The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

1. (50 points, 2+8 points each) State whether the following equalities are true or false (2 points). Explain your answer (8 points).

   (a) \(a^*b^* = a^* \cup b^*\)
   (b) \(a^* \cup b^* = b^* \cup a^*\)
   (c) \(a^*b^* = b^*a^*\)
   (d) \((a \cup b)^* = (a^*) \cup (b^*)\)
   (e) \((a \cup b)^* = (a \cup (a^*b^*))^*\)

2. (50 points, 8+2 points each) Give a regular expression for the following languages. Explain the expression by writing what each part does.

   (a) The set of strings over \(\{1, 2, 3, a, b, c\}\) that contain exactly two numbers and the sum of the numbers is even.

   (b) The set of strings over \(\{a, b, c\}\) that contains at least one \(a\) and ends in \(bb\).

   (c) The set of strings over \(\{a, b, c\}\) in which all the \(a\)'s precede the \(b\)'s, which in turn precede the \(c\)'s. It is possible that there are no \(a\)'s, or \(b\)'s, or \(c\)'s and the string is empty.

   (d) The set of strings over \(\{a, b, c\}\) in which all the \(a\)'s precede the \(b\)'s, which in turn precede the \(c\)'s. It is possible that there are no \(a\)'s, or \(b\)'s, or \(c\)'s, but \(\lambda\) is not in the language.

   (e) The set of strings over \(\{1, 2, 3, a\}\) that do not begin with 123.