Online Graph Pruning for Pathfinding on Grid Maps

Daniel Harabor and Alban Grastien, AAAI 2011
Presented by James Walker
Synopsis

- An algorithm for improving A* performance on uniform-cost grid search spaces
- Works by only expanding nodes called “jump points”
- In benchmarks, improves A* performance dramatically and outperforms other A* optimizations including hierarchical pathfinding
Characteristics

- Search performance dramatically improved
- Guaranteed optimality
- No preprocessing required
- Fairly simple to implement
- Orthogonal to existing techniques (so it can be combined with other optimizations)
- Essentially no drawbacks!!
Example

When moving in a straight line toward y, x dominates all shaded nodes so there's no need to expand them.
Example Cont.

- Repeat until another jump node is encountered
- If blocked by obstacles, conclude that searching in this direction is fruitless and generate nothing
Pruning Rules

- Goal: For the current node, identify which of its neighbors don't need to be expanded to reach the goal optimally.
- 2 cases: straight move & diagonal move
Straight move pruning

- Prune any node $n \in \text{neighbors}(x)$ which satisfies the following dominance constraint:

$$\text{len}( <p(x), \ldots, n> \setminus x ) \leq \text{len}( <p(x), x, n> )$$

- Example: Prune all neighbors except 5
Diagonal move pruning

- Almost identical to previous case, except strict dominance is required:

\[ \text{len}( \langle p(x), \ldots, n \rangle \setminus x ) < \text{len}( \langle p(x), x, n \rangle ) \]

- Example: Prune all neighbors except 2,3,5
Forced evaluation

- Sometimes an obstacle can force us to evaluate a node we would otherwise prune:

\[ \text{len( <p(x), x, n> ) < len( <p(x), ..., n> \setminus x )} \]

- Example: Evaluation of 3 is forced.
Jump Point Formal Definition

- Node $y$ is a jump point from $x$, heading in direction $d$, if $y$ minimizes the value $k$ such that $y = x + kd$ and one of the following conditions holds:

  1. $y$ is the goal
  2. $y$ has at least one neighbor with forced evaluation
  3. $d$ is a diagonal move and there exists a node $z = y + k_i d_i$ which lies $k_i \in N$ steps in direction $d \in \{d_1, d_2\}$ s.t. $z$ is a jump point from $y$ by condition 1 or 2.
Condition 3 Example

- Intuitively, it's simple: If you're on a diagonal and you turn to a non-diagonal to the next jump point, you yourself are a jump point.
Proof of Optimality

• For each optimal length path in a grid map there exists an equivalent length path which can be found by only expanding jump point nodes.

• Turning points:

• Note that straight-to-straight turning points are trivially suboptimal.
Proof of Optimality Cont.

• **Diagonal-first path:** A path $\pi$ is diagonal-first if it contains no straight-to-diagonal turning point $<n_{k-1}, n_k, n_{k+1}>$ which could be replaced by a diagonal-to-straight turning point $<n_{k-1}, n'_k, n_{k+1}>$ s.t. that the length of $\pi$ remains unchanged.

• **Lemma 1:** Each turning point along an optimal diagonal-first path $\pi'$ is also a jump point (see paper for proof of lemma).
Proof of Optimality Cont.

• Let $\pi$ be an arbitrary optimal path and $\pi'$ a diagonal-first symmetric equivalent. Every turning point in $\pi'$ is expanded optimally when searching with jump point pruning.

• Divide $\pi'$ into segments $\pi'_i$, where all moves in each $\pi'_i$ are in the same direction. Every node at beginning and end of $\pi'_i$ is a turning point.
Proof of Optimality Cont.

- Because each $\pi'_i$ consists of single-direction moves (straight or diagonal), jump points will travel from the start to end nodes of $\pi'_i$ w/o necessarily expanding all intermediate nodes; this part is guaranteed to be optimal by the pruning rules.

- **Invoke Lemma 1.** Each start and end node of $\pi'_i$ is a turning point, thus also a jump point, and is expanded.
Proof of Optimality Concluded

• Because all endpoint nodes of each segment $\pi'_i$ are expanded optimally, and all straight-line paths between endpoints are optimal, the entire path is optimal.
Experimental Results

Nodes Expanded

<table>
<thead>
<tr>
<th></th>
<th>A. Depth</th>
<th>B. Gate</th>
<th>D. Age</th>
<th>Rooms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jump Points</td>
<td>20.37</td>
<td>215.36</td>
<td>35.95</td>
<td>13.41</td>
</tr>
<tr>
<td>Swamps</td>
<td>1.89</td>
<td>2.44</td>
<td>2.99</td>
<td>4.70</td>
</tr>
<tr>
<td>HPA*</td>
<td>4.14</td>
<td>9.37</td>
<td>9.63</td>
<td>5.11</td>
</tr>
</tbody>
</table>

Computation Time

- Search Time Speedup: Adaptive Depth
- Search Time Speedup: Baldur’s Gate (512x512)
- Search Time Speedup: Dragon Age
- Search Time Speedup: Rooms
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Thank you.
Any questions?