Decision Making Under Uncertainty

- algorithms for reasoning about probabilities
- represent the world using random variables

Bayesian Belief Networks (BBNs)

- graph (directed, acyclic)
- root node
- prior
  - \(p(S=h)\)
  - \(p(S=m)\)
  - \(p(S=e)\)

The parent of a node is its "cause" 

\[
\begin{array}{c|c|c|c|c|c|c}
B & S & & & & & \\
\hline
1 & t & h & k_1 & & & \\
2 & t & m & k_2 & & & \\
3 & t & l & k_3 & & & \\
\end{array}
\]

- \(n\) RVs
- \(2^n\) entries
- \((2^n - 1)\) impossible/hard to store
- impossible/hard to figure out what the probabilities are

\[
p(D|S=h) = k_4 \\
p(D|S=m) = k_5 \\
p(D|S=e) = k_6 \\
\]

\[N \sim \beta(\theta, \alpha)\]

\[17 \pm 10^2 \quad 0^+ \]
Rain causes someone to carry an umbrella

\[ P(R) = 0.9 \]
\[ P(U|R) = 0.9 \]
\[ P(U|\neg R) = 0.1 \]

\[ P(U) = 0.9 \]
\[ P(U|\neg R) = 0.55 \]

\[ P(D|A) \]
\[ P(E|A) \]
\[ P(D|\neg A) \]
\[ P(E|\neg A) \]

\[ P(B) \]
\[ P(C) \]
\[ P(F|B) \]
\[ P(G|C) \]

\[ P(D|B,C) \]
\[ P(E|B,C) \]
\[ P(F|B,C) \]
\[ P(G|B,C) \]

CPT conditional probability tables
BN with $n$ variables
each variable has $k$ parents (at most)
max size of CP Ts: $2^k$

\[ \mathbb{P}(\neg b, \neg d, S = e) \]

\[ \mathbb{P}(\neg b \mid \neg d, S = e) \times \mathbb{P}(\neg d, S = e) \]

\[ \mathbb{P}(\neg b \mid S = e) \times \mathbb{P}(\neg d \mid S = e) \]

\[ 0.8 \]

\[ 0.8 \times 0.33 = 0.264 \]

\[ \frac{169}{800} = 0.21125 \]