A micromechanical finite element model for linear and damage-coupled viscoelastic behaviour of asphalt mixture

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SUMMARY

This study presents a finite element (FE) micromechanical modelling approach for the simulation of linear and damage-coupled viscoelastic behaviour of asphalt mixture. Asphalt mixture is a composite material of graded aggregates bound with mastic (asphalt and fine aggregates). The microstructural model of asphalt mixture incorporates an equivalent lattice network structure whereby intergranular load transfer is simulated through an effective asphalt mastic zone. The finite element model integrates the ABAQUS user material subroutine with continuum elements for the effective asphalt mastic and rigid body elements for each aggregate. A unified approach is proposed using Schapery non-linear viscoelastic model for the rate-independent and rate-dependent damage behaviour. A finite element incremental algorithm with a recursive relationship for three-dimensional (3D) linear and damage-coupled viscoelastic behaviour is developed. This algorithm is used in a 3D user-defined material model for the asphalt mastic to predict global linear and damage-coupled viscoelastic behaviour of asphalt mixture.

For linear viscoelastic study, the creep stiffnesses of mastic and asphalt mixture at different temperatures are measured in laboratory. A regression-fitting method is employed to calibrate generalized Maxwell models with Prony series and generate master stiffness curves for mastic and asphalt mixture. A computational model is developed with image analysis of sectioned surface of a test specimen. The viscoelastic prediction of mixture creep stiffness with the calibrated mastic material parameters is compared with mixture master stiffness curve over a reduced time period.

In regard to damage-coupled viscoelastic behaviour, cyclic loading responses of linear and rate-independent damage-coupled viscoelastic materials are compared. Effects of particular microstructure parameters on the rate-independent damage-coupled viscoelastic behaviour are also investigated with finite element simulations of asphalt numerical samples. Further study describes loading rate effects on the asphalt viscoelastic properties and rate-dependent damage behaviour. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: micromechanical modelling; viscoelasticity; damage behaviour; finite elements; asphalt mixture

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1. INTRODUCTION

Asphalt mixture is a very complex heterogeneous and time-dependent material, and generally contains aggregates, mastic and void space. Thus, the macro load-carrying behaviour depends on many micro-phenomena that occur at the aggregate/mastic level. Some important micro behaviours are related to mastic properties including volume percentage, elastic/viscoelastic moduli, damage/time-dependent response, aging hardening, microcracking, and debonding from aggregates. Other microstructural features include aggregate size, shape, texture and packing geometry. Because of these issues it appears that a micromechanical model would be best suited to properly simulate such a material.

Recently, many studies have been investigating the micromechanical behaviour of particulate, porous and heterogeneous materials. For example, studies on cemented particulate materials by Dvorkin et al. [1] and Zhu et al. [2] provided information on the normal and tangential load transfer between cemented particles. Applications of such contact-based micromechanical analysis for asphalt mixture behaviour have been reported by Chang and Gao [3], Cheung et al. [4] and Zhu and Nodes [5].

Numerical modelling of cemented particulate materials has generally used both finite (FEM) and discrete (DEM) element methods. The DEM analyses particulate systems by modelling the translational and rotational behaviour of each particle using Newton’s second law with appropriate interparticle contact forces. Normally, the scheme establishes an explicit, time-stepping procedure to determine each of the particle motions. DEM studies on cemented particulate materials include the work by Rothenburg et al. [6], Chang and Meegoda [7], Trent and Margolin [8], Buttlar and You [9], Ullidtz [10], and Sadd and Gao [11].

