Reminders

* New homework due Sunday
* No class next week Wednesday and Friday (Nov. 7) (Nov. 9)

* This is the 9th week of classes
  9: Ch. 13; Ch. 14 (BBN)
  10: Ch. 14 (BBN)
  11: Ch. 14 (BBN); Ch. 16 (Simple Decision Making)

Break week: Thanksgiving break

12: Ch. 16, review
   Exam 2 on Thursday
13: presentations
14: presentations

"Class savings account"
- (Sep. 6, review)
- Sep. 20, Thursday, make-up class
- Sep. 27, Thursday, make-up class
- Oct. 11, Thursday, Exam 1

Previous class
  Wumpus world - formula derivation

Career Fair
   Nov. 7 W
   Nov. 9 F
\[ P(P_{1,3}|\text{known, b}) \]

\[ = \frac{P(P_{1,3}, \text{known, b})}{P(\text{known, b})} \] 

apply definition of conditional probability

\[ = \alpha P(P_{1,3}, \text{known, b}) \]

put the denominator as \( \alpha \)

\[ = \alpha \sum_{\text{unknown}} P(P_{1,3}, \text{known, b, unknown}) \]

sum over hidden variables

\[ \rightarrow \text{a breeze is caused by a pit} \]

\[ = \alpha \sum_{\text{unknown}} P(b|P_{1,3}, \text{known, unknown}) P(P_{1,3}, \text{known, unknown}) \]

predict rule

\[ \text{unknown} = \text{frontier} \cup \text{other} \]

\[ = \alpha \sum_{\text{frontier}} \sum_{\text{other}} P(b|P_{1,3}, \text{known, frontier, other}) P(P_{1,3}, \text{known, frontier, other}) \]

\[ = \alpha \sum_{\text{frontier}} \sum_{\text{other}} P(b|P_{1,3}, \text{known, frontier}) P(P_{1,3}, \text{known, frontier, other}) \]

\[ = \alpha \sum_{\text{frontier}} P(b|P_{1,3}, \text{known, frontier}) \sum_{\text{other}} P(P_{1,3}, \text{known, frontier, other}) \]

independence

\[ = \alpha \sum_{\text{frontier}} P(b|P_{1,3}, \text{known, frontier}) \sum_{\text{other}} P(P_{1,3}) P(\text{known}) P(\text{frontier}) P(\text{other}) \]

\[ = \alpha P(\text{known}) P(P_{1,3}) \sum_{\text{frontier}} P(b|P_{1,3}, \text{known, frontier}) \sum_{\text{other}} P(\text{frontier}) P(\text{other}) \]

\[ = \alpha P(P_{1,3}) \sum_{\text{frontier}} P(b|P_{1,3}, \text{known, frontier}) \sum_{\text{other}} P(\text{frontier}) P(\text{other}) \]

\[ = \alpha P(P_{1,3}) \sum_{\text{frontier}} P(b|P_{1,3}, \text{known, frontier}) P(\text{frontier}) \sum_{\text{other}} P(\text{other}) \]

\[ = \alpha P(P_{1,3}) \sum_{\text{frontier}} P(b|P_{1,3}, \text{known, frontier}) P(\text{frontier}) \]

Results using conditional independence

\[
\begin{array}{c}
\text{1.3} \\
\frac{1.1}{\text{OK}} \\
\frac{1.2}{\text{OK}} \\
\frac{1.3}{\text{OK}} \\
\frac{1.4}{\text{OK}} \\
\frac{1.5}{\text{OK}} \\
\frac{2.1}{\text{OK}} \\
\frac{2.2}{\text{OK}} \\
\frac{2.3}{\text{OK}} \\
\frac{2.4}{\text{OK}} \\
\frac{3.1}{\text{OK}} \\
\frac{3.2}{\text{OK}} \\
\frac{3.3}{\text{OK}} \\
\frac{3.4}{\text{OK}} \\
\frac{3.5}{\text{OK}} \\
\frac{3.6}{\text{OK}} \\
\end{array}
\]

\[ 0.2 \times 0.2 = 0.04 \]

