” is modeled as “sharp” instead of the more common “thin zone” models.

(ii) The “sharp” model requires that the liquid and vapor domains are solved separately and consecutively - allowing discontinuous jumps in fluid properties and flow variables across the interface in a manner consistent with interface conditions. This allows the algorithm to focus on direct satisfaction of interface conditions as opposed to the “potentially” more efficient concurrent solution approach in a “single” fixed domain - where flow variables and fluid properties are extended smoothly across a “thin zone” interface model (with
boundary layer type variations [11-13] replacing discontinuous models) and the interface conditions get automatically satisfied by the difficult CFD for the “thin zone” interface model or the formulations (for problems without phase-change) are such that all interface conditions can be incorporated through continuum surface force method (see [8, 12, 17]).

(iii) The original hyperbolic nature of the interface tracking equation is recognized to result from one of the interface conditions and is preserved. No attempt is made to change it with phase-field models ([18]) or change the original Eulerian interior equations with equations such as those associated with Arbitrary Lagrangian Eulerian (ALE) approaches ([19, 20]).

It is recognized that these approaches may have value in dealing with other problems but not in diagnosing and addressing the recognized difficulties in implementing level-set ([5]) and/or VOF ([6, 7]) based solution approach for the problem of interest.

The sharp interface model used here involves solving the interface tracking equation which is a hyperbolic partial differential equation (PDE). For solving this well-known interface tracking equation, it is recognized that existing 5th order WENO method ([21]) combined with TVD Runge Kutta ([22]) successfully used in [13] or its semi-lagrangian variations ([23]) are already quite accurate – so it is conjectured that the aforementioned “numerical diffusion” problems do not lie here. It probably lies in interactions of the mesh-size needed for PDE solution approach on a fixed grid with the mesh-sizes needed for the well-established CFD solution approaches for the underlying thin liquid film flow. It is important to note that the liquid film is thin (in tens of micro-meters) and the interfacial velocity is low. Since the CFD length scales to be resolved are much smaller than those associated with bubble dynamics, solving the interface tracking equation by fixed-grid differencing schemes (e.g. as used in [11, 13] etc.) is avoided here – at least for diagnostic purposes - in favor of direct and “close to analytical” integration of this equation.

In this work, interface locations are obtained by an integration procedure along the characteristics curves underlying the PDE. In the proposed method of characteristics, a 4th order time-step accuracy is used for “locating” the characteristics curves on a specially designed moving grid – this allows direct numerical integration to yield the interface
location. This does three things: (i) it allows the moving grid to be coarser than the CFD grid – allowing smoothing (i.e. use of splines) for capturing curvature effects, (ii) it allows interrogation of interface locations at certain “Courant nearly equal to one” locations where it is known ([10]) that even coarse grids can correctly capture the phase (not to mention the amplitudes) of the waves governed by the interface tracking equation, and (iii) it allows the remaining interface jump conditions to be directly embedded in the separate liquid and vapor domain solutions (as boundary conditions) adopted here.

The findings from the proposed approach’s implementation as well as the algorithm’s possible level-set based 2-D/3-D extensions are discussed. This has helped, in the short run, to obtain scientific solutions of some important problems of current interest. In the long run, it is hoped that numerical analysts can pick and choose from the results and discussions – and enable development of efficient commercially available techniques that continue to employ single domain computational approaches for a broader class (than what is currently possible) of phase-change flow problems.

The dissertation presents the governing equations, computational approach, and the solution algorithm utilized to develop the unsteady computational tool on a COMSOL/MATLAB ([24]) platform. This algorithm is based on an approach that models the interface as a “sharp curve/surface” and directly utilizes the well-known interface conditions ([2, 4, 25]) for satisfying the requirements arising from flow variables’ and fluid properties’ discontinuities present across the interface. Separate finite element method (FEM) based approach leading to computational fluid dynamics (CFD) calculations for each of the two phases are performed on COMSOL. With the help of our own subroutines (on MATLAB) for interface conditions and interface tracking equation, iterative improvement leading to convergence is accomplished through proper communication between the two separate single-phase solutions and an evolving interface location. For unsteady interface tracking, the new approach for locating the interface has been implemented with the help of a suitable combination of explicit and implicit numerical methods. This unsteady approach extends an earlier steady computational tool described
in [26-30]. The 4th order time-step accuracy used in the method of characteristics for locating the characteristics curves (on a suitable spatially fixed but temporally moving grid) underlying the interface tracking equation has significantly improved an earlier first order version ([31]) of the proposed algorithm, enabling it to overcome errors in earlier reported results [32] and in more accurately locating the wavy interface (at every instant) for shear driven flows. This earlier approach ([4]), however, gave good results for gravity driven flows ([32]).

Furthermore, this work focuses on discussion of flow physics through the presentation of steady and unsteady results obtained from numerical solutions of the full two-dimensional governing equations for annular internal condensing flows in a channel. Relative to our earlier efforts ([4]), the computational methodology presented here is much improved with regard to: (i) the accuracy of the solution of the interface tracking equation, (ii) use of versatile grid choices through the use of finite element method in obtaining CFD solution over the single-phase flow domains in the solution algorithm, (iii) accurate ability to impose stress boundary conditions on the phase-change interface associated with the thin wavy liquid film flows, (iv) approaches for resolving singularities in the inlet condition (note Figure 1.1 assumes onset of condensation at the inlet), and (v) more accurate specification of “exit” cross-sectional pressure variation for the vapor-flow in a manner that makes it compatible with interface stress conditions at the exit (with an understanding that it is problem-specific and depends on the geometry further downstream).

Despite this, in essence and in all other aspects, the algorithm presented here is similar to our earlier methodology ([4, 31, 33]) implemented on a FORTRAN platform. The gravity driven flows portion of the earlier methodology has been well tested and validated by comparison with condensing flow experiments ([34, 35]). However, these earlier implementations were not accurate enough for shear driven flows. The algorithm reported here are the only ones (known to us or presented earlier by us) which are accurate for both gravity and shear driven condensing flows.
Currently, we have 3 different computational tools developed to solve the internal condensing flows as shown in Table 1-1. The home grown 2-D CFD code implemented on the FORTRAN platform uses the full governing equations and are described in [36, 37]. A 1-D Quasi steady engineering tool of [28], though more approximate is computationally more efficient and versatile over a larger parameter zone has been developed. It has been developed as an independent tool and is used as a tool that supports and improves the efficacy of the associated 2-D approaches. We also have implemented a full 2-D code on the COMSOL/MATLAB platform to overcome some of the limitations of the FORTRAN code and also to further enhance our simulation capabilities.

**Table 1-1: Summary of the computational tools and list of doctoral students responsible for its development.**

<table>
<thead>
<tr>
<th>Code</th>
<th>Code Development</th>
<th>Investigations</th>
<th>Students</th>
</tr>
</thead>
</table>
| 2-D CFD code development and implementation on FORTRAN platform | Development of steady/unsteady internal condensing flow codes | 1. Effect of gravity, shear, surface tension, noise-sensitivity on the internal condensing flows.  
|      | Development of steady/unsteady external condensing flow codes                     | 1. Comparison with classical gravity driven and shear driven analytical solutions.  
| 1-D code | Development of steady internal condensing flows simulation tool                   | Demarcation of non-dimensional parameter space into gravity and shear driven zones. | S. Mitra, R. Naik, S. Kulkarni. |
| 2-D CFD code development and implementation on | Development of steady/unsteady internal condensing flow codes | 1. Impact of transverse gravity, noise-sensitivity and surface tension. | S. Mitra, R. Naik. |
This work compares and finds good agreement of the reported steady results with the results obtained from two independent computational techniques, namely: the two-dimensional (2-D) technique implemented on FORTRAN ([33]) and a quasi one-dimensional (1-D) technique ([27, 28]). The present tool allows for the handling of a larger domain size by obtaining correctly synthesized solutions on multiple overlapping domains, and offers enhanced computational speed, higher accuracy, etc. The 1-D steady solution technique ([27, 28]) is used for computing approximate results based on an algorithm that solves the governing equations as a set of non-linear ordinary differential equations.

Two-dimensional approaches presented here suffice for modeling the fluid-physics problems of interest. This is because the interest is either in annular in-tube condensation or, as it relates to this work, in the annular/stratified condensing flows that occur on the bottom-surface of a rectangular cross-section duct in a horizontal or downward inclined orientation. Also the condensate films are thin relative to the height and width, therefore, side-walls (wetting or non-wetting) do not have any significant effects except in a small zone near the side-walls. Hence 2-D modeling of the flow is sufficient. Furthermore, in the experiments of [1], the steady and unsteady flows are parabolic and exit-zone conditions - except for special large-amplitude wavy situations to be discussed elsewhere - do not affect far upstream conditions.

The unsteady computational methodology is used for instability analyses through the investigation of the steady annular/stratified condensing flows’ response to initial
disturbances and prediction of the associated length of the annular regime. These issues are also important in understanding boundary condition (at the inlet, condensing-surface, and the exit) sensitivity issues of the shear driven flows typically realized in horizontal channel flows, micro-gravity flows and in micro-meter scale hydraulic diameter ducts. The physics underlying the instability results are discussed here and lead to a theoretical/computational estimate of the length of the annular regime $x_A$ in Figure 1.1a-b – and this identifies the flow regime’s transition from annular to plug-slug flows over the inlet Reynolds number range ($400 \leq \text{Re}_\text{in} \leq 20000$) of interest here.

The results presented here go beyond strictly laminar vapor zone of $\text{Re}_\text{in} < 1000$ and cover a parameter space of $400 \leq \text{Re}_\text{in} \leq 20000$. In the vapor core away from the interface for $400 \leq \text{Re}_\text{in} \leq 20000$, the shear driven flows’ liquid condensate is thin and liquid film is laminar in the interior – with $\text{Re}_\delta < 1000$ ([38]). Though the interface locations themselves may exhibit *laminar turbulence* in the sense that wave structures have a statistical non-deterministic nature in some of their details, a significant zone of the vapor flow near the interface of the thin condensate is expected to be laminar – and this is not only because of the continuity of tangential velocities but, also, because the normal component of interfacial vapor velocity represents suction of the vapor into the laminar condensate (with significantly reduced normal velocity on the liquid side). Furthermore, for 2-D CFD, cell-by-cell velocity determination near the interface is not affected by turbulence in the far field, the reported results correctly simulate the near interface laminar vapor flow physics and effects responsible for condensate motion and its stability. This is the reason why heat-transfer correlations based on laminar-laminar CFD theory work ([39]) with regard to predicting condensate motion (not far field vapor-core pressure-drops) and heat-transfer rates even for $400 \leq \text{Re}_\text{in} \leq 20000$. As discussed later on, the vapor core’s turbulence typically has a second order effect on: the wave-structure superposed on the steady stratified-annular solution, and the length $x_A$ of the annular regime.

The tools and results presented here are also being used for investigations of large amplitude wavy and shear driven unsteady flows that are steady-in-the-mean and result
from sustained imposition of pulsations in the boundary conditions (such as inlet mass flow rate pulsations). This is important because, as shown in [1], the pulsatile cases can be beneficial and lead to very high heat-flux wavy annular flow realizations and technologically viable pulsatile operations of the device in Figure 1.1c.
Chapter 2. Problem Statement and Governing Equations

The computational algorithm and solutions presented here are for steady and unsteady annular/stratified channel flow situations shown in Figure 2.1. These condensing flows are achieved with condensation on the bottom wall. In Figure 2.1, gravity driven cases correspond to $\alpha > 0$ and shear driven cases correspond to zero-gravity ($\bar{g} = 0$) or horizontal ($\alpha = 0$) cases. The length $L$ in Figure 2.1 typically corresponds to $L \leq x_A^P$, the length of annular regime. However, to locate $x_A^P$ by an unsteady stability analysis, $L > x_A^P$ with assumed constant pressure-plenums at different values of $L$ are also considered. This allows consideration of realistic wave-motions without the need for considering different downstream effects on wave motion (which typically exhibit weak upstream effects for certain exceptions arising in case of pulsatile annular flows) associated with different types of geometries possible for locations downstream of $L$.

Figure 2.1: Schematic of a representative condensing flow problem in a channel.

The two-dimensional computational approach employed to investigate internal condensing flows in channels and tubes is based on the full governing equations described here and in [4, 33]. The liquid and vapor phases in the flow are denoted with subscript $I = 1$ and $I = 2$ (alternatively, as $I = 'L'$ and $'V'$) respectively. Both phases are modeled as incompressible (i.e. vapor Mach numbers are low). The fluid properties (density $\rho$, viscosity $\mu$, specific
heat $C_p$, and thermal conductivity $k$) with subscript $I$ are to take their representative constant values for each phase ($I = 1$ or 2).

Let the temperature, pressure, and velocity fields over the two phases be respectively denoted as $T_I$, $p_I$, and $\vec{V}_I = u_I \hat{i} + v_I \hat{j}$ . Also, let $T_{\text{sat}}(p)$ be the saturation temperature of the vapor as a function of local pressure $p$ at the interface, $\Delta$ be the film thickness, $\dot{m}^p$ be the local interfacial mass flux (kg/m$^2$-s), $T_w(x) < T_{\text{sat}}(p)$ be a known temperature variation of the condensing surface (with its length-averaged mean value being $\bar{T}_W$). Let $g_x$ and $g_y$ be the components of gravity along the $x$ and $y$ axes, $p_0$ be the steady inlet pressure, $\Delta T \equiv T_{\text{sat}}(p_0) - \bar{T}_W$ be a representative controlling temperature difference between the vapor and the bottom plate, $h_f$ be the heat of vaporization at temperature $T_{\text{sat}}(p)$, and $U$ be the average inlet vapor speed determined by the inlet mass flow rate $\dot{M}_{\text{in}} = \rho_2 \cdot U \cdot h \cdot w$ for rectangular cross-section channel of height $h$ and width $w$, with $h/w << 1$). With $t^p$ representing the actual time and $(x^p, y^p)$ representing physical distances of a point with respect to the axes in Figure 2.1 (for which $x^p = 0$ is at the inlet and $y^p = 0$ is at the condensing surface), we introduce a new list of fundamental non-dimensiona variables, $(x, y, t, \delta, u_I, v_I, \pi_I, \theta_I, \dot{m})$ through the following definitions:

\[
\begin{align*}
[x^p, y^p, \Delta, u_I^p, v_I^p] &= [h \cdot x, h \cdot y, h \cdot \delta, U \cdot u_I, U \cdot v_I] \\
[\dot{m}^p, T_I, p_I, t^p] &= [\rho_1 \cdot U \cdot \dot{m}, T_{\text{sat}}(p_0) + \Delta T \cdot \theta_I, p_0 + p_1 \cdot U^2 \cdot \pi_I, (h/U) \cdot t]
\end{align*}
\]

(2.1)

The representative constant values of the fluid properties are obtained from Engineering Equation Solver (EES) software [40] and other data handbooks. However, there are some inherent uncertainties associated with experimental data reported in the handbook values. Therefore, key results presented in non-dimensiona terms should be considered to have some additional uncertainties in non-dimensional parameters (associated with fluid properties) over and above computational error uncertainties (associated with level of convergence, round-off, discretization/truncation, and algorithm).

2.1. Interior Equations
The differential forms of mass, momentum (x^p and y^p components), and energy equations for 2-D flow in the interior of both the incompressible phases are the well-known equations (see [4]) presented in Eq. 2.2.

The simulations emphasized here assumes laminar vapor and laminar liquid flows. For most shear driven flows, the laminar liquid flow assumption holds up to the end of the computational domain (the end is at x^p = x_L^p = L where, L < x_A^p). It has been found that the comparisons of results obtained from these simulations with the experimental results are quite good even when vapor flow far from the near interface zone and the laminar liquid is assessed to be turbulent (as indicated by local values of vapor-phase Reynolds number). This agreement is expected because near-interface vapor flow laminarizes, i.e., x and y components of the interfacial vapor velocity experience severe reduction in their values as vapor condenses across the interface. Any randomness introduced in interfacial waviness, due to far field vapor core turbulence, may contribute to “laminar interfacial turbulence” but contributes insignificantly to the strong instability mechanisms that yield an estimate for the length x_A ≡ x_A^p/h of the annular regime.

Under laminar/laminar assumption, the non-dimensional differential forms of mass, momentum (x and y components), and energy equations for the two-dimensional flow in the interior of either of the incompressible phases (I = 1 or 2) are the well-known equations:

\[
\frac{\partial u_I}{\partial x} + \frac{\partial v_I}{\partial x} = 0
\]

\[
\frac{\partial u_I}{\partial t} + u_I \frac{\partial u_I}{\partial x} + v_I \frac{\partial u_I}{\partial y} = -\left(\frac{\partial \pi_I}{\partial x}\right) + \frac{1}{\text{Re}_I} \left(\frac{\partial^2 u_I}{\partial x^2} + \frac{\partial^2 u_I}{\partial y^2}\right) + \text{Fr}_x^{-2} - \frac{1}{\text{Re}_I} \left(\frac{\partial^2 v_I}{\partial x^2} + \frac{\partial^2 v_I}{\partial y^2}\right)
\]

\[
\frac{\partial v_I}{\partial t} + u_I \frac{\partial v_I}{\partial x} + v_I \frac{\partial v_I}{\partial y} = -\left(\frac{\partial \pi_I}{\partial y}\right) + \frac{1}{\text{Re}_I} \left(\frac{\partial^2 v_I}{\partial x^2} + \frac{\partial^2 v_I}{\partial y^2}\right) + \text{Fr}_y^{-2} - \frac{1}{\text{Re}_I} \left(\frac{\partial^2 u_I}{\partial x^2} + \frac{\partial^2 u_I}{\partial y^2}\right)
\]

\[
\frac{\partial \theta_I}{\partial t} + u_I \frac{\partial \theta_I}{\partial x} + v_I \frac{\partial \theta_I}{\partial y} \approx \frac{1}{\text{Re}_I \cdot \text{Pr}_I} \left(\frac{\partial^2 \theta_I}{\partial x^2} + \frac{\partial^2 \theta_I}{\partial y^2}\right)
\]

(2.2)

where Re_I = \rho_I U_I / \mu_I, Pr_I = \mu_I C_p I / k_I, Fr_x^{-2} = g_s h / U^2 and Fr_y^{-2} = g_y h / U^2.
2.2. Interface Conditions

Superscript “i” is used for the values of flow variables at the interface. The interface, in this paper, is explicitly located by the expression $\Phi = y_p - \Delta(x_p, t_p) = 0$. The nearly exact interface conditions (see [25], [4], etc.) for condensing flows are better qualified, extended (to cover sub-micron condensate thickness values of current interest as well as for planned future investigations), and re-stated here in the Appendix A-1. The “Newtonian” fluid models for stresses $\mathbf{T}_1$ and $\mathbf{T}_2$ in Appendix A-1 define the values of the liquid and vapor phase traction vectors $\mathbf{\tau}_2^{\pi i}$ and $\mathbf{\tau}_1^{\pi i}$ at any point on the interface ($\Phi = 0$). At any point on the interface, the unit normal (directed from the liquid to the vapor phase) is denoted by $\mathbf{n}$. Note (see [41]) that

$$\mathbf{\tau}_2^{\pi i} = \mathbf{T}_2 \mathbf{n} \equiv \tau_{2x}^{\pi i} \mathbf{i} + \tau_{2y}^{\pi i} \mathbf{j} \quad \text{and} \quad \mathbf{\tau}_1^{\pi i} = \mathbf{T}_1 \mathbf{n} \equiv \tau_{1x}^{\pi i} \mathbf{i} + \tau_{1y}^{\pi i} \mathbf{j}.$$ 

The non-dimensional values of the stress vector components are respectively defined as

$$\mathbf{\tau}_2^{\pi i} = (h/\mu_2 U) \cdot \mathbf{\tau}_2^{\pi i} \equiv \tau_{2x}^{\pi i} \mathbf{i} + \tau_{2y}^{\pi i} \mathbf{j} \quad \text{and} \quad \mathbf{\tau}_1^{\pi i} = (h/\mu_1 U) \cdot \mathbf{\tau}_1^{\pi i} \equiv \tau_{1x}^{\pi i} \mathbf{i} + \tau_{1y}^{\pi i} \mathbf{j}.$$ 

Non-dimensional Cartesian co-ordinate forms of the interface conditions, for the flow in Figure 2.1, are given below:

- The non-dimensional form of the requirement of continuity of tangential component of velocities (see Eq. (A. 2)) becomes:

$$u_2^{\pi i} = u_1^{\pi i} - \delta_x (v_2^{\pi i} - v_1^{\pi i}) \quad (2.3)$$

where $\delta_x \equiv \partial \delta / \partial x$.

- The non-dimensional form of normal component of momentum balance at the interface for thick films, as modeled by Eq. (A. 3) in the Appendix, is written in Cartesian coordinates (after ignoring the normal component of viscous stresses in comparison to interfacial pressures) as:

$$\pi_1^{\pi i} = \frac{\rho_2 \pi_2^{\pi i} - \frac{1}{\text{We}} \left( \frac{\delta_{xx}}{[1 + \delta_x^2]^{3/2}} \right)}{\rho_1} + \dot{m}^2 \left( \frac{\rho_1}{\rho_2} - 1 \right), \quad (2.4)$$

where $\rho$ are the densities and $\pi$ are the pressures.
where We ≡ \( \rho_1 U^2 h / \sigma \) and surface tension \( \sigma \) for the pure vapor depends on local interfacial temperature \( T_i \) (i.e. \( \sigma = \sigma (T_i) \)).

- The tangential component of momentum balance at the interface (see Eq. (A.4)) becomes:
  \[
  \frac{\partial u_1}{\partial y} \bigg|_i = \frac{\mu_2}{\mu_1} \frac{\partial u_2}{\partial y} \bigg|_i + [t] \tag{2.5}
  \]
  where the term \([t]\) in Eq. (5) is defined as:
  \[
  [t] = \left\{ \frac{\mu_2}{\mu_1} \frac{\partial v_2}{\partial x} \right\} - \frac{2 \delta_x}{[1 + \delta_x^2]} \left\{ \frac{\partial u_1}{\partial x} - \frac{\partial v_1}{\partial y} \right\} \tag{2.6}
  \]

- The non-dimensional form of non-zero physical values of interfacial mass fluxes \( \dot{m}_{LK}^p \) and \( \dot{m}_{VK}^p \) (defined in Eq. (A.8)) arise from kinematic constraints associated with the liquid and vapor velocity values of the interface. In the non-dimensional form these are given by:
  \[
  \dot{m}_{LK}^p \equiv \left[ u_1 \left( \frac{\partial \delta}{\partial x} \right) - \left( v_1 - \frac{\partial \delta}{\partial t} \right) \right] / \sqrt{1 + \left( \frac{\partial \delta}{\partial x} \right)^2} \quad \text{and}
  \]
  \[
  \dot{m}_{VK}^p \equiv \frac{\rho_2}{\rho_1} \left[ u_2 \left( \frac{\partial \delta}{\partial x} \right) - \left( v_2 - \frac{\partial \delta}{\partial t} \right) \right] / \sqrt{1 + \left( \frac{\partial \delta}{\partial x} \right)^2}. \tag{2.7}
  \]

- The non-dimensional form of non-zero physical values of interfacial mass flux \( \dot{m}_{\text{Energy}}^p \) (as given by Eq. (A.9)) represents the constraint imposed by the dominant net thermal energy transfer rates across the interface and is given by:
  \[
  \dot{m}_{\text{Energy}}^p \equiv \frac{Ja}{Re_1 Pr_1} \left\{ \frac{\partial \theta_1}{\partial n} \bigg|_i - \frac{k_2}{k_1} \frac{\partial \theta_2}{\partial n} \bigg|_i \right\}. \tag{2.8}
  \]
where \( J_a \equiv C_{p1} \Delta T / h_{fg} \) and \( h_{fg} \equiv h_{fg}(T_{sat}(p_1)) \). Recall that liquid Reynolds number \( Re_1 \) and Prandtl number \( Pr_1 \) are given by their definitions for Eq. (2).