In regard to finite element modelling, Sepehr et al. [12] used an idealized finite element microstructural model to analyse the behaviour of an asphalt pavement layer. Soares et al. [13] used cohesive zone elements to develop a micromechanical fracture model of asphalt mixture. A particular finite element approach to simulate particulate materials has used an equivalent lattice network system to represent the interparticle load transfer behaviour. Guddati et al. [14] recently presented a random truss lattice model to simulate microdamage in asphalt mixture and demonstrated some interesting failure patterns in an indirect tension test geometry. Sadd et al. [15,16] employed a micro-frame network model to investigate the damage behaviour of asphalt mixture, this model used a special purpose finite element that incorporates the mechanical load-carrying response between neighbouring particles. Bahia et al. [17] have also used finite elements to model the aggregate-mastic response of asphalt mixture.

Damage mechanics provides a viable framework for the description of asphalt stiffness degradation, microcrack initiation, growth and coalescence, and damage-induced anisotropy. Continuum damage mechanics is based on the thermodynamics of irreversible processes to characterize elastic-coupled damage behaviours. Chaboche [18], and Simo and Ju [19] developed strain- and stress-based anisotropic continuum damage models, while Kachanov [20] proposed a microcrack-related continuum damage model for rate-independent solids. For viscoelastic materials, Taylor et al. [21] developed a computational algorithm to analyse thermomechanical behaviour. Some researches such as Simo [22] have proposed viscoelastic continuum damage models (VCDM) to describe the damage behaviour of viscoelastic materials. Schapery [23] developed detailed viscoelastic damage models based on non-equilibrium thermodynamics, viscoelastic fracture mechanics and elastic–viscoelastic correspondence principles, and a later study [24] incorporated a work potential theory. Park and Schapery [25] proposed an explicit
viscoelastic damage model for particulate composites, and Park et al. [26] applied this model to uniaxial behaviour of asphalt mixture. Recently, Schapery [27] developed constitutive equations that account for effects of viscoelastic, viscoplastic, growing damage and aging. Some recent studies were conducted on damage constitutive modelling of viscoelastic composite materials, such as Canga et al. [28], Kaliske et al. [29], Haj-Ali and Muliana [30], and Kumar and Talreja [31]. Wu and Harvey [32] recently applied a 3D continuum damage mechanics method to model the cracking behaviour of asphalt mixture.

This paper presents a micromechanical modelling scheme for the linear and damage-coupled viscoelastic behaviour of asphalt mixture by using finite element methods. The model first incorporates an equivalent lattice network structure whereby intergranular load transfer is simulated through an effective asphalt mastic zone. The previous work Sadd et al. [15,16] employed a network of user-defined micro-frame elements with a special stiffness matrix developed to predict the load transfer between cemented particles. This study incorporates the user-defined material subroutine with continuum elements for the effective asphalt mastic and rigid body defined with rigid elements for each aggregate. A unified approach for the rate-independent and rate-dependent damage behaviour has been developed using Schapery’s nonlinear viscoelastic model. Properties of the continuum elements are specified through a user material subroutine within the ABAQUS code and this allows linear and damage-coupled viscoelastic constitutive behaviour of the mastic cement to be incorporated. Section 2 of this paper introduces a finite element incremental algorithm with recursive relationships for viscoelastic behaviour. This algorithm is later used in the linear and damage viscoelastic modelling of the asphalt mastic in the proposed microstructure model. Section 3 presents microstructural modelling of heterogeneous asphalt mixture. For the viscoelastic simulation, Section 4 compares the linear viscoelastic prediction of mixture creep stiffness using calibrated mastic properties and an image computational sample with laboratory test data. In regard to damage-coupled behaviour, the cyclic loading responses of viscoelastic asphalt mixture with linear and rate-independent damage-coupled viscoelastic models are compared in Section 5. Section 6 studies the micro-parametric effects on rate-independent damage viscoelastic behaviour using finite element simulation on numerical samples with controllable microstructure. Finally, Section 7 investigates rate-dependent damage behaviour and the effect of loading rate on the viscoelastic properties and rate-dependent damage behaviour.

