Consider the set $S_1$ constructed recursively.

(i) Basis: $[1, 1] \in S_1$

(ii) Recursive step: If $[n, m] \in S_1$, then $[n + 1, m + 2(n + 1) - 1] \in S_1$.

(iii) Closure: $S_1$ consists of exactly the elements that can be obtained by starting with the basis elements of $S_1$ and applying the recursive step finitely many times to construct new elements of $S_1$.

Prove using induction that for every pair in $S_1$, the second member is the square of the first. In other words, for all $[n, m] \in S_1$, $m = n^2$.

2. (50 points) Consider the set $S_2$ constructed recursively. The first member of the basis sequence is a number, and the second member is a string. The recursive step performs an arithmetic addition on the first member and string concatenation on the second member. The symbols $a, b$ are characters, not variables.

(i) Basis: $[1, a] \in S_2$ and $[1, b] \in S_2$

(ii) Recursive step: If $[n, w] \in S_2$, then $[n + 2, awa] \in S_2$ and $[n + 2, bwb] \in S_2$.

(iii) Closure: $S_2$ consists of exactly the elements that can be obtained by starting with the basis elements of $S_2$ and applying the recursive step finitely many times to construct new elements of $S_2$.

Use induction to prove that the first member of each pair represents the length of the string which is the second member. In other words, for all $[n, m] \in S_2$, $n = \text{length}(m)$. 