- The interfacial mass balance (in Eq. (A.13) or, when necessary, by Eq. (A.14)) requires that the net mass flux (in \( kg/m^2-s \)) at a point on the interface, must be the same for all the different physical processes that impose a constraint in its value. The non-dimensional form of this requirement becomes:

\[
\dot{m}_{LK} = \dot{m}_{VK} = \dot{m}_{energy} \equiv \dot{m}
\]

(2.9)

It should be noted that negligible interfacial thermal resistance and equilibrium thermodynamics on either side of the interface is assumed to hold for \( x \) – values downstream of the origin (i.e., second or third computational cell onwards) in a CFD implementation for Figure 2.1.

- The non-dimensional thermodynamic restriction on interfacial temperatures (as given by the approximation in Eq. (A.10) as opposed to the one in Eq. (A.11)) becomes:

\[
\theta_{1i} \cong \theta_{2i} \equiv \theta_s(\pi_{2i}^1).
\]

(2.10)

Within the vapor phase, for the refrigerants and millimeter scale ducts considered here, changes in absolute pressure relative to the inlet pressure are big enough to affect vapor motion but, at the same time, they are usually too small to significantly affect saturation temperatures (except in micro-scale ducts). Therefore, we have \( \theta_s(\pi_{2i}^1) \cong \theta_s(0) \).

2.3. Boundary Conditions for Combined Consideration of the Vapor-Liquid Domain

The problem is computationally solved subject to the boundary conditions shown on a representative, not-to-scale film profile in the vapor-liquid domain of Figure 2.2.

*Top wall:* The upper wall physical temperature \( T_2(x^p, h, t^p) > T_{sat}(p_0) \) is at a superheated value (typically \( 5 - 10^\circ C \) above saturation temperature). This is close enough to the saturation temperature to allow the assumption of a nearly constant vapor temperature
$T_{\text{sat}}(p_0)$ at all locations of the vapor. This is reasonable and found to be true even if non-uniform vapor temperatures is assumed and calculated. This is because, in Eq. (8), the effects of superheat on interfacial heat-transfer are negligible considering the smallness of conduction of heat from vapor to the interface (because $K_2/K_1 << 1$) relative to the latent heat released and conductive heat-transfer rate across the liquid film.

*Bottom wall:* Besides the no-slip condition ($u_1^p(x^p, 0, t^p) = v_1^p(x^p, 0, t^p) = 0$) at the condensing surface, a steady condensing-surface temperature ($T_1(x^p, 0, t^p) = T_w(x^p) < T_{\text{sat}}(p_0)$) is assumed to be the prescribed thermal-boundary condition (although, if known, steady heat-flux values may also be prescribed).

*Inlet conditions and singularity resolution:* At the inlet $x^p = 0$, onset of condensation with zero film thickness ($\Delta(0,t^p) = 0$) and a sudden jump in physical values of condensing-surface temperature (i.e. $T_w(x^p) > T_{\text{sat}}(p_0)$ for $x^p < 0$ and $T_w(x^p) < T_{\text{sat}}(p_0)$ for $x^p > 0$) exist. This “model” singular condition of abrupt (or narrow width) onset of condensation results in infinite heat-flux, etc. which is allowed and modeled. To model this, at $x^p = \epsilon > 0$, (where $\epsilon$ is sufficiently small, say $10^{-5}$), a very small constant value $\epsilon^*$ (say $10^{-4}$) for film thickness (i.e. $\Delta (\epsilon, t^p) = \epsilon^*$) is arbitrarily chosen. Furthermore at $x^p = \epsilon$, velocity profiles $u_2^p(x^p, y^p, t^p)$ and $u_1^p(x^p, y^p, t^p)$ are prescribed in a way that the total mass-flux at $x^p = \epsilon$ is consistent with the total inlet mass flow rate at $x^p = 0$ (as given by $\dot{M}_\text{in} \equiv \rho_2 \cdot U \cdot h \cdot w$). The prescription of transverse component of velocity $v_2^p$ is not significant. Computations show that reasonably arbitrary choices at $x^p = \epsilon$ are typically unimportant as every “close to $x^p = 0$” choice leads to the same solution for all locations downstream of a certain $x^p = \epsilon^{**} > \epsilon$. If it is desired that the distance “$\epsilon^{**} - \epsilon$” be made close to zero, the arbitrary choices at $x = \epsilon$ can be judiciously and iteratively changed (with the help of backward integration and/or backward extrapolation for the somewhat unique solution available at $x > \epsilon^{**}$) to achieve a single unique solution right from $x^p \geq \epsilon$.

The actual physical value of the steady inlet pressure $p_{\text{in}} (= p_0)$ is not prescribed but appears indirectly through fluid properties and important thermodynamic properties such as $h_{\text{fg}}(p_2^i)$
\[ \approx h_f(p_0) \text{ and } T_{sat}(p_2) \approx T_{sat}(p_0). \] 

Unsteady prescription of upstream average vapor speed \( U \) (associated with unsteady inlet pressure \( p_{in} \)) is possible for investigating pulsatile cases but are not considered here.

For pulsatile inlet conditions, at \( x^p = \varepsilon \), velocity profiles \( u_2^p(x^p, y^p, t^p) \) and \( u_1^p(x^p, y^p, t^p) \) are prescribed in a way that the total mass-flux at \( x^p = \varepsilon \) is consistent with the prescribed time varying inlet mass flow rate.

**Exit conditions:**

**Steady exit condition:** For the steady problem, the flow is parabolic and no exit condition is needed. Pressure is not directly prescribed across the exit boundary for the computational simulations. Its arbitrary “reference” value \( p_{exit} \) is specified, to begin with, in the vapor domain - at the corner point at the intersection of the exit and the top wall (point B in Figure 2.2). Subsequently, for accuracy, a proper cross-sectional variation of physical vapor pressure \( p_2(x^p = x_{exit}^p, y^p) = p_{2\text{steady}}(x^p = x_{exit}^p, y^p) \) is needed and obtained as part of convergent steady solutions. The final convergent value of exit cross-sectional pressure \( p_{2\text{steady}}(x_{exit}^p, y^p) \) at \( x_{exit}^p = L \) in the subsequent calculations are obtained by requiring continuity of the stress-vector at point C – regardless of how one approaches point C (along points on the exit plane boundary condition or along points on the interface).

**Unsteady exit condition:** The wave propagation behavior for the unsteady problem is sensitive to the exit condition specification. This wave propagation behavior in the condensing flow here is somewhat analogous to the wave propagation on a string where the fixed or loose end conditions result in different wave patterns in terms of wave reflection and standing waves. The situation here is different because there are only forward moving characteristics (as opposed to forward and backward moving characteristics for a string problem). Therefore, for unsteady wave-motion on the steady solutions, the problem in principal can be considered independent of “exit condition” but may weakly depend on exit conditions depending on whether or not there is a geometry change for \( x > L \) (which can significantly speed up or retard forward moving interfacial
waves) and how close $x = L$ is to changes in geometry downstream of $x = L$. To eliminate these different possibilities that the Figure 2.2 problem formulation allows, a proper steady cross-sectional variation of physical vapor-phase pressure $p_2 (x^p = x_{sat} = L, y^p) = p_{2, steady} (x^p = x_{sat} = L, y^p)$ is typically prescribed to model the existence of a constant pressure plenum (without changes in downstream geometry) at $x^p > L$. As a caution, it is noted that for certain unsteady problems involving steady-in-the-mean pulsatile inlet flow rate conditions, experiments show ([42]) that for certain changes in geometries over $x^p > L$, the vapor-liquid phase flows’ interaction physics may change so significantly that the wave-structures may become exit-condition dependent and, as a result, the unsteady problem may become “elliptic.”

![Figure 2.2: For a representative instantaneous interface location, the figure shows the interfacial variables used for the liquid and the vapor domains.](image)

2.4. **Initial Condition**

Here $t = 0$ is not chosen to be the time when saturated vapor first comes in contact and condenses on a dry sub-cooled ($T_w(x^p) < T_{sat} (p_0)$) bottom plate. This is because the above described continuum equations do not model and incorporate various inter-molecular forces that are important in determining the time evolution of very thin (say from zero to $O(10\mu m)$) condensate film thickness $\Delta (x^p,t)$ at very early times ($t \sim 0$). Because of these modeling limitations (though some modeling approaches for very liquid thin film flows are available and described in Appendix A1), the unsteady problems considered here start
at a time \((t = 0)\) for which one has a sufficiently thick arbitrary initial guess. One starts at the steady solution, typically with an initial disturbance over the steady solution, e.g.,

\[
\delta(x, 0) = \delta_{\text{steady}}(x) + \delta'(x, 0) \quad \text{with} \quad \delta'(x, 0) \neq 0,
\]

where the modeling for steady/unsteady governing equations and boundary conditions are well established and known. From there, one obtains the “natural” large time \((t \to \infty)\) steady/quasi-steady wavy solutions with the help of the unsteady equations and assumed continued presence of steady boundary conditions (or, for pulsatile cases, one may change to suitable steady-in-the-mean but unsteady boundary conditions). Here, if \(\psi \; \psi(x,y,t)\) is any variable (such as \(u_i, v_i, \pi_i, \theta_i\), etc.), the initial values of \(\psi\) and non-dimensional film thickness \(\delta(x,t)\) are such that:

\[
\psi(x,y,0) = \psi_{\text{steady}}(x,y) \quad \text{or} \quad \psi_{\text{guess}}(x,y)
\]

\[
\text{and} \quad \delta(x,0) = \delta_{\text{steady}}(x) + \delta'(x,0) \quad \text{or} \quad \delta_{\text{guess}}(x)
\]

where \(\psi_{\text{guess}}\) and \(\delta_{\text{guess}}\) are respectively reasonable but arbitrary initial guesses whereas \(\psi_{\text{steady}}\) and \(\delta_{\text{steady}}\) are solutions of the governing equations obtained by dropping all time dependencies in Eqs. (2.2) – (2.11) and solving the resulting steady equations for parabolic steady boundary conditions (i.e. by not imposing any exit conditions except a “reference” pressure or computed cross-sectional “reference” pressure variation).
Chapter 3. Computational Approach

The computational approach consists of two parts – obtaining a steady solution and obtaining the unsteady solution for finding features of wave propagation on the steady solution. The waves arise from arbitrary non-zero initial disturbance superposed on the steady solution at a certain initial time. Though only the steady/unsteady solution approach for annular/stratified condensing flow is described here, the approach described here also works - by changing specifications for the wall temperature $T_w(x_p)$ - for adiabatic thin/film flows as well as annular flow boiling dealing with cases involving suppressed nucleation ([1, 43]).

For the steady solution approach, though not necessary, a quasi one-dimensional (1-D) computational approach ([28]) for annular/stratified condensing flow in this geometry may be used to provide initial guesses of the interface location, interfacial velocity, etc. as a function of $x$. A good initial choice expedites convergence of two-dimensional (2-D) steady solution approach described in [27].

The 2-D steady computational tool is described in [27] will not be separately described for obtaining steady solutions of the steady boundary value problem shown in Figure 2.2. It will be assumed that this solution can be obtained by the unsteady procedure described below. This is because, in the unsteady algorithm description given below, if the initial condition is replaced by an arbitrary initial guess and forward marching from time “$t$” to “$t+\Delta t$” is treated merely as steps for improving the initial guess, the steady governing equations’ solution is naturally obtained. With this understanding of intermediate solutions along with dropping of all the unsteady terms in the mathematical formulation given in Chapter 2, the unsteady algorithm described below also yields the steady solution. Alternatively, over the stable annular portion of the flow domain, the same steady solution is obtained from an arbitrary initial guess by implementing the unsteady algorithm and letting the time $t\rightarrow \infty$.

The simulation capability uses an approach of separately solving, on COMSOL, the unsteady (or steady) liquid and vapor domain governing equations over their respective
domains that result from an assumed "sharp" interface location. For the unsteady algorithm, typically \( t = 0^- \) interface location correspond to a converged steady solution and \( t = 0^+ \) interface location is known \( (\Delta(x^p, 0) = \Delta_{\text{steady}}(x^p) + \Delta'(x^p, 0) \) due to a known prescribed initial disturbance function \( \Delta'(x^p, 0) \). While accurately locating the interface at time “t,” the unsteady single domain solution approach retains all the unsteady terms in the governing equations (including interface conditions) except that the geometries for the two domains, as determined by periodically updated new location of the interface, are treated to be “fixed” between any one iteration for a forward time march associated with a short time interval of time \([t, t+\Delta t]\). The interface location for time “\( t+\Delta t \)” is thus iteratively relocated until convergence – this is achieved by tracking the interface with the help of numerical integration of its evolution equation along characteristics curves associated with it. With \( t_j \equiv j.\Delta t \) (where \( j \geq 0 \) is a positive integer) for the discretization of time, the Eulerian interface tracking procedure employs a special fixed spatial grid for a set of instants (which includes the current, three past, and one future instant), locates the time dependent characteristics curves on this grid (Figure 3.1), and temporally changes this x-t grid when the current time “\( t = t_j \)” is advanced to the future instant “\( t+\Delta t = t_{j+1}. \)” The moving grid is spatially fixed and is the same for current time \( t = t_j \), three past instants, and one future instant of time – viz. \( \{t_{j+1}, t_j, t_{j-1}, t_{j-2}, t_{j-3}\} \). The grid is used to mark the locations of points on the representative characteristic curve \( C(0)|_{i,4} \) emanating from the representative grid point associated with points \( P' \) at \( t = t_{j-1} \). For the implementation in this work, equi-spaced discretization of \( x \) (with \( e \leq x \leq L \)) at \( t = t_{j-1} \) is obtained as \( x_i \equiv e + i.\Delta x_{\text{fg}}(j) \) with positive integers \( i \geq 0 \). However, unequal spacing can be implemented and may be needed for some longer length problems. The uniform fixed grid (fg) spacing \( \Delta x_{\text{fg}}(j) \) has a superscript “\( j \),” this is to denote a space-discretization scheme (Figure 3.1) which will change when the current time \( t = t_j \) is changed. The same grid is used for reference purposes, in Figure 3.1, for points on the characteristics at all the instants \( \{t_{j+1}, t_j, t_{j-1}, t_{j-2}, t_{j-3}\} \) of interest.
On this spatially fixed grid, the “\(t+\Delta t = t_{j+1}\)” values of the interface location \(\delta(x, t)\) are interrogated and predicted at characteristic-curves’ tip points \(Q\) on each curve \(C^{(j)|_{4th}}\). This arrangement, to be further discussed, allows the proposed algorithm to correctly capture the nature of wave propagation on the interface.

Figure 3.1: Schematic showing the \(x-t\) grid associated with a given time \(t = t_{j-1}\). It also shows the characteristic curves \(C^{(j)|_{1st}}\) and \(C^{(j)|_{4th}}\) used in the computational algorithm. Only the \(x\)-axis of the indicated waves at times \(t = t_{j-3}\) and \(t_{j+1}\) belong to this figure. The wave amplitudes are along \(y\)-axis and should not be confused with the \(t\) axis in the figure.

Concurrent to each new iteration associated with improved prediction of the interface location for “\(t+\Delta t = t_{j+1}\),” updated information on the flow field is obtained by solving the unsteady liquid and vapor domain governing equations. This allows determination of new
suitable values of interfacial variables (speeds, stresses, temperatures, etc.) that are imposed at the two sides of the interface as boundary conditions appearing in the two separate single-phase formulations (on COMSOL’s CFD modules) associated with vapor and liquid domains – as needed for the implementation of the algorithm in Chapter 4. After obtaining the CFD solutions, the earlier assignments of interfacial values of the requisite flow variables are iteratively updated in the computations until converged solutions satisfying all but one of the well-known unsteady interface conditions (as described in Chapter 2) are obtained for the time “t+Δt.” COMSOL’s Finite Element Method (FEM) based fluid flow and heat transfer modules are used for separately solving the liquid and vapor domain governing equations in the interior while the two solutions are made to talk to one another, along with judicious but concurrent interface re-locations, with the help of algorithms implemented on MATLAB (that links with COMSOL). Interface evolution equation is a wave equation which arises from one remaining interface condition and is solved (with the help of the well-defined characteristics equation underlying this problem) with 4th order accuracy in time-step ∆t. The successful implementation of this approach utilizes a known fact that accurate prediction of interfacial wave’s phase is possible (without additional refinement of the x-t grid in Figure 3.1) by interrogation of δ(x, t + Δt) values at characteristic-curves’ tip points Q on each curve $C_{jth}^{(b)}$. This fact is also supported by “amplitude and phase” stability analyses of the finite-difference discretization of similar interface wave-equations ([10]).

Once the steady solution is given an initial disturbance at $t = 0$ (e.g. $δ(x, 0) = δ_{\text{steady}}(x) + δ'(x, 0)$ with $δ'(x, 0) ≠ 0$), an unsteady solution is obtained for $t > 0$. The simulation tool locates an interface ($φ(x, y, t) ≡ Φ/h = y - δ(x,t)$) by solving the interface tracking equation arising from the following part of interfacial mass-flux equalities in Eq. (A.13):

$$\dot{m}_{LK}^p = \dot{m}_{\text{Energy}}^p$$

(3.1)

Using Eqs. (A.8) and (A.9), this requirement is rewritten in the well-known popular interface evolution equation form:
\[
\frac{\partial \Phi}{\partial t^p} + \mathbf{v}_{{\text{eff}}} \cdot \nabla \Phi = 0
\]  

(3.2)

where \( \mathbf{v}_{{\text{eff}}} = \mathbf{v}_1^{\text{pli}} - (k_1 \nabla T_1 |^i - k_2 \nabla T_2 |^i) \cdot \left( \frac{1}{\rho_{1\text{fg}}} \right) \) is the modified velocity vector determined by the liquid interfacial velocity \( \mathbf{v}_1^{\text{pli}} \) and liquid and vapor temperature gradients at the interface \((\nabla T_1 |^i \text{ and } \nabla T_2 |^i)\).

The above equation is currently solved by a 2D interface tracking method for an explicit definition/representation of \( \phi \) given as: \( \phi \equiv y - \delta(x,t) = 0 \) for the unsteady case and \( \phi \equiv y - \delta_{\text{steady}}(x) = 0 \) for the steady case. In future, as outlined in section 8, an enhanced form of this interface tracking method can be carried out – at least for scientific calculations - by retaining an implicit representation of \( \phi \), through 2-D/3-D versions of the level-set type techniques ([44]) adapted for this type of special spatially “fixed” but temporally moving-grid employment for a method of characteristics solution.

### 3.1. Interface Tracking

The interface is tracked by solving a reduced non-dimensional form of Eq. (3.1) that uses \( \phi(x, y, t) \equiv \phi/h = y - \delta(x, t) = 0 \). This reduced form is given by:

\[
\frac{\partial \delta}{\partial t} + \bar{u}(x,t) \frac{\partial \delta}{\partial x} = \bar{v}(x,t)
\]  

(3.3)

where \( \bar{u}(x,t) \) and \( \bar{v}(x,t) \) definitions are:

\[
\bar{u} \equiv u_1^i + [Ja/(Re_1 \cdot Pr_1)] \frac{\partial \theta_1}{\partial x} |^i
\]  

and \( \bar{v} \equiv v_1^i + [Ja/(Re_1 \cdot Pr_1)] \frac{\partial \theta_1}{\partial y} |^i
\]

(4.4)

The domain for Eq. (3.3) is \( x \geq 0 \) and \( t \geq 0 \) and the boundary conditions are: \( \delta(0, t) = 0 \) (which is approximately \( \delta(\epsilon, t) \equiv \epsilon^* \) where \( \epsilon \) and \( \epsilon^* \) are very small fixed numbers) for all \( t \geq 0 \); and \( \delta(x,0) = \delta_{\text{guess}}(x) \), or \( \delta_{\text{Steady}}(x) \), or \( \delta_{\text{Steady}}(x) + \delta'(x,0) \) for \( x \geq \epsilon \). Known non-zero
initial disturbances $\delta'(x, 0)$ (with different spatial frequency contents) are used for unsteady simulations dedicated to stability analysis of the steady solutions.

This interface evolution equation (Eq. 3.3) is a wave equation, which is a first order hyperbolic partial differential equation (PDE). The equation is quasi-linear because $\bar{u}(x,t)$ and $\bar{v}(x,t)$ are implicitly and non-linearly dependent on $\delta(x,t)$. This is because, as seen later on, only after $\delta(x,t)$ is obtained for a new time “t+\Delta t,” can CFD/CHT yield $\bar{u}$ and $\bar{v}$ in Eq. (3.3) for that new time. From known conditions at $t = 0$, Eq. (3.3)’s characteristics $x = x_c(t)$, characteristic speed $\bar{u}$, and forcing function $\bar{v}$ can also be concurrently determined by marching forward in time.

The solution of the interface evolution Eq. (3.3) is obtained along the characteristic curves $x = x_c(t)$ given by:

$$\frac{dx_c}{dt} = \bar{u}(x_c(t), t)$$

(3.5)

Use of Eq. (3.5) in Eq. (3.3) transforms and yields the equation governing the evolution – along characteristic curves $x_c(t)$ - of the unknown interface $y = \delta(x, t)$ in the form:

$$\frac{d}{dt}\{\delta(x_c(t), t)\} = \bar{v}(x_c(t), t)$$

(3.6)

where $t \geq 0$.

Simple numerical integrations of Eqs. (3.5)-(3.6) from $t_j = t_3$ (since initial disturbance condition at $t = t_0 = 0$ in Eq. (2.11) is numerically modeled as a known disturbance imposed over $0 \leq t \leq t_3$) to future times $t_j$, $j \geq 4$ is possible. The outline of this algorithm is described next – although the actual step-by-step algorithm is given in Chapter 4.

