1. **(20+10+10 points)** Consider the following grammar $G$. Note that $G$ was obtained by transforming the grammar $S \rightarrow aSa \mid bSb \mid a \mid b \mid \lambda$ to Chomsky Normal Form.

   $$
   S \rightarrow AR \mid BX \mid AA \mid BB \mid a \mid b \mid \lambda \\
   T \rightarrow AR \mid BX \mid AA \mid BB \mid a \mid b \\
   R \rightarrow TA \\
   X \rightarrow TB \\
   A \rightarrow a \\
   B \rightarrow b
   $$

   (a) Give the upper diagonal matrix produced by the CYK algorithm when run with $G$ and the input string $abba$. **Show all your work.**

   (b) Is $abba \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

   (c) Is $abb \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

2. **(10 points)** Let $M$ be the PDA in Example 7.1.3 on page 226. $M$ accepts even length palindromes. Show the computation trees for the strings $aabbba$ and $aba$.

   $$
   M: \\
   b \, \lambda/B \\
   a \, \lambda/A \\
   b \, B/\lambda \\
   a \, A/\lambda
   $$

3. **(50 points)** Construct PDAs that accept each of the following languages.

   Explain how the PDA works: write the algorithm it follows, label the specific portions of the machine with the task performed (5 points for each machine).

   (a) $\{a^i b^j \mid 0 \leq i \leq j\}$

   (b) $\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + k = j\}$