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\[ 0.8 \times 0.2 = 0.16 \]

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\[ P(P_{1,3}|\text{known, b}) \approx < 0.31, 0.69 > \]

\[ P(P_{2,2}|\text{known, b}) \approx < 0.86, 0.14 > \]
Details of the wumpus world calculations

We know the following facts (evidence): \( b = \neg b_{1,1} \land b_{1,2} \land b_{2,1} \) known \( = \neg p_{1,1} \land \neg p_{1,2} \land \neg p_{2,1} \)

\[ P(P_{1,3} \mid \text{known, } b) = \alpha' \left( P(P_{1,3}) \sum_{\text{frontier}} P( b \mid \text{known, } P_{1,3}, \text{frontier}) P(\text{frontier}) \right) \]

First, compute with \( P_{1,3} = \text{true} \):

Compute the sum over the frontier:

\[
\begin{align*}
P(b \mid \text{known, } P_{1,3}, P_{2,2}, P_{3,1}) P(P_{2,2}, P_{3,1}) &= 1 \times 0.2 \times 0.2 = 0.04 \\
P(b \mid \text{known, } P_{1,3}, \neg P_{2,2}, P_{3,1}) P(\neg P_{2,2}, P_{3,1}) &= 1 \times 0.8 \times 0.2 = 0.16 \\
P(b \mid \text{known, } P_{1,3}, P_{2,2}, \neg P_{3,1}) P(P_{2,2}, \neg P_{3,1}) &= 1 \times 0.2 \times 0.8 = 0.16 \\
P(b \mid \text{known, } P_{1,3}, \neg P_{2,2}, \neg P_{3,1}) P(\neg P_{2,2}, \neg P_{3,1}) &= 0 \times 0.8 \times 0.8 = 0.00
\end{align*}
\]

The sum is: \( 0.04 + 0.16 + 0.16 + 0.00 = 0.36 \). \( P(P_{1,3}) = 0.2 \), therefore:

\[ P(P_{1,3}) \sum_{\text{frontier}} P( b \mid \text{known, } P_{1,3}, \text{frontier}) P(\text{frontier}) = 0.2 \times 0.36 = 0.072. \]

Then, compute with \( P_{1,3} = \text{false} \):

Compute the sum over the frontier:

\[
\begin{align*}
P(b \mid \text{known, } \neg P_{1,3}, P_{2,2}, P_{3,1}) P(P_{2,2}, P_{3,1}) &= 1 \times 0.2 \times 0.2 = 0.04 \\
P(b \mid \text{known, } \neg P_{1,3}, \neg P_{2,2}, P_{3,1}) P(\neg P_{2,2}, P_{3,1}) &= 0 \times 0.8 \times 0.2 = 0.00 \\
P(b \mid \text{known, } \neg P_{1,3}, P_{2,2}, \neg P_{3,1}) P(P_{2,2}, \neg P_{3,1}) &= 1 \times 0.2 \times 0.8 = 0.16 \\
P(b \mid \text{known, } \neg P_{1,3}, \neg P_{2,2}, \neg P_{3,1}) P(\neg P_{2,2}, \neg P_{3,1}) &= 0 \times 0.8 \times 0.8 = 0.00
\end{align*}
\]

The sum is: \( 0.04 + 0.00 + 0.16 + 0.00 = 0.20 \). \( P(\neg P_{1,3}) = 0.8 \), therefore:

\[ P(P_{1,3}) \sum_{\text{frontier}} P( b \mid \text{known, } P_{1,3}, \text{frontier}) P(\text{frontier}) = 0.8 \times 0.20 = 0.16. \]

\[ P( P_{1,3} \mid \text{known, } b) = \alpha' < 0.072, 0.16 > = \langle \frac{0.072}{0.072+0.16}, \frac{0.16}{0.072+0.16} \rangle >= < 0.31, 0.69 > \]

\[ P(P_{1,3} \mid \text{known, } b) = 0.31 \quad (= P(P_{3,1} \mid \text{known, } b) \text{ by symmetry}) \]