Two 4th order Runge-Kutta numerical integration schemes (denoted as RK4-1 and RK4-2) are used for integrating Eq. (3.5). For predicting the projected locations, at $t = t_{j+1}$, i.e. points Q on characteristic-curves $C^{(j)}_{|i|4th}$ in Figure 3.1, the starting locations $P'$ (or $x = x_i$) associated with $t = t_{j-1}$ is used to generate the equi-spaced $x_i$ grid. The same equi-spaced
grid is used for other times. For locating \( t = t_{j+1} \), point \( Q \) which is “2·\( \Delta t \)” time-step forward from \( t = t_{j-1} \) point \( P' \), the following fourth order Runge-Kutta discretization scheme (termed \( \text{RK4-2} \)) is used:

\[
x_c(t_{j+1}) = x_c(t_{j-1}) + \frac{2\Delta t}{6} \left\{ K_1 + 2K_2 + 2K_3 + K_4 \right\} + O(\Delta t^4)
\]

where \( K_1 = \bar{u}(x_c(t_{j-1}), t_{j-1}) \)

\[
K_2 = \bar{u}(x_c(t_{j-1}) + \Delta t \cdot K_1, t_{j})
\]

\[
K_3 = \bar{u}(x_c(t_{j-1}) + \Delta t \cdot K_2, t_{j}) \quad \text{and}
\]

\[
K_4 = \bar{u}(x_c(t_{j-1}) + 2\Delta t \cdot K_3, t_{j+1})
\]

Because \( \bar{u}(x, t) \) at \( t = t_{j+1} \) appearing in the \( K_4 \) term on the right side of Eq. (3.8) is not yet known, use of the \textit{implicit} scheme in Eq. (3.7) is temporarily deferred until an explicit estimate (based on first order integrations of Eqs. (3.5)-(3.6)) of \( \bar{u}(x, t) \) at \( t = t_{j+1} \) becomes available.

The 1\textsuperscript{st} order scheme based integrations of Eqs. (3.5)-(3.6), for the locations \( x_i \) in Figure 3.1, are carried out along different \( x_c(t) \) curves denoted as \( C^{(j)}_{|i|\text{1st}}. \) First, in Figure 3.1, points \( R \) (at \( t = t_i \)) associated with the \( x_i \) coordinate of point \( P' \) and then point \( S \) at \( t = t_{j+1} \) located on the same grid. This is followed by obtaining values of \( \delta(x, t_{j+1}) \) at points \( S \). Introducing the notations \( \delta(t) \equiv \delta(x_c(t), t) \) and \( \bar{u}(t) \equiv \bar{u}(x_c(t), t) \), the explicit first order forward march schemes used for this “seeding” purpose are:

\[
\Delta x^{(j)}_{RS} \equiv x_c(t_{j+1}) - x_c(t_j) = \Delta t \cdot \bar{u}(x_c(t_j), t_j) + O(\Delta t^2)
\]

\[
= \Delta t \cdot \bar{u}(t_j) + O(\Delta t^2)
\]

\[
\hat{\delta}(t_{j+1}) \equiv \delta(x_c(t_{j+1}), t_{j+1}) = \hat{\delta}(t_j) + \hat{\nu}(t_j) \cdot \Delta t + O(t^2)
\]

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Once tentative first order estimates of $\delta(x, t_{j+1}) \approx \delta^{(0)}(x, t_{j+1})$ are obtained from Eqs. (3.9)-(3.10), the revised $\bar{u}(x, t)$ and $\bar{v}(x, t)$ estimates for $t = t_{j+1}$ are obtained from COMSOL CFD solver as per algorithm in Chapter 4. Then, the implicit scheme in Eq. (3.7) is used for obtaining the locations of $x_c(t_{j+1})$, or points Q in Figure 3.1. This is followed by use of the following 4th order 4-intervals and 5-points Simpson rule ([45]) for numerical integration of Eq. (3.6).

$$\delta(t_{j+1}) - \delta(t_{j-3}) = \int_{t_{j-3}}^{t_{j+1}} \hat{v}(\tau) \, d\tau \tag{3.11}$$

$$\approx \frac{\Delta t}{3} \left[ \hat{v}(t_{j-3}) + \hat{v}(t_{j+1}) + \frac{2\Delta t}{3} \hat{v}(t_{j-1}) \right] + \frac{4\Delta t}{3} [\hat{v}(t_{j-2}) + \hat{v}(t_j)] + O(\Delta t^4)$$

The estimates of $\delta(t_{j+1})$ in Eq. (3.11) is made possible because $\bar{u}(x, t)$ and $\bar{v}(x, t)$ values are known for times $\{t_j, t_{j-1}, t_{j-2}, t_{j-3}\}$. The values of $\hat{v}(t)$ in Eq. (22) are needed at points Q, P, P', P'', and P''' on curves $C^{(i)}|_{i\text{th}}$. Starting from the most recent known locations of points Q, backward RK4-1 (an explicit single time-step “$\Delta t$” based 4th order scheme available on MATLAB [46]) numerical integration of Eq. (3.5) is used to obtain the approximate locations of points P, P', P'', and P''' on curves $C^{(i)}|_{i\text{th}}$. Now, with the use of $\hat{v}$ values at these locations, $\delta(x_c(t_{j+1}), t_{j+1})$ in Eq. (3.11) is obtained for time $t_{j+1}$ at $x_c(t_{j+1})$ location given by Eq. (3.7). The resulting $\delta$ values associated with points Q are assembled to yield a 4th order estimate of $\delta(x, t_{j+1}) \approx \delta^{(4)}(x, t_{j+1})$. This (at step (vi) in the algorithm in Chapter 4) allows repeated CFD solutions for the liquid-vapor domains obtained with the interface at $\delta(x, t_{j+1}) \approx \delta^{(4)}(x, t_{j+1})$ followed by new estimates of the location of points Q and $\delta^{(4)}(x, t_{j+1})$. The process is repeated until converged values of $x_c(t_{j+1})$ through Eq. (3.7) and $\delta(x_c(t_{j+1}), t_{j+1})$ through Eq. (3.11) have been obtained by the mixed “explicit-implicit” 4th order method described above.
For convergence of the above discretization schemes for locating curves $C^{(j)}_{i=4th}$ in Figure 3.1 and obtaining $\delta(x_c(t_{j+1}), t_{j+1})$ at points Q, naturally there are restrictions on time-step size $\Delta t$ and space-discretization size $\Delta x_{tg}^{(j)}$ for the “spatially fixed” moving-grid in Figure 3.1. These restrictions are discussed in Chapter 4.

After converged solution for $t = t_{j+1}$ have been obtained, a new x-t grid of the type shown in Figure 3.1 is created for forward marching to $t_{j+2}$. This is easily accomplished by repeating the earlier procedure using the available solutions at $t = \{t_{j+1}, t_j, t_{j-1}, t_{j-2}\}$ and using a new equi-spaced grid for the $t_j$ location (which is at a temporal distance “2.$\Delta t$” behind $t_{j+2}$.)
Chapter 4. Computational Algorithm

4.1. Solution Algorithm Details

Using “initial disturbance” information (to be described) available at times \( t_j \) \((0 \leq j \leq 3)\), the solution algorithm for \( j \geq 4 \) is as follows:

i) At discrete number of spatial \( x \)-locations, an initial guess of interface variables - \( \delta \), \( t_1^i \), \( t_2^i \), \( \theta_1^i \), \( u_2^i \), \( v_2^i \), \( \theta_2^i \) for the unsteady problem are made at \( t = t_j = t^* \) (starting with \( j = 3 \), i.e. \( t = t_3 = 3 \cdot \Delta t \)).

ii) With the known values for the interface location \( \delta(x, t) \) and other flow variables available at the current and past instants given by \( t = t^*, t^* - \Delta t, t^* - 2\Delta t, t^* - 3\Delta t \), a preliminary first order approach based prediction of the interface location for \( t = t^* + \Delta t \), associated with moving \( x-t \) grid is made. This is done on a specially defined \( x \)-discretization for the moving grid, as defined in Chapter 3, to locate the characteristics-curve \( (C^{(1)}_i)_{1st} \) in Figure 3.1) from a first order solution \( x = x_c(t) \) of Eq. (3.5). Once an estimated \( \delta(x, t^* + \Delta t) \approx \delta^{(0)}(x, t^* + \Delta t) \) is obtained, all known flow variable values at time \( t^* \) under the previous \( \delta(x, t^*) \) location of the interface are mapped, to begin with, by arbitrary stretching of the original domains (defined by \( 0 \leq y \leq \delta(x^*, t^*) \) and \( \delta(x^*, t^*) \leq y \leq h \)) to the new domains (defined by \( 0 \leq y \leq \delta(x^*, t^* + \Delta t) \) and \( \delta(x^*, t^* + \Delta t) \leq y \leq h \)). With this new predicted interface location and mapped values of flow variables, COMSOL is to be used to solve the liquid and vapor domain problems.

iii) Liquid domain problem is solved on COMSOL using the unsteady Laminar Flow model under the Single-Phase Flow model and the Heat Transfer in Fluids model. The solution uses, at the interface, the time \( t = t_j \) stress boundary conditions -i.e. axial \( (t_1^i) \) and normal \( (t_2^i) \) stress vector components, and saturation conditions (Eq. (2.10)) for the temperature \( \theta_i^j \). Besides other solution variables for time \( t = t_{j+1} \), the converged solution is post-processed on MATLAB to obtain \( u_1^i \), \( v_1^i \), and \( \theta_1^i \) values for time \( t = t_{j+1} \).

iv) Using the \( t = t_{j+1} \) liquid domain solution \{\( u_1^i \), \( v_1^i \), \( \theta_1^i \)\} values from step (iii), vapor domain interfacial velocity components \( u_2^i \) and \( v_2^i \) are obtained from the continuity of tangential
velocity (Eq. 2.3) and the remaining interfacial mass flux equality \( \dot{m}_{LK} = \dot{m}_{VK} \) in Eq. (2.9). Non-dimensional temperature \( \theta_2 \) at \( t = t_{j+1} \) is obtained from using saturation conditions (Eq. (2.10)) for the temperature at the vapor interface.

v) Using the known values of the vapor flow interfacial variables \{u_2, v_2, \theta_2\} at times \( t = t_j \) and \( t_{j+1} \), and for the \( t = t_{j+1} \) location of the interface \( \delta \), the vapor domain is solved on COMSOL using the unsteady Laminar Flow model. The solution is post-processed to obtain values of axial \( (t_{2x}) \) and normal \( (t_{2y}) \) stresses at the vapor interface. For suitably chosen and blended parts of the vapor domain away from the interface, the Turbulence Flow model can also be used - but such an algorithm is not implemented in this paper.

vi) The momentum-balance conditions at the interface and the computed \( t = t_{j+1} \) values of vapor domain interfacial stress-vector components are used together to obtain the new updated values of interfacial stress-vector components \( (t_{1x}, t_{1y}) \) at the liquid domain interface. This is done with the help of proper processing of Eqs. (2.4)-(2.6) - or their equivalent forms in the Appendix.

vii) With the known values for the interface location \( \delta(x, t) \) and other flow variables available at the time instants given by \( t = \{t^* + \Delta t, t^* - \Delta t, t^* - 2\Delta t, t^* - 3\Delta t\} \), an implicit 4th order approach described in Eq. (3.11) of Chapter - 3 is used to refine the prediction of \( \delta(x, t^* + \Delta t) \). For the same time \( t^* + \Delta t \), the 4th order approach of Chapter - 3 is also used to locate the characteristics-curve \( (C_j)_{4\text{th}} \) in Figure 3.1) and to obtain better estimates of \( \delta(x, t^* + \Delta t) \) values at the “\( t^* + \Delta t \)” tips (points Q) of the characteristics-curves in Figure 3.1. Again, with this new predicted interface location \( \delta(x, t^* + \Delta t) \approx \delta^{(4)}(x, t^* + \Delta t) \) and mapped values of flow variables for this interface location, steps (ii) - (vi) are repeated to obtain unsteady CFD solutions for the updated interface location.

viii) The process in steps (i)-(vii) is repeated until good and convergent estimates of the interface location and flow variables are obtained for the time \( t = t^* + \Delta t \).
ix) Subsequent repetition of the steps (ii) to (viii), with \( j = j + 1 \), yields converged interface locations as well as CFD solutions for liquid and vapor domains for each \( t_j = j \Delta t \) for \( j = 5, 6, \) etc.

**Initial Disturbance**

For the stability analysis of this approach, the initial disturbance – is defined for instants \( \{t_0, t_1, t_2, t_3\} \) – and this is done somewhat differently than the above. Instead of instantaneously imposing the initial disturbance at \( t = 0 \) (i.e., between \( t = 0^- \) and \( 0^+ \) with \( \delta(x,t) = \delta_{\text{steady}}(x) \) for \( t < 0 \) and \( \delta(x,t = 0^+) = \delta_{\text{steady}}(x) + \delta'(x,0) \)), the initial disturbance is imposed at \( t_0 = 0 \) and then the disturbance is defined over a small finite time interval \( [t_0, t_3] \). A meaningful but arbitrary initial “disturbance” is one which enables the ability to satisfy governing equations for \( t \geq t_j \) \( (j \geq 4) \). Such a “disturbance” is obtained here by implementing steps (i) – (vi) – i.e. omitting steps (vii) – (ix) – and implementing first order marches to times \( t = \{t_1, t_2, t_3\} \). There is no particular attention is given to “convergence” at times \( t = \{t_1, t_2, t_3\} \).

4.2 Grid-size Restrictions and Convergence Criteria

Because of the coarseness of \( \Delta x_{fg}^{(j)} \) relative to CFD mesh-size, as per discussions in the next section, smooth curve fitting operations available in [46] can be used (if needed) to generate differentiable \( \delta(x,t) \) curves on the basis of discrete \( \delta(x_c(t_{j+1}|Q,t_{j+1}) \) data obtained by the CFD approach. These smooth spline-fitted curves were used to obtain curvature terms for the evaluation of surface tension term appearing in the normal stress condition at the interface. Furthermore, for obtaining accurate spatial Discrete Fourier Transforms (DFT) with the help of direct techniques available on [46], these smoothed curves for \( \delta(x,t) \), etc. were used by transferring their values to suitable alternative spatial x-grids (often equi-spaced).

Using DFT, the dominant spatial and temporal frequencies in \( \delta(x,t) \cong \delta(x_i(t_j), \tilde{u}) \), etc. can be obtained for their values at \( t = \{t_j, t_{j-1}, t_{j-2}, t_{j-3}\} \) and the smallest spatial wave-length \( \lambda_x < L \) and temporal frequency \( f_T < 1/T \) that needs to be resolved – as per fluid-physics constraints - are ascertained. Though these frequencies can be assessed for prediction of
flow variables over $0 \leq x \leq L$ and $0 \leq t \leq T$, for the simulations reported here, it sufficed to ascertain the fluid physics constraints on $\lambda_x$ through the smallest dominant wave-length present in $\delta'(x, 0)$ and a representative frequency $f_T$ as $\bar{u}_{\text{mean}}|_{[0,L]}/\lambda_x$. Based on these considerations, spatial discretization values $\lambda_x/6 < \Delta x_{fg}^{(j=3)} < L/2$ and temporal discretization $1/6* f_T < \Delta t < 2/T$ were chosen. These choices typically satisfied the requirements of the lowest and the highest spatial and temporal frequencies (see Nyquist criteria [46]) that can be resolved and are of interest in the reported numerical solutions.

The above fluid-physics based values of $\Delta x_{fg}^{(j)}$ (e.g. $\sim 0.0002$ m for the case in Figure 4.1) are typically to be larger than the length scale $\Delta s^*$ chosen for the fine mesh-sizes that are needed for convergence of CFD solutions on COMSOL (in Figure 5.1, $\Delta s^*$ is less than the smaller of the two $\Delta s^L$ and $\Delta s^V$ mesh sizes needed for liquid and vapor domain CFD convergence). As a result of this, the CFD obtained x-variations data for $\delta(x,t)$, $\bar{u}(x,t)$, and $\bar{v}(x,t)$ can be decimated (with smoothing, if needed) for use on the coarser interface-tracking x-t grid (Figure 3.1) on MATLAB. Note that, because of micro-meter scale condensate thickness in mm-scale condensers, $\Delta s^L$ and hence $\Delta s^*$ has to be quite small – and this also adds to convergence time (as discussed later).

The choice of time-step $\Delta t$ has several other restrictions and these are discussed next.

For interface tracking, the most critical restriction on time-step size $\Delta t$ is associated with the first order explicit forward time-march discretization scheme in step-(ii) above – the step’s algorithm is also discussed has been discussed in Chapter - 3. This restriction for first order “seeding” discretization of Eq. (3.3) also requires that the Courant number $C_{ri}^{(j)} \equiv \bar{u}(x_i,t_j). \Delta t/\Delta x_{fg}^{(j)}$ associated with any time-step $t_j$ and locations $x = x_i$ satisfy the CFL criteria ([10]):

$$C_{ri}^{(j)} \equiv \bar{u}(x_i,t_j). \Delta t/\Delta x_{fg}^{(j)} < 1$$ (4.1)

Note that $\bar{u}$ values are typically dominated by their steady values and these values, for the problems considered here, monotonically increase with $x$ over $0 \leq x \leq L$. Representative
calculations of $C_{r_i}^{(j)}$ for a representative problem are shown in Figure 4.1. The $C_{r_i}^{(j)}$ versus $x_i$ curves in Figure 4.1 show two “good” choices of $\Delta t$, viz. $\Delta t_1$ and $\Delta t_2$, that satisfy Eq. (4.1) and one “non-convergent” choice that does not satisfy Eq. (4.1). Though it is not implemented here, by choosing unequal and increasing spacing $\Delta x_{fg}^{(j)}$ for the x-grid at $t = t_{j-1}$ (as opposed to equal spacing discussed in Chapter - 3), it is possible to choose larger $\Delta t$ values and yet satisfy Eq. (4.1) at larger more uniform values of $C_{r_i}^{(j)}$, say $C_{r_i}^{(j)} \approx 0.8$.

![Figure 4.1: Plot showing the Courant number variation based on definitions in Eqs. (4.1)-(4.2) along the length of the channel for a representative case.](image)

The next important and beneficial feature (which is not a requirement) of this algorithm is that once the “seed” requirement in Eq. (4.1) is met, there are no other restrictions on how “fine” the $\Delta x_{fg}^{(j)}$ in Fig. 4 must be to faithfully capture the interfacial wave dynamics – both in amplitude and phase. This is possible because the time-step march from $t = t_j$ to $t = t_{j+1}$ in Figure 3.1 is such that, for every $x = x_i$, the x-distance $\Delta x_{PQ}^{(j)}$ between points P and Q define Courant numbers $C_{r_{PQ}}^{(j)} \equiv \bar{u}(x_i,t_j). \Delta t/\Delta x_{PQ}^{(j)} \approx 1$ which satisfy:

$$C_{r_{PQ}}^{(j)} \equiv \bar{u}(x_i,t_j). \Delta t/\Delta x_{PQ}^{(j)} \approx 1$$

The special feature of Eq. (4.2) is essential to achieve convergence in interfacial waves’ amplitude and phase on coarse grids (also see [10]) – thereby significantly reducing convergence times associated with interface-tracking part of this algorithm. The fact that
the criteria in Eq. (4.2) is met by this algorithm is also shown by representative calculations in Figure 4.1. As discussed in the next section, this feature of the algorithm removes the well-known “numerical diffusion” issues ([see discussions in Chapter - 5 and [23]]) associated with the solution of Eq. (3.2).

As far as convergence of unsteady CFD solver in steps (iii), (v), and (vii) are concerned, again there are no restrictions on ∆t. This is because COMSOL’s unsteady solves uses its own in-built optimal sub-divisions of ∆t to obtain convergent solutions for their temporarily fixed vapor/liquid domains – provided spatial mesh refinements are adequate (this is discussed in Chapter - 5).

The use of sophisticated COMSOL solvers in steps (iii), (v), and (vii) always lead to convergent solutions on a sufficiently refined CFD mesh, however these are the most time consuming operations in the above described algorithm. Our own interface-tracking algorithm on MATLAB, relative to the in-built COMSOL operations, take comparatively negligible time.

4.3. Solution Representation Formats

The numerical solution of the steady and unsteady condensing flow problem depicted in Figure 2.1 (and formulated in Chapter - 2) is obtained as per the above described procedures. In this dissertation, specific sample solutions are presented in physical variables to give a clear idea of the fluid used and the physical dimensions involved. However generalized correlated results representing solution variables of interest are presented in non-dimensional forms - with clearly marked boundaries of the non-dimensional parameter-space covered by the solutions underlying the correlations. Clearly, specifications of: the fluid, inlet pressure p₀, channel height h, channel inclination α, bottom wall cooling conditions (e.g. specification of wall temperature $T_w(x^0) < T_{sat}(p_0)$), and inlet mass flow rate per unit channel width (or average inlet speed U) define the steady problem. The unsteady problems are defined by evolution of initial disturbance that are imposed on the steady solution.
The physical variables are non-dimensionalized as per the relationships specified in Eq. (2.1). An inspection of all the non-dimensional governing equations, interface conditions, and boundary conditions reveal the fact that the steady and unsteady flows considered here are parabolic and are affected by the following set of non-dimensional parameters:

\[
\{ \text{Re}_{\text{in}}, J_a, F_{r-x}^{-2}, F_{r-y}^{-2}, \frac{\rho_2}{\rho_1}, \frac{\mu_2}{\mu_1}, Pr_1, We \} 
\]

where \( \text{Re}_{\text{in}} \equiv \rho_2 u_h / \mu_2 \), \( J_a \equiv C_{p1} \Delta T / h_{fg} \), \( Pr_1 \equiv \mu_1 C_{p1} / k_1 \), \( F_{r-x}^{-2} \equiv g x h / U_2^2 \), \( F_{r-y}^{-2} \equiv g y h / U_2^2 \), and \( We \equiv \rho_1 U^2 h / \sigma \). Note that the variable \( "J_a/Pr_1" \) in the set on the right side of Eq. (4.3) replaces the variables \( J_a \) and \( Pr_1 \) appearing on the left side of the equation. This is because, for mm-scale condensers considered here, the liquid condensate is \( \mu \text{m}-\)scale thick and one can ignore the liquid flows’ convection term appearing in the differential form of its energy equation (see Eq. (2.2)).

The inlet pressure \( p_0 \) is important in determining the fluid properties appearing in the definition of the non-dimensional numbers in Eq. (4.3).

It should be noted that the solution obtained for any particular non-dimensional form of a flow variable such as interfacial shear, interfacial speed, wall heat-flux, etc. may depend, in a non-linear way on the parameters in Eq. (4.3) along with non-dimensional distance \( x \) and time \( t \). Other equivalent combinations of the non-dimensional numbers are possible. This is why it is not uncommon to see different non-dimensional numbers appearing in engineering correlations for heat-flux and the length of the annular regime \( x_A \equiv x_A^p / h \) (see discussions in [39]). Non-dimensional form of physical wall heat-flux \( q_{w}''(x) \equiv -k_1 \left( \frac{\partial T_1}{\partial y} \right) \bigg|_{y_p=0} \equiv h_x [T_{sat}(p_0) - T_w(x)] \) for any given wall temperature variation \( T_w(x) \) is typically defined through:
for a fixed non-dimensional function \( \theta(x) \equiv \theta_1(x, 0, t) = \frac{T_\text{sat}(x) - T_w(x)}{\Delta T} \) that represents a fixed “method of cooling” for different values of \( \Delta T = T_\text{sat} - T_w \). It is found in [39] that \( \text{Nu}_x(x) \) as well as non-dimensional annular length \( x_A \), generally depends on the parameters in Eq. (4.3) or any equivalent non-dimensional representation of these parameters that may appear to look like a different set of non-dimensional numbers. For example, it is popular to correlate \( \text{Nu}_x(x) \) as a function of quality \( X(x) \) instead of distance \( x \).