2. DAMAGE-COUPLED VISCOELASTIC MODEL

2.1. One-dimensional linear viscoelastic model

A generalized Maxwell model is commonly used to simulate the constitutive behaviour of linear solid viscoelastic material. This model consists of an elastic spring with constant $E_\infty$ in parallel with $M$ Maxwell elements. The stress–strain relationship for this model can be expressed as a hereditary integral

$$\sigma = E_\infty \varepsilon + \int_0^t E_i \frac{d\varepsilon(\tau)}{d\tau} d\tau$$  \hspace{1cm} (1)

where $E_i$ is expressed with a Prony series

$$E_i = \sum_{m=1}^{M} E_m e^{-(t-\tau)/\rho_m} \quad \text{and} \quad \rho_m = \frac{\eta_m}{E_m}$$  \hspace{1cm} (2)
In these equations, $E_\infty$ is the relaxed elastic modulus, $E_t$ is the transient modulus as a function of the time, $E_m$, $\eta_m$ and $\rho_m$ are the spring constant, dashpot viscosity and relaxation time, respectively, for the $m$th Maxwell element.

The reduced time (effective time) is defined by using time–temperature superposition principle as

$$\tilde{\xi}(t) = \int_0^t \frac{1}{\tau_T} \, d\tau$$

where the term $\tau_T = \tau_T(T(\tau))$ is a temperature-dependent time-scale shift factor.

A displacement-based incremental finite element modelling scheme with constant strain rate over each increment has been developed. An incremental numerical algorithm for the linear viscoelastic integral has been used from the work of Zocher and Groves [33]. This algorithm was developed in closed form and results in a recursive relationship. This method creates an incremental formulation of the current stress state from recursive variables stored in the previous step, and current variables of time and strain increments.

Assuming the stress is known at the reduced time $\tilde{\xi}_{n-1}$, the current stress at reduced time $\tilde{\xi}_n$, according to (1), is given by

$$\sigma(\tilde{\xi}_n) = E_\infty \varepsilon(\tilde{\xi}_n) + \int_0^{\tilde{\xi}_n} E_t(\tilde{\xi}_n - \tilde{\xi}) \frac{d\varepsilon(\tilde{\xi})}{d\tilde{\xi}} \, d\tilde{\xi}$$

Its incremental form can be written in two parts,

$$\Delta \sigma = E' \cdot \Delta \varepsilon + \Delta \sigma^R$$

The first part includes the integration from the previous step $\tilde{\xi}_{n-1}$ to the current step $\tilde{\xi}_n$. The reduced time increment is defined as $\Delta \tilde{\xi} = \tilde{\xi}_n - \tilde{\xi}_{n-1}$. This formulation assumes the incremental strain $\Delta \varepsilon = \varepsilon_n - \varepsilon_{n-1}$ is known, and the strain changes with a constant strain rate $\tilde{R}_e$ during each interval $\tilde{\xi}_{n-1} \leq \tilde{\xi}_n \leq \tilde{\xi}_n$. So the incremental modulus $E'$ and strain rate $\tilde{R}_e$ can be expressed by

$$E' = E_\infty + \sum_{m=1}^M \frac{E_m \rho_m}{\Delta \tilde{\xi}} \left(1 - e^{-\Delta \tilde{\xi}/\rho_m}\right) \quad \text{and} \quad \tilde{R}_e = \frac{\Delta \varepsilon}{\Delta \tilde{\xi}}$$

Note that the incremental stiffness $E'$ is not dependent on the time if $\Delta \tilde{\xi}$ remains constant.

The second part of relation (5) is formulated with integration from 0 to the previous step $\tilde{\xi}_{n-1}$, and this leads to a recursive relation with the history variables $S_m$.

$$\Delta \sigma^R = \sum_{m=1}^M \left(1 - e^{-\Delta \tilde{\xi}/\rho_m}\right) S_m(\tilde{\xi}_n) \quad \text{and} \quad S_m(\tilde{\xi}_n) = E_m R_e \rho_m (1 - e^{-\Delta \tilde{\xi}/\rho_m}) + S_m(\tilde{\xi}_{n-1}) e^{-\Delta \tilde{\xi}/\rho_m}$$

For the initial increment, the history variable $S_m(\tilde{\xi}_1)$ equals to $E_m R_e \rho_m (1 - e^{-\Delta \tilde{\xi}/\rho_m})$ and is similar to the following formulations.