Typically the correlations (see [39]) for \( \text{Nu}_x(X) \) has a weak dependence on the spatial variation of \( T_w(x) \) for different “methods of cooling” specified by \( \theta(x) \). Despite this, the physical/raw values of \( q''_w(x) \equiv h_x[T_\text{sat}(p_o) - T_w(x)] \) may have a strong dependence on the spatial variation in \( T_w(x) \) and can easily be obtained (see [39]) from \( \text{Nu}_x(X) \) correlation.

Certain alternative combinations of the parameter set in Eq. (4.3) may more accurately specify a particular physical variable’s dependence on these parameters. For example, physics based stability analysis results of the type described in this paper may, in future, be used to define different, but more natural, combination of non-dimensional parameters that more directly affect the values of \( x_A/h \) – the length of annular regime.
Chapter 5. Convergence and Grid Independence

Accuracy ([27]) of any solution can be inferred from the following:

- The convergence and convergence rate of flow variables in the interior of each of the two single-phase fluid domains.
- Grid independence of the converged solutions.
- Satisfaction of all the interface conditions for the condensing flow problem.

5.1. Convergence and Convergence Rates

During the solution procedure, COMSOL operations indicated the requisite convergence of flow variables for parts of the algorithm where steady or unsteady single-phase fluid domain solutions were obtained under temporarily “fixed” interface locations. The spatial grid independence for these operations has been shown, for representative steady solutions, in [27]. Since the number of operations and time taken by the COMSOL CFD solver dominate those taken by steady or unsteady algorithms for any given interface location, the relevant CFD convergence rates with mesh-size (for triangular two-dimensional meshes used here) is important. For representative liquid domain and vapor domain steady CFD solutions (obtained for prescribed boundary conditions – including interface location and interface boundary-condition variables), the convergence with mesh-size are respectively shown in Figures 5.1a-b. The CFD solvers ensure convergence and grid-independence if the chosen mesh size is smaller than the smallest of the two mesh sizes $\Delta s^L$ and $\Delta s^V$ marked on Figures 5.1a-b. The representative convergence versus grid-size curves (vapor domain in Figure 5.1a and liquid domain in Figure 5.1b) for the average of x and y components of velocities were calculated at $x = 0.06$ m for the transverse gravity shear driven case in Figure 6.1. The averages are calculated for the velocity values across the cross-section of liquid and vapor domains.
Figure 5.1: Plot of variation of average values of x and y components of the velocity as a function of the average element size in: (a) vapor domain, and (b) liquid domain. The minimum values of mesh sizes $\Delta s^L$ and $\Delta s^V$ that need to selected are indicated based on the convergence shown in the flow variables. The chosen values of mesh size is marked as $\Delta s^*$. 

(a) 

(b)
5.2. **Grid Independence**

Using sufficiently refined mesh size obtained for representative steady solutions (such as the ones in Figures 5.1a-b), the temporal grid independence for the unsteady solutions is discussed next.

For a representative case, the unsteady film thickness at a specific time \( t = t^* \) is predicted for two different time-steps (\( \Delta t_1^p \) and \( \Delta t_2^p \)) such that \( t^* \approx N_1 \cdot \Delta t_1^p \approx N_2 \cdot \Delta t_2^p \) for certain integers \( N_1 \) and \( N_2 \). A representative case and its plot of the computed film thickness \( \Delta(x^p,t^p) \) is shown with the help of Table 5-1 and Figure 5.2. Besides showing the steady and disturbed solutions for \( t^p = 0^- \) and \( t^p = t^p_4 \), the plot in Figure 5.2 shows good agreement for the two solutions at time \( t^p = t^p* \approx 30 \cdot \Delta t_1^p \approx 60 \cdot \Delta t_2^p \).

<table>
<thead>
<tr>
<th>Mesh 1</th>
<th>Mesh 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_1 )</td>
<td>( \Delta t_1 )</td>
</tr>
<tr>
<td>30</td>
<td>0.01 s</td>
</tr>
</tbody>
</table>

**Table 5-1: Mesh parameters used in the unsteady grid independence study.**
Figure 5.2: Plot of unsteady film thickness values in response to an initial disturbance is obtained, by using two different time-steps, and shown for t = 0.3 s. (Run parameters: Fluid – FC 72, U = 1 m/s, ΔT = 20 °C, channel height = 2 mm)

5.3. Spatio-temporal Convergence Rates

Once the convergence criteria in Eq. (4.1) is satisfied by a chosen Δt and the mesh-size meets the smallness criteria obtained by representative values in Figure 5.1, the convergence rate is independent of Δt and the mesh-size. This is because COMSOL’s unsteady solver uses its own in-built optimal sub-divisions of “Δt” to obtain convergent solutions for these temporarily fixed vapor/liquid domains. COMSOL’s unsteady solver uses, for each phase, an internal assessment of convective time-scale restrictions arising from x and y component of velocity magnitudes it is trying to resolve for the mesh-sizes that cover the solution domains.

As a result of the above, overall convergence time is mainly determined by the unsteady COMSOL solvers’ convergence (or partial convergence) in steps (iii), (v), and (vii) of the algorithm in Chapter - 4.
5.4. Convergence, “Numerical Diffusion,” and Satisfaction of Interface-Conditions

The ability to satisfy the interface conditions is an extremely critical aspect of simulations for free-surface problems. The existing volume-of-fluid ([6, 47]), and level-set techniques ([11, 44, 48-51]), rely on convergence of CFD computations within a “finite thickness band” model of the interface to satisfy interface conditions as the approaches’ implement progressively smaller thicknesses of these “bands.” This is needed for accurately resolving fluid flow variables’ (and their derivatives’) discontinuities present on either side of the “bands.” Furthermore, these approaches’ algorithms for solving Eq. (3.2) require, for convergence, spatial mesh-sizes for the interface equation solvers (analogues of $\Delta x_{fg}^{(i)}$ sizes in this paper) to shrink. As these mesh-sizes shrink and become comparable to the “near interface” CFD mesh sizes for the liquid and vapor domains, “numerical diffusion” in locating the interface typically becomes unavoidable (at least on commercially available versions of these codes). Perhaps, as a result of this “numerical diffusion” problem, commercially available VOF and level-set methods based codes are currently unable to accurately satisfy some of the interface conditions – particularly the interfacial mass flux equalities in Eq. (2.9) – while concurrently and correctly resolving wave-dynamics (both in amplitude and phase) of interest to stability analyses.

In the “sharp” interface model and moving grid solution approach for the interface equation of this paper, there is no need of continuous reduction of $\Delta x_{fg}^{(i)}$ sizes associated with the solution of the interface equation. This and the fact that satisfaction of the remaining mass-flux equality and other interface conditions are embedded into the algorithm, the earlier described “numerical diffusion” issues are not expected for the proposed algorithm.

The equalities in Eq. (2.9) require that, at any point (e.g. point “P” in Figure 5.3) on the interface, the three different mass fluxes $\dot{m}_{LK}$, $\dot{m}_{VK}$ and $\dot{m}_{\text{Energy}}$ as obtained, respectively, from three different considerations of liquid kinematics, vapor kinematics, and thermal energy transfer rate constraints (see schematic in Figure 5.3) must be equal to one another. The representative unsteady simulation case result in Figure 5.4 shows that this paper’s
numerical approach – known to be capable of resolving the interfacial waves’ amplitude- and-phase – also allows excellent satisfaction of this vital mass-flux equality requirement. Note that accurate satisfaction of all the remaining interface conditions (not just Eq. (2.9)) described in the problem formulation of Chapter - 2 is built into the algorithm and therefore, by design, satisfaction of the remaining interface conditions are also expected. For a representative unsteady case, this is shown in Table 5-2 and Table 5-3 (see [27] for similar tabular representation for a representative steady case).

\[
\dot{m}_{vk} = -\rho_2 \left( v_2^i - v_s \right) \hat{n}
\]

\[
\dot{m}_{lk} = -\rho_1 \left( v_1^i - v_s \right) \hat{n}
\]

\[
\dot{m}_{\text{Energy}} = \frac{1}{h_{fg}} \cdot k_1 \left[ \partial T_1 / \partial n \right]^i
\]

\[\text{Inlet} \quad \uparrow \quad \text{Vapor} \quad \downarrow \quad \text{Liquid} \quad \downarrow \quad \text{Condensing Surface}\]

\[\text{Velocity,}U\]

\[\text{Interface}\]

\[\text{Pressure,}P\]

\[\text{Figure 5.3: Schematic of a condensing flow showing the method of computing the interfacial mass fluxes (kg/m}^2\text{-s) based on liquid kinematics (}\dot{m}_{lk}\text{), vapor kinematics (}\dot{m}_{vk}\text{), and energy constraints (}\dot{m}_{\text{Energy}}\text{).}\]
Figure 5.4: Spatial variations resulting from three different interfacial mass flux (kg/m²-s) expressions for a specific time instant $t = 0.15$ s. The results show very good agreement among the three mass-fluxes. (Run parameters: Fluid – FC72, $U = 1$ m/s, $\Delta T = 20$ °C, $h = 2$ mm)

Figure 5.5 shows the satisfaction of the requirement of continuity of tangential velocity at the interface as required by Eq. (2.3). Figure 5.6 shows the satisfaction of the requirement of the normal component of momentum balance at the interface for thick films as given by Eq. (2.4). These figures shows excellent satisfaction of the interface conditions.
Figure 5.5: Spatial variations of the requirement of continuity of tangential velocities at a specific time instant $t = 0.15$ s. The results show very good agreement. (Run parameters: Fluid – FC72, $U = 1$ m/s, $\Delta T = 20$ °C, $h = 2$ mm)

Figure 5.6: Spatial variations of the requirement of normal component of momentum balance in Cartesian coordinates at a specific time instant $t = 0.15$ s. The
results show very good agreement. (Run parameters: Fluid – FC72, U = 1 m/s, ΔT = 20 °C, h = 2 mm)

Accurate satisfaction of all the interface conditions described in the problem formulation of Chapter - 2 have been accomplished. For a representative case, Table 5-2 shows the satisfaction of these conditions at different locations along the length of the channel at particular time instant \( t = t^* = 0.15 \) s and Table 5-3 shows the satisfaction of these conditions at different instants of time at a particular distance from the inlet at \( x = x^* = 0.05 \) m.

Table 5-2: Table showing the satisfaction of interface variables for different locations along the length of the condenser at \( t = 0.15 \) s. (Run parameters: Fluid – FC72, U = 1 m/s, ΔT = 20 °C, h = 2 mm)

<table>
<thead>
<tr>
<th>Location x [m]</th>
<th>( \dot{m}_{\text{LK}} ) [kg/m²s]</th>
<th>( \dot{m}_{\text{VK}} ) [kg/m²s]</th>
<th>( \dot{m}_{\text{Energy}} ) [kg/m²s]</th>
<th>Max % Diff.</th>
<th>( u_i ) [m/s]</th>
<th>( u_i + \ldots ) [m/s]</th>
<th>% Diff.</th>
<th>( p_i ) [Pa]</th>
<th>( p_i + \ldots ) [Pa]</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0223</td>
<td>0.1199</td>
<td>0.1199</td>
<td>0.1199</td>
<td>0.002</td>
<td>0.0361</td>
<td>0.0361</td>
<td>1E-07</td>
<td>1.3186</td>
<td>1.3167</td>
<td>0.141</td>
</tr>
<tr>
<td>0.0486</td>
<td>0.0819</td>
<td>0.0824</td>
<td>0.0824</td>
<td>0.644</td>
<td>0.0426</td>
<td>0.0426</td>
<td>2E-06</td>
<td>1.7975</td>
<td>1.7572</td>
<td>2.241</td>
</tr>
<tr>
<td>0.0749</td>
<td>0.0662</td>
<td>0.0662</td>
<td>0.0662</td>
<td>0.003</td>
<td>0.0435</td>
<td>0.0435</td>
<td>6E-07</td>
<td>1.9991</td>
<td>1.9992</td>
<td>0.007</td>
</tr>
<tr>
<td>0.1012</td>
<td>0.0554</td>
<td>0.0554</td>
<td>0.0554</td>
<td>0.001</td>
<td>0.0435</td>
<td>0.0435</td>
<td>6E-07</td>
<td>2.0155</td>
<td>2.0156</td>
<td>0.003</td>
</tr>
<tr>
<td>0.1275</td>
<td>0.0477</td>
<td>0.0477</td>
<td>0.0477</td>
<td>0.005</td>
<td>0.0428</td>
<td>0.0428</td>
<td>5E-07</td>
<td>1.8991</td>
<td>1.8992</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 5-3: Table showing the satisfaction of interface variables at a distance \( x = 0.05 \) m from the inlet at different time instants. (Run parameters: Fluid – FC72, U = 1 m/s, ΔT = 20 °C, h = 2 mm)

<table>
<thead>
<tr>
<th>Location t [s]</th>
<th>( \dot{m}_{\text{LK}} ) [kg/m²s]</th>
<th>( \dot{m}_{\text{VK}} ) [kg/m²s]</th>
<th>( \dot{m}_{\text{Energy}} ) [kg/m²s]</th>
<th>Max % Diff.</th>
<th>( u_i ) [m/s]</th>
<th>( u_i + \ldots ) [m/s]</th>
<th>% Diff.</th>
<th>( p_i ) [Pa]</th>
<th>( p_i + \ldots ) [Pa]</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0533</td>
<td>0.0846</td>
<td>0.0847</td>
<td>0.0847</td>
<td>0.102</td>
<td>0.0426</td>
<td>0.0426</td>
<td>1E-07</td>
<td>1.8311</td>
<td>1.8449</td>
<td>0.754</td>
</tr>
<tr>
<td>0.1367</td>
<td>0.0784</td>
<td>0.0774</td>
<td>0.0774</td>
<td>1.271</td>
<td>0.0415</td>
<td>0.0415</td>
<td>4E-06</td>
<td>1.8072</td>
<td>1.7851</td>
<td>1.226</td>
</tr>
<tr>
<td>0.2200</td>
<td>0.0832</td>
<td>0.0833</td>
<td>0.0833</td>
<td>0.096</td>
<td>0.0408</td>
<td>0.0408</td>
<td>2E-08</td>
<td>1.8658</td>
<td>1.9156</td>
<td>2.670</td>
</tr>
<tr>
<td>0.3033</td>
<td>0.0809</td>
<td>0.0808</td>
<td>0.0808</td>
<td>0.090</td>
<td>0.0428</td>
<td>0.0428</td>
<td>1E-08</td>
<td>1.8504</td>
<td>1.7926</td>
<td>3.120</td>
</tr>
<tr>
<td>0.3867</td>
<td>0.0745</td>
<td>0.0744</td>
<td>0.0744</td>
<td>0.107</td>
<td>0.0395</td>
<td>0.0395</td>
<td>4E-09</td>
<td>1.9308</td>
<td>1.9591</td>
<td>1.468</td>
</tr>
</tbody>
</table>

Note that Tables 5.2-5.3 show the satisfaction of the following: interfacial mass flux equalities in Eq. (2.9), continuity of tangential velocities in Eq. (2.3), and the normal component of momentum balance in Eq. (2.4). The variables’ definitions in the Tables are
the same as described in Chapter - 2. Table 5-2 shows the satisfaction at a particular time instant at different x locations whereas Table 5-3 shows the satisfaction at specific x location at different time instants of time.

5.5. **Convergence times**

These simulations were run on a Intel(R) Core(TM) i7 920 @ 2.6 GHZ processor machine with 12 GB RAM. The run times for the each time-step is a function of the kind of the initial disturbance given to a particular steady flow. The predominant time consuming factor in the entire algorithm is associated with the use of CFD solvers - which currently take about 90% of the time for an iteration or any one cycle of the steps described in the Chapter - 4. However this time can be significantly reduced by not requiring complete CFD convergence for intermediate steps (iii), (v), and (vii) - as convergence in step (viii) is all that is required. This being a scientific study, we have not explored such an optimization.

For a representative run of the grid independence study presented in Table 5-1 (Mesh1) and Figure 5.2, the total run time and its details are as follows: (i) time step = \( \Delta t^0_1 = 0.01 \) s, no of time steps considered, \( N = 30 \); (ii) average number of internal iterations for 30 time steps was 12 and total run time was 20.26 hrs; (iii) run time of COMSOL CFD solvers was approximately 17.9 hrs or 88% of the total run time; (iv) run time of other MATLAB operations i.e., interface tracking, interface variable calculations, etc. was approximately 2.3 hrs or 12% of the total run time.

As mentioned earlier, these total run times can be optimized by playing with COMSOL mesh sizes and partial convergences (as opposed to the complete convergence in the reported numbers) in steps (iii), (v), and (vii) of the algorithm – as a result significant reduction in the overall run time is expected.
Chapter 6. Results and Discussions

This chapter presents steady and unsteady results relating to stability and sensitivity of the internal condensing flows. The run parameters for the test runs reported in this chapter and the corresponding non-dimensional numbers for these cases are presented in Table 6-1.

Table 6-1: Table showing the specifications of the run case parameters and associated non-dimensional numbers for the results presented in the Chapter 6.

<table>
<thead>
<tr>
<th>Case</th>
<th>$p_e$ (kPa)</th>
<th>$T_{sat}$ (K)</th>
<th>$\Delta T$ (K)</th>
<th>$h$ (mm)</th>
<th>$U$ (m/s)</th>
<th>$g_x$ (m/s$^2$)</th>
<th>$g_y$ (m/s$^2$)</th>
<th>$\rho_2/\rho_1$</th>
<th>$\mu_2/\mu_1$</th>
<th>$Re_{in}$</th>
<th>$Ja$</th>
<th>$Pr_1$</th>
<th>$Fr_x^{-2}$</th>
<th>$Fr_y^{-2}$</th>
<th>$We$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Driven - Absence of Transverse Gravity</td>
<td>100</td>
<td>329.42</td>
<td>20</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.0082</td>
<td>0.0235</td>
<td>2261.4</td>
<td>0.2735</td>
<td>8.0053</td>
<td>0</td>
<td>0</td>
<td>385.98</td>
</tr>
<tr>
<td>Shear Driven - Presence of Transverse Gravity</td>
<td>100</td>
<td>329.42</td>
<td>20</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-9.81</td>
<td>0.0082</td>
<td>0.0235</td>
<td>2261.4</td>
<td>0.2735</td>
<td>8.0053</td>
<td>0</td>
<td>-9.81</td>
<td>385.98</td>
</tr>
<tr>
<td>Gravity Driven - Inclined Channel</td>
<td>100</td>
<td>329.42</td>
<td>20</td>
<td>2</td>
<td>1</td>
<td>0.3424</td>
<td>-9.804</td>
<td>0.0082</td>
<td>0.0235</td>
<td>2261.4</td>
<td>0.2735</td>
<td>8.0053</td>
<td>6.85E-04</td>
<td>-0.0196</td>
<td>385.98</td>
</tr>
<tr>
<td>Gravity Driven - Vertical Channel</td>
<td>100</td>
<td>329.42</td>
<td>10</td>
<td>2</td>
<td>1</td>
<td>9.81</td>
<td>0</td>
<td>0.0082</td>
<td>0.0235</td>
<td>2261.4</td>
<td>0.1358</td>
<td>8.0053</td>
<td>0.0196</td>
<td>0</td>
<td>385.98</td>
</tr>
</tbody>
</table>

6.1. Steady flow simulation results

The steady solutions obtained from the current simulation technique ([26, 27]) has already been found to be in agreement with both a 1-D Engineering technique ([27, 28]) and the previous 2-D Fortran tool based approaches ([27, 28]).

For inclined ($g_x > 0$) cases, the steady computational results compatibility with experimental results is also known [28]. For horizontal shear driven cases, compatibility of results with experiments are reported later on in section 6.1.3.
6.1.1. Shear-driven flows ($g_x = 0$) with ($g_y = -g$) and without ($g_y = 0$) transverse gravity

Comparative effects of transverse gravity are discussed through results presented in Figs. 6.1 - 6.3.

Figure 6.1 shows the comparison of the steady film thickness values along the length of the channel for two shear driven ($g_x = 0$) flows – with (horizontal channel, i.e. $g_y = -g$) and without (space based zero gravity, i.e. $g_y = 0$) transverse gravity. For the two moderate temperature difference cases ($\Delta T = 20 \, ^\circ C$), the transverse gravity component is seen to have negligible impact on steady film thickness values.

Figure 6.2 shows that the differences in the predicted flow variables for a representative shear driven flow, in the presence and absence of the transverse gravity component is also negligible - except for cross-sectional variations in relative pressure “$p_1 - p_0$.” The relative pressure is mostly uniform in the absence of gravity; however, as expected, it shows a non-uniform hydrostatic variation (typically in the denser liquid phase) in the presence of transverse gravity.

As discussed later on, unsteady stability analyses show that the effects of transverse gravity (resulting from differences in the cross-sectional variations in the liquid phase’s relative pressures) has a more significant impact on the unsteady dynamics and the length $x_A$ of the annular regime.
Figure 6.1: Plot of the two steady film thickness profiles show negligible effects of transverse gravity. (Run parameters: Fluid – FC-72, \( U = 1 \text{m/s} \), \( \Delta T = 20 \text{ °C} \), \( h = 2 \text{ mm} \)).
Figure 6.2: Cross-sectional profiles for x-component of velocity $u_I$, relative pressure “$p_I - p_0$,” and physical temperature $T_I$ at $x = 0.08$ m location in Fig. 8. The solid black line curve is for the absence of transverse gravity whereas the dashed grey line curve represents the presence of transverse gravity.

6.1.2. Comparison of shear driven and gravity driven flows

Figure 6.3 shows the comparison of the steady film thickness values along the length of the channel for the two shear driven flows (horizontal channel and the channel in zero-gravity) and a gravity driven flow ($\alpha = 2$ deg, downward inclination of the channel). For these three moderate temperature difference cases ($\Delta T = 20$ °C), Fig. 6.3 clearly shows a difference in the film thickness values – with faster gravity driven cases being thinner and associated with higher heat-flux ($\sim 1/\Delta$) values - along the length of the channel.

![Figure 6.3: Plot of steady film thickness showing the differences for shear driven and gravity driven flows (Run parameters: Fluid – FC-72, $U = 1$m/s, $\Delta T = 20$ °C, $h = 2$ mm).](image)

Figure 6.4 compares the streamline patterns for a shear-driven horizontal flow and a gravity assisted ($2^\circ$ downward inclined channel) annular condensing flow - indicating the tendency
of the film to fly-off from the surface in the absence of a gravitational component in the direction of the flow. This is an indication of an earlier transition to non-annular regimes for the shear-driven flow - and hence, in comparison to the gravity assisted situation, a shorter stable annular regime length \( x_A \) is expected.
Figure 6.4: Streamline patterns of a steady flow for: (a) a horizontal channel, and (b) a 2° downward inclined channel. The color map shows the velocity magnitude distributions. (Run parameters: Fluid – FC72, $U = 1 \text{ m/s}$, $\Delta T = 20 \degree \text{C}$, channel height $= 2 \text{ mm}$)

6.1.3. Comparison of steady flow predictions with experimental and other computational results

Table 6-2 and Figure 6.5 show the comparison of the heat-flux values obtained from the simulation approach when experimentally measured quasi-steady values of inlet mass flow rate, inlet pressure, and condensing-surface temperature variations are used. The experimental results are for the non-pulsatile run cases given in [52] and [1]. Table 6-2 shows good agreement for heat-flux values measured experimentally at a 40 cm location from the inlet of the test section. Although only limited number of comparisons with in-house experimental runs are shown here, the code’s prediction efficacy relative to large amount of shear driven condensing flow data (involving mm-scale circular cross-sections [53] and similar supportive simulation tools [28]) are discussed in [39]. The chosen comparisons in Table 6-2 are special in the sense that they also show the code’s ability to handle different spatially varying (as opposed to uniform) $T_w(x^p)$ prescriptions that model more realistic thermal boundary conditions. Prescribed heat-flux thermal boundary conditions can also be used with minor changes to the code.

Table 6-2: Comparison of steady 2-D simulation results with experimental run cases in [52] and [1]
The simulations used in the above reported comparisons with experiments assumed laminar vapor and laminar liquid. For most shear driven flows, the laminar liquid flow assumption holds for sufficient distances. It has been reported in [39] that the comparisons between
steady simulations and experiments are good even when the vapor flow - away from the laminar liquid and the interface zone – is turbulent (as per simple single-phase local vapor Reynolds number estimates). As a result, steady flow simulation results for \( \text{Nu}_x \) in Eq. (4.3) has been correlated (see [39]) to yield the following:

\[
\text{Nu}_x = 0.113 \ x^{-0.433} \mathcal{R}_{\text{in}}^{0.503} \left( \frac{Ja}{Pr_1} \right)^{-0.308} \left( \frac{\rho_2}{\rho_1} \right)^{-0.537} \left( \frac{\mu_2}{\mu_1} \right)^{0.443},
\]

where \( 0 \leq x \leq x_A, 800 \leq \mathcal{R}_{\text{in}} \leq 23000, 0.005 \leq Ja/Pr_1 \leq 0.021, 0.0013 \leq \rho_2/\rho_1 \leq 23000, 0.012 \leq \rho_2/\rho_1 \leq 0.034 \), and correlation for \( x_A \) is reported later.

Similarly, in future works of our group, a method for improving the turbulent vapor field associated predictions (such as pressure drops, etc.) will be implemented.

6.2. **Unsteady Flow Simulation Results – Shear Driven Flows**

Though the steady solutions’ results for shear driven cases are of the type shown in Figs. 6.1-6.4 and they indicate insignificant differences (except for hydrostatic pressure variations) between cases with and without transverse gravity, their unsteady responses indicate that wave structures, length of the annular regime, most energetic temporal and spatial frequencies (or time-periods and wave-lengths), etc. are quite different.

6.2.1. **Shear Driven Flow with Transverse Gravity**

A representative shear driven case, with transverse gravity (Run parameters: Fluid – FC72, \( U = 1 \text{m/s}, \Delta T = 20 \ ^\circ\text{C}, \) channel height, \( h = 2 \text{ mm} \)), is studied here to understand its transition from an annular to a non-annular regime. Two different disturbed interface locations are initially (i.e. \( \Delta(x,0) = \Delta_{\text{Steady}}(x) + \Delta'(x,0) \)) given to the steady base flow and its time evolution is predicted. The resulting \( \Delta'(x,t) \) time evolutions, and their understanding, are presented next - through the results in Figures 6.6 – 6.16.

Such shear driven cases’ unsteady solutions, using the first order discretization scheme for Eq. (3.3), were attempted and reported in an earlier paper ([32]). However, because of numerically induced disturbances, these studies could only suggest sensitivity of shear
driven flows and a non-linear stability analysis was not possible and not implemented. The high sensitivity of such flows to numerical-error induced interfacial waves are such that they are significantly self-amplified in lower vapor speed downstream zones. The nature and presence of these waves led to making of some wrong conjectures regarding exit-condition dependencies ([32]). To successfully undertake a study based on interface’s wave-dynamic response, the shear driven flows’ representative result (from this 4th order algorithm), as shown in Figure 6.6, is essential. The result shows that in the absence of externally imposed initial disturbances, the unsteady simulations over $0 \leq x \leq x^* \approx 0.08$ m ($> x_A$) remain close to the steady solution for $0 \leq t \leq t^* \approx 0.4$ s - and there are no numerical error induced interfacial waves (this was not the case in ([32])). This knowledge allows - based on known imposition of initial disturbances within this spatial interval - a spatio-temporal evolution study over the intervals identified in Figure 6.6.

![Figure 6.6: Plot of the steady $\Delta_{\text{steady}}(x)$ and unsteady film thickness evolution $\Delta(x,t)$ for a case with zero externally imposed initial disturbance, i.e. $\Delta'(x,0) = 0$. (Run parameters: Fluid – FC-72, $U = 1$ m/s, $\Delta T = 20$ °C, channel height = 2 mm, $g_y = -g$)](image)

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Figure 6.7a shows interfacial film thickness evolution \(\Delta(x,t) = \Delta_{\text{Steady}}(x) + \Delta'(x,t)\) resulting from superposition of a “Type-1” initial disturbance \(\Delta'(x,0) \neq 0\) that has a presence of a long wave-length (larger than the most destabilizing wave-length to be determined). The initial disturbance is of the type \(\Delta'(x,0) \approx a_1 \cdot \sin\left(\frac{2\pi x}{\lambda_1}\right)\) where \(\lambda_1 = 0.01 \text{ m}\) and \(a_1 = 1.2 \times 10^{-5} \text{ m}\). Figure 6.7b shows the magnitude of the Discrete Fourier Transform (termed DFT is as defined in [46]) with respect to \(x\) of \(\Delta'(x,t)\) - with \(t\) as a parameter.
Figure 6.7: (a) Unsteady flow $\Delta(x,t)$ evolution for a condensing flow situation resulting from imposition of the “Type-1” initial disturbance $\Delta'(x,0) \approx a_1 \sin(2\pi x/\lambda_1)$ with $\lambda_1 = 0.01$ m and $a_1 = 1.2e-5$ m superposed at the indicated location on the steady solution. (b) Plot of the magnitude of DFT of “$\Delta'(x,t) \equiv \Delta(x,t) - \Delta_{\text{Steady}}(x)$” with respect to x and time t as a parameter. The plot shows high growth at a predominant wavenumber, $k_{md} \approx 390$ m$^{-1}$. (Run parameters: Fluid – FC-72, U = 1 m/s, $\Delta T = 20$ °C, channel height = 2 mm, $g_y = -g$)

Figure 6.7a indicates a rapid growth in the region of $(x_{AL} \approx 0.06$ m $< x < (x_{AU} \approx 0.07$ m) indicating the presence of instability beyond an approximate single representative length of $x_A \approx (x_{AL} + x_{AU})/2 \approx 0.065$ m. Figure 6.7b shows the presence of a dominant fast growing wave number $k_{md} \approx 390$ m$^{-1}$. The two other growing wave-numbers $k_{me1}$ and $k_{me2}$ below 75 m$^{-1}$ as $t = 0.46$ s is indicative more of the changing mean ($k_{me} \to 0$) of $|\Delta'(x,t)|$ because of its asymmetric growth in Figure 6.7 as opposed to any presence and growth of associated wave lengths. The fact that the wave-number $k_{me}$ do not represent significant flow physics
phenomena is also independently confirmed, later on, by the spectrum of the disturbance kinetic energy associated with condensate motion.

The instability results of Figure 6.8 are better visualized by the % growth in interfacial disturbance indicated by curves for \( |\Delta'(x,t)|/\Delta_{\text{steady}}(x) \). Figure 6.8a through a 3-D plot and Figure 6.8b through a contour plot of \( |\Delta'(x,t)|/\Delta_{\text{steady}}(x) \) clearly indicate catastrophic growth in disturbance values in the vicinity of \( x \approx 0.06 \) m and \( t \approx 0.5 \) m.

![3-D plot of magnitude of \( \Delta'(x,t)/\Delta_{\text{steady}}(x) \)](image)

(a)
Figure 6.8: (a) 3-D Plot showing $|\Delta'(x,t)|/\Delta_{\text{steady}}(x)$ as a function of distance and time, and (b) Contour plot of $|\Delta'(x,t)|/\Delta_{\text{steady}}(x)$ with higher contour levels showing the film breaking up and leading to transition from annular to non-annular regimes.

To better understand the above described dynamics associated with the growth of the film thickness and relate it to the spectrum of initial disturbance, several different spectrums for $\Delta'(x,0)$ were chosen – and each time a rapid growth at the most dangerous wavenumber $k_{md} = 1/\lambda_{md} \approx 390$ m$^{-1}$ was observed. For further unsteady response studies, a different and special $\Delta'(x,0)$ spectrum containing the most dangerous wave-length $\lambda_2 = \lambda_{md} = 1/k_{md}$ - identified earlier through imposition of “Type-1” spectrum – and smaller amplitude $a_2$ is given to the steady base flow and is termed “Type-2” disturbance. In this example, its definition becomes $\Delta'(x,0) \equiv a_2 \cdot \sin\left(\frac{2\pi x}{\lambda_2}\right)$ over $0.01 \leq x \leq x_L$, $a_2 = 9.28 \times 10^{-7}$ m, where $1/\lambda_2 = k_{md} = 390$ m$^{-1}$ is obtained from Figure 6.7.

Figure 6.9a shows the interfacial film thickness evolution ($\Delta(x,t) \equiv \Delta_{\text{steady}}(x) + \Delta'(x,t)$) resulting from the “Type – 2” initial disturbance $\Delta'(x,0)$ and Figure 6.9b shows the
magnitude of the Discrete Fourier Transform with respect to x of $\Delta'(x,t)$ - with t as a parameter. The results in Figure 6.7b and Figure 6.9b, when compared, show that the same $k = k_{md}$ remains the most dangerous wavenumber but the growth rates – as indicated by “change, in time $T \approx 0.46s$, of the magnitude of DFT of $\Delta'(x,t)$ associated with $k \approx k_m$” divided by “time $T$· initial-disturbance amplitude ($a_1$ or $a_2$)” are significantly higher for “Type-2” disturbances. This indicates that: i) the growth rates for a flow situation are significantly affected by the $\Delta'(x,0)$ spectrum, (ii) because of the larger growth rate and lower redistribution of energy to other wave-lengths, it is easier to locate the instability associated “blow-up” in $\Delta'(x,t)$ values of the solution if the $\Delta'(x,0)$ spectrum is of “Type-2,” and (iii) despite statistical variations typically present in nature with regard to inherent randomness in characterizing initial disturbances and associated randomness in the dynamic response leading to flow instabilities and transitions, the values obtained for the location $[x_{AL}, x_{AU}]$ of onset of instability and the most dangerous wave-number $k_{md}$ remain rather insensitive and change little.
Figure 6.9: a) Plot of unsteady film thickness evolution $\Delta(x,t)$ for a condensing flow situation with a Type-2 initial disturbance ($\Delta'(x,0) \equiv a. \sin(2\pi k \Delta x)$). b) Plot of the magnitude of DFT of “$\Delta'(x,t) \equiv \Delta(x,t) - \Delta_{\text{steady}}(x)$” with respect to $x$ and time $t$ as a parameter. The plot shows the predominant wavenumber ($k_{\text{md}}$). (Run parameters: Fluid – FC-72, $U = 1 \text{ m/s}$, $\Delta T = 20 \text{ °C}$, channel height = 2 mm, $g_y = -g$).

The identification of lower and upper bounds for annular to non-annular transitions - [$x_{AL}$, $x_{AU}$] associated with the reported instability phenomena is also confirmed by the characteristic curves shown in the Figure 6.10. These curves are obtained by RK4-1 solution of Eq. (3.4) for $x_c(0) = x^*$ and a discrete range of choices of $x^*$ over $0 \leq x^* \leq L$. These curves clearly indicate intersection, with the first intersection being at $x \approx 0.065 \text{ m}$ if time $t$ is extrapolated beyond $t \approx 0.6 \text{ s}$. This type of wave break-up clearly indicate multiple-valued nature of $\Delta(x,t)$ beyond $x > x_A$ and the inability of the explicit representation “$\varphi(x, y, t) \equiv y - \delta(x, t) = 0$” to handle the interface profile for $x > x_A$.

Therefore, from here and henceforth, for this flow situation, the instability analyses will be discussed by further considering the unsteady response to “Type-2” initial disturbance - as
in Figure 6.9. It should be noted, and this follows from later observations for Figure 6.22, that such wave break-ups occur when faster moving information transfer from the rear catches up with the slower moving information transfer at the front. This phenomenon is also well known in other areas of wave-phenomena (e.g. see [54, 55] for shallow water wave-breaking).

Figure 6.10: Plot of unsteady characteristics curves showing the intersection $x \approx 0.065$ m.
To better understand the physics and to relate to classical kinetic energy approach to stability analyses ([56, 57], etc.), the kinetic energy associated with *disturbances* in any liquid control volume (see Figure 6.11), whose upstream edge is at \( x \) and has a width of \( \Delta x \), is denoted as “\( \text{KE}'_L(x,t) \).” This value is obtained by subtracting the corresponding steady cases’ kinetic energy from the disturbed unsteady cases’ kinetic energy.

\[ \text{KE}'_L(x,t) = \ldots \]

**Figure 6.11: Energy analysis for the condensing flow control volumes \( CV_1(t) \) and \( CV_2(t) \) of width “\( \Delta x \).”**

The time rate of change in \( \text{KE}'_L(x,t) \) associated with the kinetic energy in the condensate control volume \( CV_1(t) \) of Figure 6.11, is denoted as \( \frac{d}{dt} (\text{KE}'_L) \) and calculated as per mechanical energy transfer rates across the control-volume faces in accordance with terms appearing in Eqs. (A.17)- (A.20) of Appendix A2.

The growth rate in \( \text{KE}'_L(x,t) \) can also be inferred by considering its evolution under “Type-2” initial disturbance associated with \( k \approx k_{md} \) and by considering *time evolution* of its spatial averages over certain \( x_1 \leq x \leq x_2 \). The spatial averages \( \overline{\text{KE}}'_L\big|_{x_1,x_2}(t) \equiv \left[ \int_{x_1}^{x_2} \text{KE}'_L \cdot dx \right] / (x_2 - x_1) \) of \( \text{KE}'_L(x,t) \) are plotted in Figure 6.12, as a function of time, for
\[ [x_1, x_2] \cong [0, x_A] \] and \([x_1, x_2] \cong [x_A, L]\). Figure 6.12 shows a plateauing of kinetic energy over \([0, x_A]\) and an exponential growth feature for \(x > x_A\).

Figure 6.12: Spatially averaged value of \(\text{KE}'_L(x, t)\) over (a) \([0, x_A]\) and (b) \([x_A, L]\). For \([0, x_A]\) it shows a tendency to plateau and for \([x_A, L]\), the average shows a tendency for exponential growth and eventual breakup of the annular regime.

Figure 6.13 plots time rate of change of kinetic energy per unit width (computed through Eq. (A.20) in Appendix A2) with time for specific \(x\) locations before and after the transition point \(x_A\). The results in Figure 6.13 supplement the result in Figure 6.12 because, for a
representative \( x < x_A \) location, the time rate of kinetic energy quickly dies out indicating a stable annular regime. However, in the non-annular zone where \( x > x_A \), the rate of growth of kinetic energy is considerable at large times and this would lead to break up of the film thickness profile in the current model – indicative of the observed ([1]) transition to plug/slug if an implicit interface equation “\( \Phi (x, y, t) = 0 \)” could be used to represent interface locations.

![Figure 6.13](image)

**Figure 6.13:** Plot of \( \frac{d}{dt}(KE'L) \) as a function of time. (a) For a representative \( x \approx 0.0354 \) m location before \( x_A \approx 0.065 \) m with results consistent with plateauing of \( KE'L \) values. (b) For a representative \( x \approx 0.075 \) m location after \( x_A \approx 0.065 \) m - with results consistent with exponential growth in \( KE'L \) values.

To understand the spectral content of these disturbances, we consider time dependent DFT of \( KE'_L(x, t) \) over: i) \( 0 < x < x_{AL} \) and ii) \( x_{AU} < x < L \). The DFT results are shown in Figure 6.14. The results in Figure 6.14a for \( 0 < x < x_{AL} \) show the presence of several wavelengths in the vicinity of the most energetic and rapidly growing wavelength (\( \lambda_{md} \) or corresponding wavenumber \( k_{md} \)). The increasing and subsequently decreasing nature of the DFT’s amplitude is seen in the vicinity of the wavenumber \( k_{md} \). This is consistent with persistent
waviness of the stable zone \(0 < x < x_{AL}\). The results in Figure 6.14b for \(x_{AU} < x < L\) zone show exponential growth at the predominant wavenumber \(k_{md}\). Again, this implies instability of the annular zone.

(a)
Further, we also consider the 2-Dimensional DFT of $KE'_L(x, t)$ over: i) $0 < x < x_{AL}$ and ii) $x_{AU} < x < L$. The 2-D DFT results are shown in Figure 6.15 and Figure 6.16. The results in Figure 6.15 for $0 < x < x_{AL}$ show the presence of several wavelengths in the vicinity of the most energetic and rapidly growing wavelength and frequency (wavenumber $k_{md}$ and frequency $f_{md}$). The increasing and subsequently decreasing nature of the DFT’s amplitude is seen in the vicinity of the wavenumber $k_{md}$ and frequency $f_{md}$. This is consistent with persistent waviness of the stable zone $0 < x < x_{AL}$. The results in Figure 6.16 for $x_{AU} < x < L$ zone show exponential growth at the predominant wavenumber $k_{md}$ and frequency $f_{md}$. Again, this implies instability of the annular zone.
Figure 6.15: a) Plot of 2-Dimensional DFT of KE’\_L (x,t) over 0 < x < x AL showing the presence of several wavelengths in the vicinity of the most energetic and rapidly growing wavelength and frequency. b) Plot of the planar view of 2D DFT of (a).
Figure 6.16: a) Plot of 2-Dimensional DFT of $\text{KE}'_L(x,t)$ over $x_{AU} < x < L$ showing the presence the most energetic and rapidly growing wavelength and frequency. b) Plot of the planar view of 2D DFT of (a).

With regard to realizing stable/annular flows, the approach’s ability to indicate plateauing but persistent waviness is unique to the first of its kind non-linear analysis in this work. Furthermore, such spectral results are unique and different from linear stability analyses ([57-60], etc.) on the following counts: (i) these are not stability studies of stationary thin films ([58]), (ii) we do not make the typical assumption on the nature of the time evolution of disturbance amplitudes (i.e. exponential growth or damping functions assumed in [57, 59, 60], etc.), and (iii) our result showing persistence waviness and its predominant wavelength, as seen through long time plateauing of sapatial average of $\text{KE}'_L(x, t)$ over $[0, x_A]$, etc. is a more genuine non-linear stability result (that characterizes persistent waves seen in experiments, a result that cannot be obtained from linear stability analyses).
Thus, through three different methods (interface evolution, characteristics intersection, and liquid condensate’s kinetic energy evolution), consistent instability mechanisms have been identified that show: (i) large disturbance growth over short times, if x is greater than the identified \( x_A \) and (ii) insignificant growth leading to plateauing over long times if x is less than the identified \( x_A \).

6.2.2. Shear Driven Flows – Absence of Transverse Gravity

The above results are for a case in the presence of transverse gravity. Considering the analogous case in the absence of transverse gravity, a difference in dynamic response is observed and is shown in Figs. 6.17 – 6.21. Besides the differences in dynamics and associated \( k_{md} \) and \( f_{md} \) value, there is a clear impact on the length of the annular zone \( x_A \) (which now reduces to \( x_A \approx 0.035 \) m).

On similar line of analysis to the one used in Section 6.2.1, to understand the sensitivity of dynamics and growth of the film thickness to the spectrum of initial disturbance, several different spectrums for \( \Delta'(x,0) \) were chosen – and each time a rapid growth at the most dangerous wavenumber \( k_{md} \approx 190 \text{ m}^{-1} \) was observed. For further unsteady response studies, a special \( \Delta'(x,0) \) spectrum containing the most dangerous wavenumber \( k_{md} \) is given to the steady base flow with its definition being \( \Delta'(x,0) \equiv a. \sin(2\pi k_{md}x) \) over \( 0.01 \leq x \leq x_L \), \( a = 2.68e-6 \text{ m} \), and \( k_{md} = 190 \text{ m}^{-1} \).

Figure 6.17 shows the interfacial film thickness evolution (\( \Delta(x,t) \equiv \Delta_{\text{Steady}}(x) + \Delta'(x,t) \)) resulting from the initial disturbance \( \Delta'(x,0) \equiv a. \sin(2\pi k_{md}x) \). The results also show that the same \( k = k_{md} \approx 190 \text{ m}^{-1} \) remains the most dangerous wavenumber but the growth rates obtained for amplitudes associated with the \( k \approx k_{md} \) are different.
(a) Film Thickness, \( \Delta (x,t) \)

- Steady Film
- Initial Disturbance
- Film at \( t = 0.2 \) s
- Film at \( t = 0.4 \) s

Distance along the length of the channel, \( x \) (m)

Distance from the condensing surface, \( y \) (m)

(b) DFT of \( \Delta'(x,t) \) for \( 0 < x < L \)

- \( t = 0 \) s
- \( t = 0.07 \) s
- \( t = 0.29 \) s
- \( t = 0.38 \) s

Magnitude of \( \Delta'(x,t) \)

Wavenumber, \( k \) (1/m)

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Figure 6.17: a) Plot of unsteady film thickness evolution $\Delta(x,t)$ for a condensing flow situation with a Type-2 initial disturbance ($\Delta'(x,0) \equiv a. \sin (2\pi k_{md}x)$). b) Plot of the magnitude of DFT of “$\Delta'(x,t) \equiv \Delta(x,t) - \Delta_{\text{steady}}(x)$” with respect to $x$ and time $t$ as a parameter. The plot shows the predominant wavenumber ($k_{md}$). (Run parameters: Fluid – FC-72, $U = 1$ m/s, $\Delta T = 20$ °C, channel height = 2 mm, $g_y = 0$).

The characteristic curves obtained by RK4 solution of Eq. (3.4) for $x_c(0) = x^*$ and a discrete range of choices of $x^*$ over $0 \leq x^* \leq L$ is shown in Figure 6.18. These curves clearly indicate intersection, with the first intersection being at $x \approx 0.035$ m. This type of wave break-up clearly indicate multiple-valued nature of $\Delta(x,t)$ beyond $x > x_A$ and the inability of the explicit representation “$\varphi(x, y, t) \equiv y - \delta(x, t) = 0$” to handle the interface profile for $x > x_A$. 
Figure 6.18: Plot of unsteady characteristics curves showing the intersection $x \approx 0.035$ m.

The spatial average of $KE'_L(x, t)$, $\bar{KE}'_L[x_1, x_2](t) \equiv \left[ \int_{x_1}^{x_2} KE'_L \cdot dx \right]/(x_2 - x_1)$ of $KE'_L(x, t)$ are plotted in Figure 6.19, as a function of time, for $[x_1, x_2] \approx [0, x_A]$ and $[x_1, x_2] \approx [x_A, L]$. Figure 6.19 shows a plateuing of kinetic energy over $[0, x_A]$ and an exponential growth feature for $x > x_A$. 
Figure 6.19: Spatially averaged value of KE_L′(x, t) over (a) [0, x_A] and (b) [x_A, L]. For [0, x_A] it shows a tendency to plateau and for [x_A, L], the average shows a tendency for exponential growth and eventual breakup of the annular regime.