### 2.2. One-dimensional damage-coupled viscoelastic model

Rate-independent failure processes are characterized by the fact that damage growth is the only dissipative mechanism and that the current state does not depend on rate effects [34]. The rate-independent damage variable is defined as a function of maximum equivalent strain [22]. A second damage mechanism referred to as rate-dependent damage is caused by the viscoelastic time-dependent material property and loading rate effects, and its damage variable is based on
the equivalent strain rate. A unified approach is presented for both failure mechanisms by using the Schapery model.

The original Schapery non-linear viscoelastic model [35] was given as

$$
\sigma(\xi) = h_E E_{\infty} \epsilon(\xi) + \int_0^\xi h_1 E_t(\xi - \xi') \frac{d(h_2 \hat{\epsilon}(\xi'))}{d\xi'} d\xi' 
$$

(8)

This model incorporates three different non-linear parameters: \( h_E \) is the non-linear factor of the relaxed elastic modulus \( E_{\infty} \), \( h_1 \) measures the non-linearity effect in the transient modulus \( E_t \), and \( h_2 \) accounts for the loading rate effect.

Following form (8), a unified damage-coupled viscoelastic model for both failure mechanisms is proposed by replacing three non-linear parameters with damage variables which would be expressed with damage evolution functions.

$$
\sigma(\xi) = h_E(\epsilon_{\text{max}}) E_{\infty} \epsilon(\xi) + \int_0^\xi h_1(\epsilon_{\text{max}}) E_t(\xi - \xi') \frac{d(h_2 \hat{\epsilon}(\xi'))}{d\xi'} d\xi' 
$$

(9)

where \( h_E \) and \( h_1 \) are the elastic and viscoelastic damage variables for the rate-independent failure behaviour, \( h_2 \) is the rate-dependent damage variable dependent on the strain rate \( \dot{\epsilon} \). The rate-independent variables \( h_E \) and \( h_1 \) are functions of the maximum strain \( \epsilon_{\text{max}} \), which is defined as the maximum value over the past history up to the current time \( \xi \),

$$
\epsilon_{\text{max}} = \max(\epsilon(\xi')) \quad \xi' \in [0, \xi] 
$$

(10)

The elastic damage variable \( h_E = 1 - \Omega \) measures the relaxed elastic stiffness reduction, and can be described by using the inelastic damage evolution law in Sadd et al. [15,16],

$$
h_E(\epsilon_{\text{max}}) = e^{-b(\epsilon_{\text{max}}/\epsilon_0)} 
$$

(11)

where the material parameters \( \epsilon_0 \) and \( b \) are related to the softening strain and damage evolution rate, respectively.

The viscoelastic variable \( h_1 \) measures the damage effect in the transient modulus, and is chosen with the following exponential form by Simo [22],

$$
h_1(\epsilon_{\text{max}}) = \beta + (1 - \beta) \frac{1 - e^{-\epsilon_{\text{max}}/\epsilon_0}}{\epsilon_{\text{max}}/\epsilon_0} \quad \beta \in [0, 1] 
$$

(12)

The variable \( h_1 \) will reduce from 1 to \( \beta \) as the maximum strain \( \epsilon_{\text{max}} \) increases, and \( \epsilon_0 \) is also the softening strain.