Further, we also consider the 2-Dimensional DFT of KE_L′(x, t) over: i) 0 < x < x_{AL} and ii) x_{AU} < x < L. The 2-D DFT results are shown in Figure 6.20 and Figure 6.21. The results in Figure 6.20 for 0 < x < x_{AL} show the presence of several wavelengths in the vicinity of the most energetic and rapidly growing wavelength and frequency (wavenumber k_{md} and frequency f_{md}). The increasing and subsequently decreasing nature of the DFT’s amplitude is seen in the vicinity of the wavenumber k_{md} ≈ 190 m^{-1} and frequency f_{md} ≈ 7 Hz. This is
consistent with persistent waviness of the stable zone $0 < x < x_{AL}$. The results in Figure 6.21 for $x_{AU} < x < L$ zone show exponential growth at the predominant wavenumber $k_{md}$ and frequency $f_{md}$. Again, this implies instability of the annular zone.
Figure 6.20: a) Plot of 2-Dimensional DFT of $\text{KE}'_L (x,t)$ over $0 < x < x_{\text{AL}}$ showing the presence of several wavelengths in the vicinity of the most energetic and rapidly growing wavelength and frequency. b) Plot of the planar view of 2D DFT of (a).
Figure 6.21: a) Plot of 2-Dimensional DFT of KE'₇ (x,t) over xₐ < x < L showing the presence of the most energetic and rapidly growing wavelength and frequency. b) Plot of the planar view of 2D DFT of (a).

6.3 Shear Driven Flows - Signatures of Instability in Features of the Steady Base Flow

The approximate location of the transition point xₐ, despite expected minor statistical variations in its location (as is expected for all such transitions), appears to be fairly robust – both experimentally (see non-pulsatile fully condensing flow cases in [34]) and in the computations above. This leads to often sought features in the steady base flow that can be modeled/considered to be trigger(s) for the observed instability features that are robust and repeatable – and also repeatable in their experimental realizations.

To decipher such signatures of instability leading to transition from annular to non-annular regimes, the spatial variations in the various mechanical energy transfer rate mechanisms...
and the characteristic-speeds/wave-speeds, along with other flow features, were investigated for the steady base flow.

Figure 6.22 shows the plot of the characteristics/wave speed $\bar{u} = \bar{u}_{\text{steady}}(x)$ with distance $x$. The $\bar{u}$ value first increases with distance, peaks, and then begins to decrease. The peak in the value at $x = x^*2 \equiv x^*1 + \Delta x_{cr} \approx 0.08$m (just before decrease in the wave-speed takes place) has been found to consistently correspond roughly to be slightly larger than the location where transition from annular to non-annular regimes take place in the presence of gravity. The instability analyses (as shown through Figs. 6.7-6.14) shows a length of annular regime as $x_{A|g} \approx 0.065$m. Thus, the peaking and drooping nature of the characteristic speed provide an upper bound for the transition criteria in the presence of transverse gravity. This fact has also been favorably compared with our own experimental runs of ([1]).

![Figure 6.22: Plot of the characteristic wave speed showing a peak at $x^*2 \approx 0.08$ m.](image)

(Run parameters: Fluid – FC-72, $U = 1$ m/s, $\Delta T = 20$ °C, channel height = 2 mm, $g_y = -g$ or $g_y = 0$)

However, in the absence of transverse gravity, the transition from annular to non-annular regimes happens much earlier than the corresponding case with the transverse gravity. By
plotting for a representative case, in Figure 6.23, the steady base flows’ various mechanical energy transfer rate terms with distance – one can identify the significant terms that contribute to the zero kinetic energy growth (see Eq. (A.19) in Appendix 2). It is seen that the net interface viscous work per unit width for the control volume, as transferred through the interface work term $VW_{L\text{-int}}$, and net internal viscous dissipation per unit width term $VD_{L}$ for the control volume are the predominant terms. It is clear that these two terms tend cancel each other for ensuring that the equality in Eq. (A.19) holds. They show an extremum at around $x \equiv x^{*1} \approx 0.04$ m which is found to provide a good estimate for the approximate transition (to non-annular regimes) location for this case in the absence of transverse gravity. Similar locations for most cases involving absence of transverse gravity provide the characterizing feature of $x_{A|0g}$. 
6.4. Generalized and Correlated Results for the Length $x_A$ of the Annular Regime

Based on the steady and unsteady analyses presented here, the length of annular regime in the absence of transverse gravity is identified by $x_A|_{0g}$ and in the presence of transverse gravity is identified by $x_A|_{1g}$. Through the steady analyses, an approximate value for the distances $x^*_1$ and $x^*_2 \equiv x^*_1 + \Delta x_{cr}$ (and therefore for their difference $\Delta x_{cr}$) were correlated with their dependence on non-dimensional parameters as given in Eq. (4.1). The relationship among $x_A|_{0g}$, $x_A|_{1g}$, $x^*_1$, $x^*_2$, and $\Delta x_{cr}$ are given below in Eqs. (6.2)-(6.3). The
actual values, though somewhat uncertain because of the inherent behavior of a transition zone, are estimated by:

\[ x^* \leq x_{A|0g} \leq x^* + \beta \Delta x_{cr} \quad (6.2) \]

and \( x_{A|1g} \approx x_{A|0g} + \bar{\beta} \Delta x_{cr} \) \( (6.3) \)

It is typically found that, in Eq. (6.2), \( \beta \) is in the range \(-0.2 \leq \beta \leq 0.2 \) whereas \( \bar{\beta} \) in Eq. (6.3) is in the range \( 0.7 \leq \bar{\beta} \leq 1 \) However, for approximate single-value estimates, one can set \( \beta \approx 0 \) in Eq. (6.2), i.e. \( x_{A|0g} \approx x^* \) and \( \bar{\beta} \approx 1 \) in Eq. (6.3), i.e. \( x_{A|1g} \approx x_{A|0g} + \Delta x_{cr} \).

The approximate single-value estimates developed for \( x_{A|0g} \) (with 6.433 % average error) and \( x_{A|1g} \) (with 5.791 % average error) were presented in [39], it is part of the Siddharth Ravikumar’s Master Thesis [ ] and are reproduced here as Eqs. (6.4-6.5).

\[ x_{A|0g} = 0.0155 \ast Re_{in}^{0.9616} \left( \frac{Ja}{Pr_1} \right)^{-1.1859} \left( \frac{\rho_2}{\rho_1} \right)^{0.4425} \left( \frac{\mu_2}{\mu_1} \right)^{0.3} \quad (6.4) \]

\[ x_{A|1g} = 2.9413 \ast Re_{in}^{0.8514} \left( \frac{Ja}{Pr_1} \right)^{-2.1714} \left( \frac{\rho_2}{\rho_1} \right)^{1.031} \left( \frac{\mu_2}{\mu_1} \right)^{1.6366} \quad (6.5) \]

6.5. Gravity Driven Flows

6.5.1. Inclined Channel Flow

Figure 6.3 shows the comparison of the steady film thickness values along the length of the condenser for the shear driven flow in horizontal channel with the gravity driven flow in a 2° inclined channel for the same run parameters. It shows the gravity driven flow to be much thinner and have an associated higher heat-flux values.

To perform a stability analysis for the inclined flow, an initial disturbance of the type \( \Delta'(x,0) \equiv \sum_{i=1}^{3} a_i \sin (2\pi k_i x) \) (where \([k] = [306 \ 65 \ 32] \) m\(^{-1}\) and \([a] = [0.0328 \ 0.1522 \ 0.304] \times 1e-5 \) m) was chosen and the response was studied. Figure 6.24 shows the interfacial film
thickness evolution \( \Delta(x,t) = \Delta_{\text{Steady}}(x) + \Delta'(x,t) \) resulting from this initial disturbance. The results show that the disturbance in the range of wavelengths tend to die out and the flow to be stable in the zone (where the horizontal channel condensing flow showed a tendency to loose stability) under consideration.

Figure 6.24: Plot of unsteady film thickness evolution \( \Delta(x,t) \) for an inclined channel condensing flow situation with an initial disturbance \( \Delta'(x,0) = \sum_{i=1}^{3} a_i \sin(2\pi k_i x) \) showing the initial disturbance to be dying out and the flow to be stable in the length under consideration. (Run parameters: Fluid – FC-72, \( U = 1 \text{ m/s} \), \( \Delta T = 20 ^\circ \text{C} \), channel height = 2 mm, inclination = 2\(^\circ\))

Figure 6.25 shows the DFT plot of the film thickness in Figure 6.24 and clearly shows the dying out of the imposed initial disturbance.
Figure 6.25: Plot of the magnitude of DFT of “Δ’(x,t) ≡ Δ(x,t) - ΔSteady(x)” in Fig. 6.22 with respect to x and time t as a parameter. (Run parameters: Fluid – FC-72, U = 1 m/s, ΔT = 20 °C, channel height = 2 mm, inclination = 2°).

The spatial average of $KE'_L(x,t)$, $\overline{KE'_L}_{[x_1,x_2]}(t) \equiv \left[ \int_{x_1}^{x_2} KE'_L \cdot dx \right]/(x_2 - x_1)$ of $KE'_L(x,t)$ are plotted in Figure 6.26, as a function of time, for $[x_1,x_2] \equiv [0,L]$. Figure 6.26 shows the decreasing tendency of the kinetic energy leading to a highly stable and annular regime over $[0,L]$. 
Figures 6.24 - 6.26 shows the gravity driven flow in a 2° inclined channel for the same run parameters as for the shear driven cases in Section 6.2 remains stable and shows no signature through the growth of instability.

### 6.5.2. Vertical Channel Flow

To investigate a vertical channel flow, the following run case parameters are considered: Fluid – FC-72, $U = 1$ m/s, $\Delta T = 10$ °C, channel height = 2 mm, $g_x = 9.81$ m/s² and $g_y = 0$.

Figure 6.27 shows a plot of the steady film thickness values and the local film Reynolds number ($Re_\delta$) along the length of the condenser. The definition of the local Reynolds number is given by:

$$Re_\delta = \frac{4 \rho_1 u_m \delta}{\mu_1} \tag{6.6}$$

where $u_m$ is the average velocity in the film, and $\delta$ is the film thickness. As explained in [38], the film is laminar wave-free for $Re_\delta \leq 30$ and beyond it, the film becomes wavy and ripple formation takes place in the condensate. Further, beyond $Re_\delta \approx 1800$, the film transitions from laminar to turbulence. Figure 6.27b shows the laminar wave-free and laminar wavy zones marked along the length of the length of the condenser.
(a) Film Thickness

Distance from the condensing surface, $y$ (m)

Distance along the length of the condenser, $x$ (m)

(b) Local Reynolds Number, $Re_\delta$

Distance along the length of the condenser, $x$ (m)

Laminar Wave-Free
Laminar Wavy
Figure 6.27: a) Plot of steady film thickness values along the length of the condenser for a vertical down flow case b) Plot the local film Reynolds number along the length of the condenser evolution showing in the laminar wave-free and laminar wavy zones. (Run parameters: Fluid – FC-72, \( U = 1 \) m/s, \( \Delta T = 10 \) °C, channel height = 2 mm, \( g_x = 9.81 \) m/s² and \( g_y = 0 \))

To investigate the stability of the flow, an initial disturbance of the type \( \Delta'(x,0) \equiv \sum_{i=1}^{3} a_i \sin(2\pi k_i x) \) (where \( [k] = [350 \ 196 \ 98] \) m\(^{-1}\) and \( [a] = [0.0412 \ 0.0736 \ 0.1473] \times 10^{-6} \) m) was chosen and the unsteady response was studied. Figure 6.28 shows the response of the flow to the initial disturbance. It is clearly observed for \( x < 0.035 \) m or \( \text{Re}_\delta \leq 35 \), the initial disturbance dies out and the flow is stable. However, for \( \text{Re}_\delta \geq 35 \) the flow tends to be wavy and leads to wave breakups.

Figure 6.28: Plot of unsteady film thickness evolution \( \Delta(x,t) \) for an inclined channel condensing flow situation with an initial disturbance \( \Delta'(x,0) \equiv \sum_{i=1}^{3} a_i \sin(2\pi k_i x) \) showing the initial disturbance to be laminar wave-free for \( \text{Re}_\delta \leq 35 \) (or \( x < 0.035 \) m) and wavy for \( \text{Re}_\delta \geq 35 \). (Run parameters: Fluid – FC-72, \( U = 1 \) m/s, \( \Delta T = 10 \) °C, channel height = 2 mm, \( g_x = 9.81 \) m/s² and \( g_y = 0 \))
Figure 6.29 shows the DFT plot of the film thickness in Figure 6.28 for the two different zones of $\text{Re}_\delta \leq 35$ and $\text{Re}_\delta \geq 35$. It shows the diminishing tendencies and growth of the initial disturbance in the different zones respectively. This confirms the well-known behavior of gravity driven vertical condensing flows and further provides confidence in our algorithm.
Figure 6.29: Plot of the magnitude of DFT of \( \Delta'(x,t) \equiv \Delta(x,t) - \Delta_{\text{Steady}}(x) \) with respect to \( x \) and time \( t \) as a parameter for (a) \( \text{Re}\delta \leq 35 \) and (b) \( \text{Re}\delta \geq 35 \).

6.6 Sensitivity of Shear Driven Flows

To analyze the sensitivity of the shear driven flows, small amplitude pulsations along with bottom wall vibrations was introduced and the flow response was studied. For the shear driven flow with the presence of transverse gravity previously described in Section 6.2, an inlet oscillation in mass flow rate of the type \( \dot{M}_{in}(t) = \dot{M}_{in\text{Steady}} + a_{\text{Min}} \sin(2.\pi. f_p. t) \) was introduced. Here \( a_{\text{Min}} \) was 5% of the steady inlet mass flow rate, \( \dot{M}_{in\text{Steady}} \) and \( f_p = 20 \) Hz. Also, the condensing wall is vibrated as \( V_{\text{wall}}(t) = a_{\text{wall}} \sin(2.\pi. f_p. t) \) with \( a_{\text{wall}} = 3 \times 10^{-7} \) m and \( f_p = 20 \) Hz. The run case parameters of the shear driven flow used for the study is: Fluid – FC-72, \( U = 1 \) m/s, \( \Delta T = 20 \) °C, channel height = 2 mm, \( g_y = -9.81 \) m/s². Figure 6.30a shows the plot of the inlet mass flow rate and Figure 6.30b shows the plot of wall vibrations used to study the sensitivity of the shear driven flow.
Figure 6.30: a) Plot of the time varying inlet mass flow rate for a representative (2mm x 15 mm cross-section) channel as a function of time b) Plot of the imposed vibrations on the bottom condensing wall as function of time.

For this study, along with the imposed mass flow rate fluctuations and wall vibrations, an initial disturbance of the type an initial disturbance of the type \( \Delta'(x,0) \equiv \sum_{i=1}^{3} a_i \sin(2\pi k_i x) \) (where \([k] = [255 59 30] \text{ m}^{-1} \) and \([a] = [0.0437 0.1874 0.3748] \times 10^{-5} \text{ m} \)) was chosen and the response was studied. Figure 6.31 shows the interfacial film thickness evolution \( \Delta(x,t) \equiv \Delta_{\text{Steady}}(x) + \Delta'(x,t) \) resulting from this initial disturbance. The results show that the disturbance in the range of wavelengths tend to die out and the flow to be stable in the zone and not sensitive to the disturbances.

Figure 6.31: Plot of unsteady film thickness evolution \( \Delta(x,t) \) for a channel condensing flow situation with an initial disturbance \( \Delta'(x,0) \equiv \sum_{i=1}^{3} a_i \sin(2\pi k_i x) \) showing the initial disturbance to be dying out and the flow to be stable in the length under consideration. (Run parameters: Fluid – FC-72, \( U = 1 \text{ m/s} \), \( \Delta T = 20 \degree \text{C} \), channel height = 2 mm, \( gy = -9.81 \text{ m/s}^2 \))
Figure 6.32 shows the DFT of $\Delta'(x,t)$ with respect to $x$ at different time instants. This clearly shows the damping out of the initial disturbance in the presence of imposed low amplitude pulsations. This shows that the flow is sensitivity of the flow to the typically present noise in condensing flow operations imposed here through low amplitude pulsations as well as small vibrations at the bottom wall.

![DFT of $\Delta'(x,t)$ for 0 < $x$ < $L$](image)

**Figure 6.32:** Plot of the magnitude of DFT of “$\Delta'(x,t) \equiv \Delta(x,t) - \Delta_{\text{Steady}}(x)$” in Fig. 6.29 with respect to $x$ and time $t$ as a parameter. (Run parameters: Fluid – FC-72, U = 1 m/s, $\Delta T = 20 \, ^\circ\text{C}$, channel height = 2 mm, $g_y = -9.81 \, \text{m/s}^2$)
Chapter 7. Other Outstanding Issues and Forthcoming Works

7.1. Level-set extension approaches of the proposed algorithm for 2-D/3-D problems under implicit representation $\Phi(x^p, t^p) = 0$ for locating the interface

This work is not focussed on 3-D vapor/gas-liquid flow formulations and numerous associated issues (convergence, efficiency, multiply connected domains for handling multiple vapor bubbles ([8, 9], etc.) or plugs/slugs, etc.) that are better handled by researchers in the field and/or software developers.

However a key level-set type 2-D/3-D extension of the proposed approach in Chapter 3-4 is outlined here to indicate how the main advantages of a level-set technique ([5]) – that of capturing phenomena such as “breaking of interface” into multiple parts (between a pair of times, say $t_j$ and $t_{j+1}$) can be retained for 2-D problems of this paper. Such an extension to capture the interface shapes at $x \approx x_A$ where the flow transitions from annular to plug/slug regimes is proposed here. It retains the following two principal advantages of the algorithm in Chapters 3-4: (i) direct use of *method of characteristics* for solving Eq. (3.2) on a *relatively coarse* spatially “fixed” but temporally “moving” $x^p$-$t^p$ grid – for cases where proper determination of the wave-structure on the interface is important, and (ii) a “sharp” interface model with separate liquid and vapor domain CFD solutions obtained with “embedded” interface conditions designed to automatically satisfy interface conditions in the presence of discontinuities across the interface. For many problems, the second feature of sequential CFD solutions for the liquid and vapor phases could be dropped, after addressing “numerical diffusion” problems, by retaining suitable variations of existing single-domain level-set ([5]) or VOF ([6]) solution techniques. However the proposed “sharp” interface approach (in place of “finite thickness interface” models) with embedded interface conditions and separate liquid-vapor domain solution techniques is likely to remain necessary for mm-scale boilers and condensers with “micro-meter” scale liquid-film flows.

In this sub-section, we will be dealing exclusively with physical variables ($x^p$, $t^p$, etc.) and because of the location of this sub-section, the superscripts “p” on all such variables will
be dropped – the reader should therefore consider the variables to be dimensional. Implicit representations of the interface $\Sigma(t)$ in Figure 7.1a (2-D case) and Figure 7.1b (3-D case) means that the 2-D/3-D contour plots (obtainable by standard procedures in MATLAB [46]) of $\Phi(\vec{x}, t) = 0$ yields the surfaces $\Sigma(t)$ (shown for $t = t_{j-1}$, $t_j$, and $t_{j+1}$). Furthermore, an assembly of surfaces satisfying $-\delta_\varphi \leq \Phi(\vec{x}, t) \leq \delta_\varphi$ for any small $\delta_\varphi > 0$ yields a small volume “Vol(t)” that encapsulates $\Sigma(t)$. For known $\Sigma(t)$ at $t = t_{j-1}$ and $t_j$, it is possible to define “Vol(t)” at $t = t_{j-1}$ and $t_j$ such that $\Phi(\vec{x}, t)$ have well defined positive and negative values inside these volumes - while $\Phi(\vec{x}, t)$ is zero at the interface locations. One such method is to use the fact that most points on a known $\Sigma(t)$ at $t = t_{j-1}$ and $t_j$ have well defined unit normals $\hat{n}$ and one can assign “signed distance” values of $\Phi(\vec{x} + \Delta n \cdot \hat{n}, t) \approx \Delta n$ at all “$\vec{x} + \Delta n \cdot \hat{n}$” points for small positive or negative scalar distance $\Delta n$ along the chosen normals $\hat{n}$. Using this method, Figure 7.1a (2-D case) and Figure 7.1b (3-D case) show these volumes “Vol (t)” with well-defined $\Phi$ for $t = t_{j-1}$ (and at $t = t_j$, though these $t = t_j$ volumes are not shown for convenience).
As in the earlier explicit interface representation based 2-D algorithm presented in Chapters 3-4, assume that the flow variables and interface $\Sigma(t)$ are known up to a certain time $t \leq t^* = t_j$ (e.g. steady solutions for $t = t^* \leq 0$ or steady solutions superposed with “known” disturbance variables for $0 \leq t \leq t^* = t_3$). Necessarily, by virtue of requiring a solution, the interface condition represented by the Eulerian form of interface-tracking equation Eq. (3.2) is to be satisfied for all $t^* \geq t_j$ (and $j \geq 4$).

Next, as in level-set approaches ([5]), it is proposed that the interface equation Eq. (3.2) be mathematically extended (for PDE based solution purposes) from $\Sigma(t)$ to the volume “Vol($t_j$)” such that the following is satisfied for all $\vec{x} \in \text{Vol}(t_j)$ and $t \geq t_j$ (with $j \geq 4$):

Figure 7.1: (a) Grid schematic for march along characteristics for an implicit 2-D level-set algorithm. (b) Grid schematic for march along characteristics for an implicit 3-D level-set algorithm.
\[ \frac{\partial \Phi}{\partial t} + \mathbf{v}_{\text{eff}} \cdot \nabla \Phi = 0 \]  \hfill (7.1)

where \( \mathbf{v}_{\text{eff}} = \mathbf{v}_1 - (k_1 \mathbf{V}_1 - k_2 \mathbf{V}_2) \cdot \left( \frac{1}{\rho_1 h_{tg}} \right) = \mathbf{u} + \mathbf{v}_j + \mathbf{w}k \equiv [\mathbf{u}, \mathbf{v}, \mathbf{w}]^T. \)

Once again, the characteristics-curves \( \mathbf{X} = \mathbf{X}_c(t) \equiv x_c(t)\mathbf{i} + y_c(t)\mathbf{j} + z_c(t)\mathbf{k} \) originating at points \( P'(t_{j-1}) \) on \( \Sigma(t_{j-1}) \) in Figs. 26a-b or characteristics-like curves \( \mathbf{X}_c(t) \) originating at points \( P_A(t_{j-1}) \in \text{Vol}(t_{j-1}) \) are such that they satisfy:

\[ \frac{d\mathbf{X}_c}{dt} = \mathbf{v}_{\text{eff}}(\mathbf{X}_c(t), t), \]  \hfill (7.2)

The curves satisfying Eq. (7.2) are such that the interface-tracking Eq. (7.1)’s representation along these curves become:

\[ \frac{d}{dt} \{ \Phi(\mathbf{X}_c(t), t) \} = 0 \]  \hfill (7.3)

Since Eqs. (7.1)-(7.2) remain valid, as one analytically integrates these two equations to advance from \( t = t^* \) to \( t = t^* + \Delta t, \Sigma(t^*) \) given by \( \Phi(\mathbf{X}, t^*) = 0 \) will move to \( \Sigma(t^* + \Delta t) \) in a fashion that it will be given by \( \Phi(\mathbf{X} = \mathbf{X}_c(t^* + \Delta t), t^* + \Delta t) = 0. \)

The method of characteristics based level-set approach that parallels the explicit representation approach of sections 3-4 requires that, instead of numerical integration of both Eqs. (3.5) and (3.6), only the scalar Eq. (3.5)’s vector analogue, Eq. (7.2), be numerically integrated. This is because integration of Eq. (7.3) is much simpler and it has already yielded: \( \Phi(\mathbf{X}_c(t^* + \Delta t), t^* + \Delta t) = 0. \)

The method of characteristics based numerical integration of Eq. (7.2) then requires a 2-D/3-D version of the x-t grid depicted in Figure 3.1 – this is for implementing the analogue of the 4th order “explicit-implicit” discretizations given in Eqs. (3.7)-(3.8). The 4th order march along characteristics curves \( \mathbf{X}_c(t) \) – from points \( P' \) on \( \Sigma(t_{j-1}) \) (co-ordinates are “\( x_k, y_l \)” on the rectangular grid of Figure 7.1a and “\( x_k, y_l, z_m \)” on the rectangular grid of Figure
7.1b) to points Q on $\Sigma(t_{j+1})$ (see $C^{(j)}|_{P-4th}$ in Figure 7.1) is given by the following vector form ([45, 46, 61]) of the fourth order Runge-Kutta scheme between $t_{j-1}$ and $t_{j+1}$ (that are time instants “2·$\Delta t$” apart from one another):

$$\hat{x}_c(t_{j+1})|_Q = \hat{x}_c(t_{j-1})|_{P'} + \frac{2\Delta t}{6} \left[ \bar{K}_1 + 2 \cdot \bar{K}_2 + 2 \cdot \bar{K}_3 + \bar{K}_4 \right] + O(\Delta t^4) \quad (7.4)$$

where

$$\bar{K}_1 = \bar{v}_{eff}(\hat{x}_c(t_{j-1}), t_{j-1}),$$

$$\bar{K}_2 = \bar{v}_{eff}(\hat{x}_c(t_{j}), t_{j}),$$

$$\bar{K}_3 = \bar{v}_{eff}(\hat{x}_c(t_{j}), t_{j} + \Delta t)$$

and

$$\bar{K}_4 = \bar{v}_{eff}(\hat{x}_c(t_{j+1}) + 2\Delta t \cdot \bar{K}_3, t_{j+1}).$$

Since $\bar{K}_4$ on the right side of Eq. (7.4) is to be evaluated at $t = t_{j+1}$, the representation in Eq. (7.5) is implicit and preliminary explicit estimates for $\bar{v}_{eff}(\hat{x}_c(t_{j+1}))$ is to be obtained by the following two steps. First, a first order march (as in Chapters 3-4) from points R on $\Sigma(t_j)$ to S on $\Sigma(t_{j+1})$ - along characteristics $C^{(j)}|_{R-1st}$ in Figure 7.1 – is undertaken. For points R on $\Sigma(t_j)$, it should be noted that it is to be located on “xk, yl” rectangular grid of Figure 7.1a or “xk, yl, zm” rectangular grid of Figure 7.1b. These grids are to be created by placements of points $P'$ on $\Sigma(t_{j-1})$ – then developing the grids in the vicinity of the surface as well as defining/fixing their extensions away from the surface. This mesh generation technique could be as simple as a simple refined equi-spaced Cartesian grid that is pre-defined and points $P'$ on $\Sigma(t_{j-1})$ are simply located on them. Alternatively, depending on the shapes and curvatures of $\Sigma(t_{j-1})$, points $P'$ on $\Sigma(t_{j-1})$ can be selected and corresponding fixed-grids can be generated in a non-trivial manner by more advanced mesh-generation techniques that are currently available (e.g. say with [24]). Then the same grid is used for $t = t_j$ and $t_{j+1}$. Therefore the position and coordinates of R in Figure 7.1a is denoted as $\hat{x}_c(t_j)|_R = (xk|P', yl|R)$ and is obtained by using the coordinates of a point $P'$ on $\Sigma(t_{j-1})$ and extending its y-axis until it intersects with the known surface $\Sigma(t_j)$ at R. Similarly, the position and
coordinates of R in Figure 7.1b is denoted as $\mathbf{\bar{x}}_c(t_j)|_R = (x_k|P', y_l|P', zmR)$ and is obtained by using the coordinates of a point $P'$ on $\Sigma(t_{j-1})$ and extending its z-axis until it intersects with the known surface at $\Sigma(t_j)$ at R. The 1st order estimate of the positions $\mathbf{\bar{x}}_c(t_{j+1})|_S$ of points S on $\Sigma(t_{j+1})$ are given through the relation:

$$\mathbf{\bar{x}}_c(t_{j+1})|_S = \mathbf{\bar{x}}_c(t_j)|_R + \mathbf{\bar{v}}_{\text{eff}}(\mathbf{\bar{x}}_c(t_j), t_j) \cdot \Delta t + O(\Delta t^4) \quad (7.6)$$

The collection of points S on $\Sigma(t_{j+1})$ yield its first order estimate denoted by $\Sigma(1)(t_{j+1})$. Mapping the earlier liquid-vapor flow variables’ CFD (on COMSOL) estimates to the new domains formed by the new interface location $\Sigma(1)(t_{j+1})$ and obtaining new unsteady CFD (on COMSOL) estimates for $t = t_{j+1}$, analogous to the algorithm in Chapter 4. This will yield a first CFD estimate of $\mathbf{\bar{v}}_{\text{eff}}(\mathbf{\bar{x}}_c(t_{j+1})$. This enables use of Eq. (7.4) for obtaining $\mathbf{C}^{(0)}|_{P'}$. 4th order equations and positions of points Q on $\Sigma(t_{j+1})$ - which yields an improved 4th order location of the interface at $t = t_{j+1}$, denoted as $\Sigma(4)(t_{j+1})$.

For use of the 3-D analogue of section 4’s algorithm – which involves repeated CFD solutions for the two separate fluid phases’ domains – defined by the knowledge of $\Sigma(4)(t_{j+1})$ and repeated updating of $\Sigma(4)(t_{j+1})$ by the above procedure is needed. This is likely to give a good and convergent solution for many cases (as in the earlier explicit algorithm). However this approach is not expected to be the best when the 2-D/3-D interface may experience a complex change between $t = t_j$ and $t = t_{j+1}$ (e.g. annular waves breaking-up to form plug-slug regimes over $x > x_A$ in Fig. 1b). To capture such phenomena, in place of repeated updates of $\Sigma(4)(t_{j+1})$, another level-set (ls) based update $\Sigma(4\text{-ls})(t_{j+1})$ is recommended and using this update corresponding new CFD solutions are to be found – before the process is repeated to obtain convergent $\Sigma(4\text{-ls})(t_{j+1})$ and associated solutions. The procedure for this next update for $\Sigma(4\text{-ls})(t_{j+1})$ uses the well-known level-set principles ([5, 12]) within this algorithm’s context – and these are described next.
For implementing the level-set extension, the *relatively coarse* (with respect to CFD mesh sizes) spatially “fixed” but temporally “moving” x-t grid (that involves rectangular grid points shown in Figure 7.1) are to be used for mapping known \( \Phi(\vec{x}, t) \) values at points \( P', P_A, \) etc. in “Vol(\( t_{j-1} \))” to points \( Q, Q_A, \) etc. in “Vol(\( t_{j+1} \)).” Now Eqs. (7.2)-(7.3) imply that, a combination of analytical integration of Eq. (7.3) with the numerical integration of Eq. (7.2) given by Eq. (7.4), states that for bounded derivatives of \( \Phi(\vec{x}, t) \) with respect to \( \vec{x} \) - the assignments of \( \Phi \) in “Vol(\( t_{j+1} \))” will be such that:

\[
\Phi(\vec{x}_{Q_A}, t_{j+1}) = \Phi(\vec{x}_{P_A}, t_{j-1}) + O(\Delta t^4)
\]

(7.7)

for every pair of points on characteristics curves \( \vec{x}_c(t) \) that map points \( P_A \in \text{Vol}(t_{j-1}) \) to \( Q_A \in \text{Vol}(t_{j+1}). \)

Assembling the information on \( \Phi(\vec{x}_{Q_A}, t_{j+1}) \), next consider \( \Phi(\vec{x}, t_{j+1}) \) for all \( \vec{x} \) in a certain “Vol(\( t_{j+1} \))” shown in Figure 7.1. Although well-known algorithms ([46]) for contour plots of \( \Phi(\vec{x}, t_{j+1}) \) can specifically yield the specific surface \( \Sigma^{(4-ls)}(t_{j+1}) \) of interest – which must satisfy \( \Phi(\vec{x}, t_{j+1}) = 0 \) - it is prudent to make this process efficient ([12]) by first obtaining reinitialized values \( \Phi^{\text{reinit}}(\vec{x}, t_{j+1}) \) of the computed \( \Phi(\vec{x}, t_{j+1}) \) through:

\[
\Phi^{\text{reinit}}(\vec{x}, t_{j+1}) = \frac{1}{|\nabla \Phi(\vec{x}, t_{j+1})|} \cdot \Phi(\vec{x}, t_{j+1})
\]

(7.8)

and then finding the specific surface \( \Sigma^{(4-ls)}(t_{j+1}) \) of interest through satisfaction of:

\( \Phi^{\text{reinit}}(\vec{x}, t_{j+1}) = 0 \). The renormalization in Eq. (7.8) is recommended because: (i) \( |\nabla \Phi(\vec{x}, t_{j+1})| \neq 0 \) for most points on a regular \( \Sigma(t_{j+1}) \) - as unit normal \( \hat{n} \) are well defined, and, because of this, a zero of \( \Phi^{\text{reinit}}(\vec{x}, t_{j+1}) \) corresponds to a zero of \( \Phi(\vec{x}, t_{j+1}) \); and (ii) the non-zero contour plots of \( -\delta \phi \leq \Phi^{\text{reinit}}(\vec{x}, t_{j+1}) \leq \delta \phi \) are generally better behaved (i.e., well separated from the zero contour) than those associated with \( \Phi(\vec{x}, t_{j+1}) \). This second feature of \( \Phi^{\text{reinit}} \) follows from the fact that at a small positive or negative scalar distance
$\Delta n$ along the chosen normal $\mathbf{n}$ of $\Sigma^{(4-1s)}(t_{j+1})$, one has: $\Phi_{\text{reinit}}^\ast (\mathbf{x} + \Delta n \cdot \mathbf{n}, t_{j+1}) \approx \Delta n$. This uses the same underlying principle as the one used for level-set approach’s re-initialization scheme (often PDE based, see [12]) which yields the signed distance values of $\Phi_{\text{reinit}}^\ast (\mathbf{x}, t_{j+1})$.

A cost/benefit analysis of utilizing the above interface-tracking algorithm with suitable extensions of the remaining algorithms in those Chapters 3-4 is best evaluated by those with experience with other existing 3-D algorithms. Needless to say, there will be different “good” choices for different phase-change problems of interest. The main advantage of the above approach is that, for gas-liquid flows involving micron-scale annular liquid films, the $\Phi_{\text{reinit}}^\ast (\mathbf{x}, t_{j+1})$ values are primarily interrogated at “tip” points $Q$ of the characteristics-curves $C^{0}_{p-4th}$ that emanate from points $P'(x_k, y_l, z_m)$ – which are on a coarse grid and the grid can remain coarse relative to a CFD grid up until convergent solutions are obtained for $t = t_{j+1}$. As a result of this, the spacing $\Delta x_{PQ}^{(j)}, \Delta y_{PQ}^{(j)}$, and $\Delta z_{PQ}^{(j)}$ that define the vector $\overline{PQ}$ in Figure 7.1 are such that the corresponding Courant numbers $C_{r-x-PQ}^{(j)} \equiv \overline{u}(x_i, t_j). \Delta t/\Delta x_{PQ}^{(j)}, C_{r-y-PQ}^{(j)} \equiv \overline{v}(x_i, t_j). \Delta t/\Delta y_{PQ}^{(j)}$, and $C_{r-z-PQ}^{(j)} \equiv \overline{w}(x_i, t_j). \Delta t/\Delta z_{PQ}^{(j)}$ automatically satisfy the desired relationship (see discussions following Eq. (4.2)):

$$C_{r-x-PQ}^{(j)} \approx C_{r-y-PQ}^{(j)} \approx C_{r-z-PQ}^{(j)} \approx 1 \quad (7.9)$$

Satisfaction of Eq. (7.9) on grids coarser than CFD mesh sizes is important for reducing computational time, removing typical “numerical diffusion” associated with other sophisticated integration techniques (e.g. 5th order WENO [13]) for Eq. (7.1). For these class of problems, “numerical diffusion” arises when the grid-size associated with Eq. (7.1) become comparable to the CFD mesh-size, and (as a result) the real time satisfaction of the mass-flux condition (Eq. (2.9)) – viz., $\dot{m}_{LK} \approx \dot{m}_{VK} \approx \dot{m}_{\text{Energy}}$ – becomes difficult for locating an interface which must also concurrently resolve the wave-dynamics associated with its location.
7.2. Vapor core turbulence

The correlations presented here and in [39], cover parameter space where $800 \leq \text{Re}_{in} \leq 20000$ and vapor core away from the interface is turbulent while the shear driven flow itself is annular/stratified. The vapor core’s turbulence typically has a second order effect on the amplitude of the wave-structure in the annular zone as well on the length $x_A$ of the annular regime. The low impact of turbulence on $x_A$ is easily inferred because it is shown here that the destabilizing energy contents on the laminar film (whether it is within $x < x_A$ or $x \approx x_A$) are rather “sharp” and “peaky” at a particular de-stabilizing spatial frequency $k_{md}$ identified as the most dangerous wave-length ($\lambda_{md} = 1/k_{md}$) and associated time-scale for the temporal frequency $f_{md}$. However turbulence related energy content is diffused and spread out (see [41]) over a much larger range of length and time scales.

7.3. Role of exit conditions

In an actual shear driven condenser or condensing flow experiments (see [1, 3, 62]), a condenser’s exit geometry is more likely to be something similar to the arrangement in Figure 1.1c rather than the one shown in Figures 1.1a-b. The speed with which interfacial waves move forward, i.e. whether they speed up or slow down when they reach the exit, depend on exit conditions and this affects the long time wave-structure and its energy-content present on the mean-interface location. For typical instantaneous film thickness values at wave-troughs (if they are greater than 100 μm), this wave structure dependence on exit geometry is not too large and simple modeling of its effects ([39]) will suffice.

Pulsatile cases and exit conditions

However, for certain pulsatile cases ([1]), vapor phase imposes oscillatory shear on the interface (whose values are dependent on acoustic wave reflections at the exit geometry of the condenser in Figure 1.1c) and, in addition, film-thickness values at wave-troughs decrease below $O(10 \text{ μm})$ values - necessitating special modeling of wall stress boundary conditions (see Appendix A1). Under these conditions, the wave-troughs “stick” on a wetting condensing-surface and the wave-structures are significantly “elliptic” or exit-
condition dependent ([1]). As a result, the earlier presented heat-transfer correlations and the length of the annular regime correlations do not hold.
Chapter 8. Conclusions

1. Fundamental 2-D steady/unsteady predictive tools for flow condensation have been developed.

2. With regard to convergence and satisfaction of interfacial conditions in the presence of waves, the algorithm and the tool show excellent accuracy.

3. The demonstrated compatibility of the scientific tool’s ability to predict results in agreement with other simulation tools and its ability to predict results in agreement with the experimentally obtained data on heat-flux further supports the validity of: the computational tool, the physics assumptions used for the underlying model, and the newly proposed heat-transfer correlations.

4. The flow physics differences for shear driven condensing flows – in the presence and absence of transverse gravity is identified and it is also compared with the inclined channel gravity driven flows.

5. The simulation tool is used to predict vertical channel internal condensing flows and the flow-physics is found to be consistent with the known behavior of this class of flows.

6. A method to identify the transition (from annular to non-annular regimes) has been proposed on the basis of certain identifying features of the steady solution and these features have been used to estimate the length of annular regimes. The estimates have been used to develop correlations for the length of annular regime in the presence and in the absence of transverse gravity.

7. Level-set extension approaches of the algorithm for 2-D/3-D problems under implicit representation $\Phi(x^p, t^p) = 0$ for locating the interface is proposed.

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Appendices

Appendix A1

The interface conditions that apply at any point $\mathbf{x}^p$ and time $t^p$ at the interface are described. The interface is located implicitly by a scalar valued function $\Phi(\mathbf{x}^p, t^p) = 0$. The values of a physical variable at the interface are denoted by a superscript “i.” If the physical variable already has a superscript “p” then its value is denoted with superscript “pi” (e.g. $\mathbf{x}^p$ at the interface is denoted as $\mathbf{x}^{pi}$).

The sign appearing in the definition of $\Phi$ in $\Phi = 0$ is adjusted such that the unit normal at any point on the interface, when directed from the liquid towards the vapor, is denoted by $\mathbf{n} \equiv \nabla \Phi / |\nabla \Phi |$.

At any point on the interface, the plane perpendicular to $\mathbf{n}$ defines the tangent plane. This tangent plane forms a line where it intersects the plane formed by the interfacial vapor velocity vector $\mathbf{v}^{pi}_2$ and the unit normal $\mathbf{n}$. Along this line, a unit tangent vector $\mathbf{t}$ is defined to be in the direction of increasing $x^p$ values (see Fig. 2). Each phase ($I = 1$ or $2$) is modeled as a viscous and incompressible Newtonian fluid with stress tensor $\mathbf{T}_I = -p_I \mathbf{1} + \mathbf{S}_I$, where $\mathbf{1}$ is the identity tensor, $p_I$ is the pressure, and $\mathbf{S}_I \equiv 2\mu_I \mathbf{D}_I$ is the viscous stress tensor. In the definition of $\mathbf{S}_I$, $\mu_I$ is the viscosity of phase $I$, and $\mathbf{D}_I \equiv \frac{1}{2} \left[ (\nabla \mathbf{v}_I^p)^T + (\nabla \mathbf{v}_I^p) \right]$ is the rate of deformation tensor whose Cartesian components are $(\mathbf{D}_I)_{ij} = \frac{1}{2} \left( \frac{\partial v_i^p}{\partial x_j} + \frac{\partial v_j^p}{\partial x_i} \right)$.

Interface Kinematics

The surface velocity $\mathbf{v}_s$ of a point on the interface ($\Phi = 0$) at time $t$ is associated with this point’s movement to a new mapped position on the interface at time $t + \Delta t$. All such mappings must be such that the normal component of this $\mathbf{v}_s$ is given by:
Continuity of Tangential Velocities

The tangential component of the vapor and the liquid velocities at the interface must be continuous, i.e,

\[ \mathbf{v}_1^{pl} \cdot \mathbf{\hat{t}} = \mathbf{v}_2^{pl} \cdot \mathbf{\hat{t}} \]  \hspace{1cm} \text{(A. 2)}

Interfacial Stresses and Interfacial Momentum Balance for Thick Films

Allowing for variations in surface tension \( \sigma \) over the interface such that the vector \( \nabla_s \sigma \) is primarily in the tangent plane, the normal component of momentum balance at a point on the interface is given by:

\[
p_1 = p_2 + \left( \frac{\dot{m}^b}{\rho_2} - \frac{1}{\rho_1} \right) + \sigma \nabla_s \mathbf{\hat{n}} - \nabla_s \sigma \mathbf{\hat{n}} + \left( \mathbf{S}_1^{1} - \mathbf{S}_2^{1} \right) \mathbf{\hat{n}} \cdot \mathbf{\hat{n}} \]  \hspace{1cm} \text{(A. 3)}

The symbols for the vector \( \nabla_s \sigma \) and the curvature \( \nabla_s \mathbf{\hat{n}} \) in Eq. (A.3) respectively denote surface-gradient operator and surface divergence operator and their meanings are well defined in differential geometry textbooks (\cite{63}). The meanings are the same if these operators were defined in 3-D space surrounding the interface as per their classical 3-D definitions and then, subsequently, their values are evaluated at the interface location.

The tangential component of momentum balance at any point on the interface, which allows for surface variations in the surface tension \( \sigma \), reduces to:

\[ \mathbf{S}_1^{1} \mathbf{\hat{n}} \cdot \mathbf{\hat{t}} = \mathbf{S}_2^{1} \mathbf{\hat{n}} \cdot \mathbf{\hat{t}} + \nabla_s \sigma \mathbf{\hat{t}} \]  \hspace{1cm} \text{(A. 4)}

In the phase-change flow problems considered here, interfacial temperature variations are negligible and there are no interfacial impurities. Hence the Marangoni term \( \nabla_s \sigma \mathbf{\hat{t}} \) can be ignored.
Interfacial Stresses and Interfacial Momentum Balance for Thin Films

When the liquid film in Fig. 2 becomes sufficiently thin – either steadily or unsteadily with $|\Delta(x, t)| < \Delta_{cr}$, where $\Delta_{cr}$ could be less than 10–15 nm - depending on the material constituting the fluid and the condensing surface (issues associated with wettability or equilibrium contact angles), the liquid side interfacial pressure $p_{1i}$ in Eq. (A.3) and interfacial shear in Eq. (A.4) start getting affected by van der Waals forces which are strongest at the liquid-solid interface (or the adsorbed layer at $y_p \approx 0$ in Fig. 2). This effect is often modeled as “disjoining pressure” $p_{\text{disj}}$ effect (see [2, 15, 64]) and leads to Eq. (A.3) being rewritten as:

$$p_{1i}^i = p_{2i}^i + p_{\text{disj}} + \left(\frac{\dot{m}p}{\rho_2^i - \rho_1^i}\right) + \sigma \nabla_s \cdot \mathbf{n} - \nabla_s \sigma \cdot \mathbf{n} + \left(S_{1i}^i - S_{2i}^i\right) \mathbf{n} \cdot \mathbf{t},$$

(A. 5)

where the disjoining pressure is typically negative for very thin steady films on hydrophilic surfaces - though its modeled magnitude can vary under different dynamic tendencies (towards or away from the solid wall) of the interface. The modeling for disjoining pressure, under equilibrium or steady flow assumption, is currently an active research area. Some of the popular models are of the type given below:

$$p_{\text{disj}} = -A \Delta^{-B} + C_1 \exp(-K\Delta) + A_1 (C_2 \Delta + 2)/B_1 \Delta^3 [1 + B_2 \Delta]^2.$$  

(A. 6)

The constants $A$ and $B$ in the first expression are discussed in [2, 64] whereas the constants $A_1$, $K$, $B_1$, $B_2$, $C_1$, and $C_2$ appearing in the second expression are discussed in [65].