The rate-dependent damage variable \( h_2 \) is proposed as

$$
h_2 = \left(1 + \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_C} - 1\right)^{-\gamma}\right)^{-1} \Rightarrow h_2 = \begin{cases} 
\left(\frac{\dot{\epsilon}}{\dot{\epsilon}_C}\right)^{-\gamma} & \text{if } \dot{\epsilon} \geq \dot{\epsilon}_C \\
1 & \text{if } \dot{\epsilon} < \dot{\epsilon}_C 
\end{cases} 
$$

(13)

where \( \dot{\epsilon}_C \) is the threshold strain rate that determines the start of the rate-dependent damage, and \( \gamma \geq 0 \) is related to the rate-dependent damage evolution rate. When the parameter \( \gamma = 0 \) or loading rate \( \dot{\epsilon} < \dot{\epsilon}_C \), \( h_2 = 1 \) and this model has only rate-independent damage behaviour. The influence of parameters \( \gamma \) and \( \dot{\epsilon}_C \) on model rate-dependent damage behaviour will be illustrated in the following loading rate effect study.

The incremental formulation of the 1D damage-coupled viscoelastic behaviour can be established by using a similar procedure as used in the previous section. According to
relation (9), the current stress at reduced time \( \xi_n \) is given by

\[
\sigma(\xi_n) = \sigma_e(\xi_n) + \int_0^{\xi_n} \sigma(\xi_n) \, \frac{d(h_e(\xi_n) \xi(\xi_n))}{d\xi_n} \, d\xi_n
\]

and its incremental form can also be formulated into two parts

\[
\Delta \sigma = E' \cdot \Delta e + \Delta \sigma^R
\]

Assuming again that the strain changes with constant rate \( R_e \) during each interval \( \xi_{n-1} \leq \xi' \leq \xi_n \), the first term in Equation (15) gives the incremental modulus \( E' \) as

\[
E' = h_e(\xi_{n-1}) E'_e + h_e(\xi_{n-1}) h_2(R_e) \sum_{m=1}^{N} E_m \rho_m (1 - e^{-\Delta \xi/\rho_m})
\]

where

\[
R_e = \frac{\Delta e}{\Delta \xi} \quad \text{and} \quad h_2(R_e) = \begin{cases} \left( \frac{R_e}{\hat{e}_c} \right)^{-2} & \text{if } R_e \geq \hat{e}_c \\ 1 & \text{if } R_e < \hat{e}_c \end{cases}
\]

This incremental modulus \( E' \) includes both rate-independent and rate-dependent damage variables. The second term of (15) also leads to a recursive relation with the history variables \( S_m \), which includes the viscoelastic damage variable \( h_1 \) and rate-dependent damage variable \( h_2 \),

\[
\Delta \sigma^R = \sum_{m=1}^{N} -(1 - e^{-\Delta \xi/\rho_m}) S_m(\xi_n)
\]

and

\[
S_m(\xi_n) = E_m h_1(\xi_{n-1}) h_2(R_e) R_e \rho_m (1 - e^{-\Delta \xi/\rho_m}) + S_m(\xi_{n-1}) e^{-\Delta \xi/\rho_m}
\]

This model includes two damage mechanisms when the strain rate \( \hat{e} \geq \hat{e}_C \), and has only rate-independent damage behaviour if the strain rate is less than this threshold value.

2.3. Three-dimensional damage-coupled viscoelastic model

The numerical formulations for 1D damage-coupled viscoelastic behaviour are now to be generalized to a multiaxial (3D) constitutive formulation for isotropic matrix material. As employed by Simo [22], and Haj-Ali and Muliana [30], uncoupled volumetric and deviatoric stress–strain relations are assumed. The formulation also assumes the incremental strain tensor is known and the strains change linearly during each interval \( \xi_{n-1} \leq \xi' \leq \xi_n \).