Furthermore, for thin liquid film flows where disjoining pressure effects become important or significantly non-zero through Eq. (A.5), the shear stress condition in Eq. (A.4) and at the wall ($y_p = 0$) may also get modified (see [64]) because of solid like tendencies near the adsorbed layer and one may re-write it as:

$$S_{1i}^i \mathbf{n} \cdot \mathbf{t} = S_{2i}^i \mathbf{n} \cdot \mathbf{t} + \nabla_s \sigma \cdot \mathbf{t} + \tau_{\text{disj}}$$

(A. 7)
However, effective general purpose models for $p_{\text{disj}}$ and $\tau_{\text{disj}}$ are currently not available. It is expected that future modeling efforts for $p_{\text{disj}}(y)$ and $\tau_{\text{disj}}(y)$ will be in terms of their variations over $0 \leq y \leq \Delta$ and their values at $y = \Delta$ and $y = 0$ will not only be important to Eqs. (A.5) and (A.7) but also for complementing the no-slip condition at the adsorbed wall layer at $y^p = 0$ - where additional and consistent prescriptions of stress boundary conditions (i.e., $p_1|_{y^p=0}$ and $\mathbf{S}_i|_{y^p=0}$) may also be needed. This is because, in many applications such as the one in this paper, the interfacial velocity components of wavy thin liquid film flows are quite small and close to no-slip boundary conditions relative to the characteristic speed of the flow. Therefore it becomes essential that future models, such as the ones suggested above, use wall stress boundary conditions to supplement the no-slip condition at $y^p = 0$. Such modeling must also account for thicker film situations (say order 10 µm, as in pulsatile flow experiments [1]) where interfacial velocity components are close to zero relative to their values at wave-crests and/or characteristic inlet speed (and are therefore comparable to the zero values at $y^p = 0$). In such O(10 µm) thickness situations, although $p_{\text{disj}}(y)$ and $\tau_{\text{disj}}(y)$ are effectively zero at $y^p = \Delta = O(10 \mu m)$, the $p_{\text{disj}}(y)$ and $\tau_{\text{disj}}(y)$ at $y^p = 0$ cannot entirely be determined by the van der Waals forces that relate only to $\Delta$ values. The modeling should take into account the fact that solid like adsorbed layers may demand that wall stresses also depend on the interfacial stress values $p_1$ and $\mathbf{S}_1\hat{n}$. 

### Interfacial Mass Fluxes for Thick or Thin Films

The mass-flux $m^p$ is denoted respectively as $m_{\text{VK}}^p$ and $m_{\text{LK}}^p$ to denote kinematic restrictions respectively imposed by interfacial values of vapor and liquid velocities. Thus, the definitions are:

\[
\begin{align*}
\dot{m}_{\text{VK}}^p & \equiv - \rho_2(\mathbf{v}_{\text{2}}^\text{pi} - \mathbf{v}_s).\hat{n}, \quad \text{and} \\
\dot{m}_{\text{LK}}^p & \equiv - \rho_1(\mathbf{v}_{\text{1}}^\text{pi} - \mathbf{v}_s).\hat{n}
\end{align*}
\]
The energy balance at a point on the interface, with energy fluxes being relative to the moving interface, also imposes a restriction on the interfacial mass flux \( \dot{m}_{\text{Energy}}^p \), and this restriction is given by:

\[
\dot{m}_{\text{Energy}}^p = \frac{1}{h_{fg}} \left[ (k_1 \nabla T_1 \cdot \hat{n} - k_2 \nabla T_2 \cdot \hat{n}) \right] + \frac{d\sigma}{dt}\left|_{s} \right| \frac{1}{2} \dot{\bar{m}}_s^p \left( |\bar{v}^p_1 - \bar{v}_s|^2 - |\bar{v}^p_2 - \bar{v}_s|^2 \right) + \left( S^i_{\hat{n}} \cdot (\bar{v}^p_i - \bar{v}_s) - S^i_{\bar{v}_s} \cdot (\bar{v}^p_2 - \bar{v}_s) \right) \right] \approx \frac{1}{h_{fg}} \left[ k_1 \frac{\partial T_1}{\partial n}\left|_{s} \right| - k_2 \frac{\partial T_2}{\partial n}\left|_{s} \right| \right]
\]  

(A. 9)

In deriving the first equality in Eq. (A.9), the equality of surface energy per unit area to surface tension force per unit length is assumed as per usual assumption regarding equilibrium interfacial thermodynamics. The symbol \( \frac{d\sigma}{dt}\left|_{s} \right| \) denotes rate of change of surface energy per unit area per unit time and equals \( \frac{\partial \sigma}{\partial t}\left|_{s} \right| + \bar{v}_s \cdot \nabla \sigma \). This term along with interfacial kinetic energy exchanges and exchange associated with the workings of the normal components of the viscous stresses are considered negligible relative to the net interfacial heat transfer. The resulting simplification is indicated by the second equality in Eq. (A.9).

**Equilibrium Thermodynamics and Negligible Interfacial Thermal Resistance**

The term “\( h_{fg} \equiv h_g - h_f \)” appearing in Eq. (A.9) arises from the appearance of difference in the vapor phase enthalpy \( h^v_2 \equiv h_g \) and liquid phase enthalpy \( h^l_1 \equiv h_f \). As described next, the assumption of equilibrium thermodynamics at the interface allows one to use thermodynamics tables ([40]) to estimate “\( h_{fg} \)” as \( h_{fg} \equiv h_{fg}(T_s(p^l_2)) \) for condensing flow situations and as \( h_{fg} \equiv h_{fg}(T_s(p^l_1)) \) for boiling situations. The equilibrium thermodynamics assumption is typically retained for this term even if non-equilibrium
thermodynamic considerations are included for modeling other thermodynamic effects at the interface.

Whenever the film thickness is sufficiently large (say > 10 µm) over most of the condensing flow regime in Fig. 2 (i.e. $x > \varepsilon$, where $\varepsilon << L$ is small), equilibrium thermodynamic assumption as well as negligible interfacial thermal resistance assumption typically hold. For most problems, as is assumed here, these assumptions are good from second and third computational cells (in the x-direction) in any grid used to resolve the singularity at $x_p \approx 0$ in Fig. 2. This is because, from second to third cell onwards, interfacial mass transfer rates $\dot{m}_p$ is sufficiently small in non-dimensional terms, i.e. $\dot{m} \equiv \dot{m}_p/\rho_1 U \ll 1$. Under these conditions, for $x \geq \varepsilon$, if $T^i_1$ and $T^i_2$ respectively denote the liquid and vapor temperatures at the interface, the following scientific and engineering “model of equilibrium thermodynamics” holds at the interface:

$$T^i_1 \cong T^i_2 \equiv T_{sat}(p_2^i) \quad \text{(A.10)}$$

However, for some “thin film” situations, Eq. (A.10) assumption of negligible interfacial thermal resistance, i.e. $\Delta T^i/\Delta T \ll 1$ (where $\Delta T^i \equiv T^i_2 - T^i_1$ and $\Delta T \equiv T_{sat}(p_0) - T_w$), does not hold and $\Delta T^i$ needs to be modeled.

**Equilibrium Thermodynamics on One Side and Non-negligible Interfacial Thermal Resistance Due to Non Equilibrium Thermodynamics on the Other Side**

One of these conditions of interest here correspond to conditions where the liquid film is “thin” over most or significant parts of the length of the channel in Fig. 2. These are conditions where $\dot{m} \equiv \dot{m}_p/\rho_1 U$ is significant along with the fact that the disjoining pressure term “$p_{disj}$” in Eqs. (A.5 – A.6) cannot be ignored. Though there are no rigorously established scientific approach for modeling $\Delta T^i$ under these conditions. A commonly recommended model uses kinetic theory of gases (see, [2] or [66]) and one introduces another restriction on interfacial mass-flux $\dot{m}_p$ through $\dot{m}_p = \dot{m}_{\text{kinetic}}^p$ - where $\dot{m}_{\text{kinetic}}^p$
has a new and separate definition. For defining \( \dot{m}_{\text{kinetic}}^p \), one assumes \( T_2^i \) and \( T_1^i \) are related to one another and to \( \dot{m}_{\text{kinetic}}^p \) through the definition:

\[
\dot{m}_{\text{kinetic}}^p \approx \frac{2\sigma_c}{2 - \sigma_c} \frac{p_{\text{sat}}(T_2^i) - p_{\text{sat}}(T_1^i)}{(2\pi RT_2^i)^{\frac{1}{2}}} - \frac{p_{\text{sat}}(T_1^i)}{(2\pi RT_1^i)^{\frac{1}{2}}},
\]

(A. 11)

where \( \sigma_c \) is an “accommodation” coefficient ([2, 66]) and \( R \equiv R_u/M \) is a gas constant related to the universal gas constant \( R_u \) and fluid’s molecular weight \( M \). In addition to Eq. (A.11), one may either assume \( T_1^i < T_2^i \) and specify \( T_2^i \) through the equilibrium thermodynamics approximation:

\[
T_2^i \approx T_{\text{sat}}(p_2^i), \quad \text{or}
\]

(A. 12)

or treat both \( T_1^i \) and \( T_2^i \) as unknowns and require satisfaction of another slightly superior equilibrium thermodynamics model (see [15]) given as:

\[
p_2^i \approx p_{\text{sat}}(T_2^i) \exp \left[ \rho \left( \frac{p_1^i - p_{\text{sat}}(T_2^i)}{RT_2^i} \right) \right],
\]

(A. 13)

where \( p_1^i \) is given by Eq. (A.5).

**Interfacial Mass Balance for Thick and Thin Films**

Mass balance at any point on the interface requires a single-valued interfacial mass-flux. That is, when \( \Delta T^i/\Delta T \ll 1 \), one must satisfy (as in this paper)

\[
\dot{m}_{\text{LK}}^p = \dot{m}_{\text{VK}}^p = \dot{m}_{\text{Energy}}^p \equiv \dot{m}^p
\]

(A. 14)

If \( \Delta T^i/\Delta T \) is not insignificant, one can either assume the model in Eq. (A.12) or the model in Eq. (A.13) and replace the requirement in Eq. (14) by the new interfacial mass-balance requirement:
\[ \dot{m}_{LK} = \dot{m}_{VK} = \dot{m}_{\text{Energy}} = \dot{m}_{\text{kinetic}} = \dot{m} \quad (A. 15) \]

**Non Equilibrium Thermodynamics on Both Sides**

Though Eqs. (A.12) - (A.15) suffice for some “very” thin film flow condensation situations of interest, it is known and expected that for certain unsteady conditions (e.g. pulsatile flows [1] at sufficiently large externally imposed frequency), \( \partial p_1 / \partial t \) and \( \partial T_1 / \partial t \) values may become so large that the assumption of equilibrium thermodynamics will not be adequate on either side of the interface. Under these non-equilibrium time-varying conditions, “spinodal region” models ([2]) with \( p^1_2(t) \neq p_{\text{sat}}(T^1_2), \rho^1_2(t) \neq \text{constant} \), etc. may be necessary to allow for metastable super-cooled or super-heated liquid states (e.g., as at the “wave troughs” for some pulsatile flow situations of the type described in [1, 15]).

**Appendix A2 - Post-processing with regard to mechanical energy transfer mechanism**

Mechanical energy transfer rates to and from any time-varying control volume \( CV_l(t) \) (with \( l = 1 \) or \( L \) for liquid and \( l = 2 \) or \( V \) for vapor) shown in Figure 6.11 can be obtained, from well-known principles ([67], [41]).

In essence, one begins with the well-known differential form of momentum balance:

\[ \rho_l \frac{d\vec{v}_l}{dt} = -\nabla p_l + \rho \vec{g} + \text{div}(\mathbf{S}_l) \quad (A. 16) \]

for each of the two phases (\( l = 1 \) or 2). The fluid in these phases are modeled as Newtonian and incompressible. Newtonian fluids’ viscous stress tensors are \( \mathbf{S}_l = 2\mu_l \mathbf{D}_l \) and \( \mathbf{D}_l \equiv \frac{1}{2} \left[ (\mathbf{\nabla} \vec{v}^l_1) + (\mathbf{\nabla} \vec{v}^l_1)^T \right] \) are the strain rate tensors whose Cartesian components are \( (\mathbf{D}_l)_{ij} = \frac{1}{2} \left[ \frac{\partial v^l_i}{\partial x_j} + \frac{\partial v^l_j}{\partial x_i} \right] \).

Taking a dot product between the velocity vector \( \vec{v}_l \) and Eq. (A.16) leads to the differential form of the mechanical energy equation:
\[ \frac{d}{dt} \left[ \frac{1}{2} \rho |\vec{v}_I|^2 \right] = \rho \vec{g} \cdot \vec{v}_I - \nabla p \cdot \vec{v}_I + \vec{v}_I \cdot \text{div}(\mathbf{S}_I) \quad (A.17) \]

By integrating Eq. (A.17) over the liquid or vapor control volumes (CV$_1$ or CV$_2$) in Fig. 19, one obtains:

\[ \frac{d}{dt} \int_{CV_I} \rho I \left( \frac{1}{2} |\vec{v}_I|^2 \right) \, d\text{vol} + \int_{CS_I} \left( \frac{1}{2} |\vec{v}_I|^2 \right) \left\{ (\vec{v}_I - \vec{v}_s) \cdot \mathbf{n} \right\} \cdot \text{da} = \int_{CV_I} \rho \vec{g} \cdot \vec{v}_I \cdot d\text{vol} \]

\[ - \int_{CS_I} p_I (\vec{v} \cdot \mathbf{n}) \cdot \text{da} + \int_{CS_I} \{ \mathbf{S}_I \cdot \mathbf{n} \cdot \vec{v} \} \cdot \text{da} - \int_{CV_I} \varphi_{\text{visc} - I} \cdot d\text{vol} \]

where \( \vec{v}_s \) is the surface velocity of a point on the control surface and \( \varphi_{\text{visc} - I} \equiv \text{tr} \left( \mathbf{D}_I \mathbf{D}_I^T \right) \geq 0 \) is the viscous dissipation per unit volume. With simplifications for the flow in Figure 6.11, resulting from Eq. (A.18)'s division by \( \Delta x \) and letting \( \Delta x \to 0 \), one obtains, for either of the two phases, the following reduced form:

\[ \frac{d}{dt} \left\{ \text{KE}_I(x^p, t^p) \right\} = \text{KE}_{\text{conv}}(x^p, t^p) + \text{KE}_{\text{int}}(x^p, t^p) + \text{PE}_{\text{grav}}(x^p, t^p) + \text{PW}_{\text{conv}}(x^p, t^p) + \text{PW}_{\text{int}}(x^p, t^p) + \text{VW}_{\text{conv}}(x^p, t^p) + \text{VW}_{\text{int}}(x^p, t^p) + \text{VD}_I(x^p, t^p) \quad (A.19) \]

The physical meaning of the terms in the Eq. (A.19) (which are defined to be positive if the energy is added into the control volume and negative otherwise) are presented in Table-A.1 whereas their mathematical definitions are presented in Table-A.2.
Table A.1. Definitions of terms in the mechanical energy transfer equation \((I = L \text{ or } V)\) respectively for liquid and vapor phases). SI units of these terms are W/m.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{d}{dt} KE_I(x_p, t_p))</td>
<td>Local time rate of change of control-volume kinetic energy per unit width, at locations (x_p).</td>
</tr>
<tr>
<td>(KE_I-\text{conv})</td>
<td>Net kinetic energy per unit width transferred (convected) into the control-volume by the movement of the fluid through the two vertical sides (of CV(_I)) in Figure 6.11.</td>
</tr>
<tr>
<td>(KE_I-\text{int})</td>
<td>Net kinetic energy transfer across the interface per unit width of the CV.</td>
</tr>
<tr>
<td>(PE_I-\text{grav})</td>
<td>Net rate of change of gravitational potential energy per unit width of the CV.</td>
</tr>
<tr>
<td>(PW_I-\text{conv})</td>
<td>Net pressure working per unit width transferred (convected) into the control-volume by the movement of the fluid through the two vertical sides (of CV(_I)) in Figure 6.11.</td>
</tr>
<tr>
<td>(PW_I-\text{int})</td>
<td>Net pressure working per unit width of the CV transferred through the interface.</td>
</tr>
<tr>
<td>(VW_I-\text{conv})</td>
<td>Net viscous working per unit width transferred (convected) into the control-volume by the movement of the fluid through the two vertical sides (of CV(_I)) in Figure 6.11.</td>
</tr>
<tr>
<td>(VW_I-\text{int})</td>
<td>Net viscous working per unit width of the CV transferred through the interface.</td>
</tr>
<tr>
<td>(VD_I)</td>
<td>Net viscous dissipation per unit width of the CV (negative number as this energy disappears in thermal forms).</td>
</tr>
</tbody>
</table>
Table A.2. Specific liquid (I = L) and vapor (I = V) domain mathematical definitions for the terms of the mechanical energy transfer rate Eq. (A.19)

<table>
<thead>
<tr>
<th>Liquid Definitions</th>
<th>Vapor Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ KE_L(x^p, t^p) \equiv \int_0^{\Delta(x)} \left( \frac{1}{2} \rho \left( (u_1^p)^2 + (v_1^p)^2 \right) \right) , dy ]</td>
<td>[ KE_V(x^p, t^p) \equiv \int_0^{h} \left( \frac{1}{2} \rho \left( (u_2^p)^2 + (v_2^p)^2 \right) \right) , dy ]</td>
</tr>
<tr>
<td>[ KE_{L-conv}(x^p, t^p) \equiv -\frac{\partial}{\partial x^p} \int_0^{\Delta(x)} \left( \frac{1}{2} \rho \left( (u_1^p)^2 + (v_1^p)^2 \right) \right) , dy + \left( v_1^p \right)^2 \cdot u_1^p , dy ]</td>
<td>[ KE_{V-conv}(x^p, t^p) \equiv -\frac{\partial}{\partial x^p} \int_0^{h} \left( \frac{1}{2} \rho \left( (u_2^p)^2 + (v_2^p)^2 \right) \right) , dy + \left( v_2^p \right)^2 \cdot u_2^p , dy ]</td>
</tr>
<tr>
<td>[ KE_{L-int}(x^p, t^p) \equiv \hat{m}^p \sqrt{1 + \Delta x^p} \cdot \frac{1}{2} \left( (u_1^p)^2 + (v_1^p)^2 \right) ]</td>
<td>[ KE_{V-int}(x^p, t^p) \equiv -\hat{m}^p \sqrt{1 + \Delta x^p} \cdot \frac{1}{2} \left( (u_2^p)^2 + (v_2^p)^2 \right) ]</td>
</tr>
<tr>
<td>[ PE_{L-grav}(x^p, t^p) \equiv \int_0^{\Delta(x)} \left( p_1 g_x u_1^p + p_1 g_y v_1^p \right) , dy ]</td>
<td>[ PE_{V-grav}(x^p, t^p) \equiv \int_0^{h} \left( p_2 g_x u_2^p + p_2 g_y v_2^p \right) , dy ]</td>
</tr>
<tr>
<td>[ PW_{L-conv}(x^p, t^p) \equiv -\frac{\partial}{\partial x^p} \int_0^{\Delta(x)} p_1 u_1^p , dy ]</td>
<td>[ PW_{V-conv}(x^p, t^p) \equiv -\frac{\partial}{\partial x} \int_0^{h} p_2 u_2^p , dy ]</td>
</tr>
<tr>
<td>[ PW_{L-int}(x^p, t^p) \equiv -(p_1 \frac{\partial}{\partial t} - \hat{m}^p \sqrt{1 + \Delta x^p} \cdot \frac{p_1^1}{\rho_1}) ]</td>
<td>[ PW_{V-int}(x^p, t^p) \equiv (p_2 \frac{\partial}{\partial t} - \hat{m}^p \sqrt{1 + \Delta x^p} \cdot \frac{p_2^1}{\rho_2}) ]</td>
</tr>
<tr>
<td>[ VW_{L-conv}(x^p, t^p) \equiv \frac{\partial}{\partial x^p} \int_0^{\Delta(x)} (S_{11} u_1^p + S_{21} v_1^p) , dy ]</td>
<td>[ VW_{V-conv}(x^p, t^p) \equiv \frac{\partial}{\partial x^p} \int_0^{h} (S_{11} u_2^p + S_{21} v_2^p) , dy ]</td>
</tr>
<tr>
<td>[ VW_{L-int}(x^p, t^p) \equiv (S_{12} u_1^{pl} + S_{22} v_1^{pl}) - \Delta x (S_{11} u_1^{pl} + S_{21} v_1^{pl}) ]</td>
<td>[ VW_{V-int}(x^p, t^p) \equiv \Delta x (S_{11} u_2^{pl} + S_{21} v_2^{pl}) - (S_{12} u_2^{pl} + S_{22} v_2^{pl}) ]</td>
</tr>
<tr>
<td>[ VD_L(x^p, t^p) \equiv -\int_0^{\Delta(x)} \varphi_{visc-1} , dy ]</td>
<td>[ VD_V(x^p, t^p) \equiv -\int_0^{h} \varphi_{visc-2} , dy ]</td>
</tr>
<tr>
<td>[ \equiv -\int_0^{\Delta(x)} 2\mu_1 \cdot tr(D_1 D_1^T) , dy ]</td>
<td>[ \equiv -\int_0^{h} 2\mu_2 \cdot tr(D_2 D_2^T) , dy ]</td>
</tr>
</tbody>
</table>
For steady solutions, there is no time dependence and Eq. (A.19) can be written as:

\[
0 \equiv KE_{I-\text{conv}}(x^p) + KE_{I-\text{int}}(x^p) + PE_{I-\text{grav}}(x^p) + PW_{I-\text{conv}}(x^p) \\
+ PW_{I-\text{int}}(x^p) + VW_{I-\text{conv}}(x^p) + VW_{I-\text{int}}(x^p) \\
+ VD_l(x^p)
\]  

(A. 20)

For performing stability analyses, an initial disturbance is given and the unsteady evolution of the flow variables (including the interface) are observed. Similarly, time evolutions of energy transfer terms related to disturbance kinetic energy \(KE_L(x^p, t^p) \equiv KE_{L-\text{st}}(x^p) + KE_L'(x^p, t^p)\), are obtained from term by term subtractions of Eq. (A.20) from Eq. (A.19). The resulting equation, for \(I = L\), is given below as Eq. (A.21). The flow variables are also decomposed in steady and disturbance components. For example, in the decomposition as \(\Delta(x^p, t^p) \equiv \Delta_{st}(x^p) + \Delta'(x^p, t^p)\) where \(\Delta'(x^p, t^p)\) is the disturbance component.

\[
\frac{d}{dt}KE_I'(x^p, t^p) \equiv KE_{I-\text{conv}}(x^p, t^p) + KE_{I-\text{int}}(x^p, t^p) \\
+ PE_{I-\text{grav}}(x^p, t^p) + PW_{I-\text{conv}}(x^p, t^p) \\
+ PW_{I-\text{int}}(x^p, t^p) + VW_{I-\text{conv}}(x^p, t^p) \\
+ VW_{I-\text{int}}(x^p, t^p) + VD_l'(x^p, t^p)
\]  

(A. 21)

The terms on the left and right sides of the steady Eq. (A.19), unsteady Eq. (A.18), and disturbance mechanical energy evolution equation Eq. (A.20), along with the sum of the right side terms can be plotted to understand the importance of various mechanical energy transfer mechanisms underlying the behavior of the flows.

The individual terms as well as good agreements between the summed terms on the left and right sides of Eqs. (A.19) and (A.20) are observed. The post processed agreement (at least approximate equality in terms of relative errors) between the left and right sides of these equations represent an additional verification of the quality of the solution and their convergence.
References

[27] S.A. Mitra, Development of one-dimensional and two-dimensional computational tools that simulate steady internal condensing flows in terrestrial and zero-gravity environments, Mechanical Engineering, Michigan Technological University, Houghton, MI, 2012.