The stress and strain tensors can be decomposed into the usual sum of volumetric and deviatoric parts,

\[
\sigma_{ij} = 1/3 \sigma_{kk} \delta_{ij} + \sigma_{ij}, \quad \varepsilon_{ij} = 1/3 \varepsilon_{kk} \delta_{ij} + \varepsilon_{ij}
\]

The elastic stress–strain relations for the volumetric and deviatoric behaviour are expressed by

\[
\sigma_{kk} = 3K \varepsilon_{kk}, \quad \sigma_{ij} = 2G \varepsilon_{ij}
\]

where \( K = E/(1-2v) \) and \( G = E/(2(1+v)) \) are the elastic bulk and shear modulus. The terms \( K_{\infty} \) and \( G_{\infty} \) are the relaxed bulk and shear moduli, and \( K_m \) and \( G_m \) are the bulk and shear constants for the spring in the \( m \)th Maxwell element.
By applying the previous 1D damage-coupled viscoelastic model, the volumetric constitutive relationship is expressed with the volumetric stress \( \sigma_{kk}(\xi) \) and strain \( \dot{\epsilon}_{kk} \) in the general form

\[
\sigma_{kk}(\xi) = 3K_\infty h_t(e_{\text{max}}^{kk}) \dot{\epsilon}_{kk}(\xi) + \int_0^\xi 3K_t(\xi - \xi')h_1(e_{\text{max}}^{kk}) \frac{d(h_2(\dot{\epsilon}_{kk}) \dot{\epsilon}_{kk}(\xi'))}{d\xi'} d\xi'
\]

(20)

where \( K_t(\xi - \xi') = \sum_{m=1}^M K_m e^{-(\xi - \xi')/\rho_m} \) is the transient bulk modulus, \( e_{\text{max}}^{kk} \) is the maximum volumetric strain, and \( \dot{\epsilon}_{kk} \) is the volumetric strain rate. It is assumed that tension and compression damage behaviours are independent, so two different history variables \( e_{\text{max}}^{kk} \) are used for the tension and compression volumetric behaviours.

Following the previous formulation procedures, the incremental formulation of the volumetric behaviour is obtained with constant volumetric strain rate \( R_{kk} = \Delta \dot{\epsilon}_{kk}/\Delta \xi \),

\[
\Delta \sigma_{kk} = 3 \left[ K_\infty h_t(e_{\text{max}}^{kk}) + \sum_{m=1}^N h_1(e_{\text{max}}^{kk}) h_2(R_{kk}) \frac{K_m \rho_m}{\Delta \xi} (1 - e^{-\Delta \xi/\rho_m}) \right] \Delta \dot{\epsilon}_{kk} + \Delta \sigma_{kk}^R
\]

(21)

where

\[
h_2(R_{kk}) = \begin{cases} \left( \frac{R_{kk}}{\dot{\epsilon}_C} \right)^{-x} & \text{if } R_{kk} \geq \dot{\epsilon}_C \\ 1 & \text{if } R_{kk} < \dot{\epsilon}_C \end{cases}
\]

and the residual part \( \Delta \sigma_{kk}^R \) can be expressed in a recursive relation with the history variable \( S_m(\xi) \),

\[
\Delta \sigma_{kk}^R = \sum_{m=1}^M (1 - e^{-\Delta \xi/\rho_m}) S_m(\xi)
\]

(22)

For the deviatoric behaviour, the constitutive relationship is written using deviatoric stress \( \dot{\sigma}_{ij} \) and strain \( \dot{\epsilon}_{ij} \),

\[
\dot{\sigma}_{ij}(\xi) = 2G_t h_t(e_{\text{max}}^{ij}) \dot{\epsilon}_{ij}(\xi) + \int_0^\xi 2G_t(\xi - \xi') h_1(e_{\text{max}}^{ij}) \frac{d(h_2(\dot{\epsilon}_{ij}) \dot{\epsilon}_{ij}(\xi'))}{d\xi'} d\xi'
\]

(23)

where \( G_t(\xi - \xi') = \sum_{m=1}^N G_m e^{-(\xi - \xi')/\rho_m} \) is the transient shear modulus, \( e_{\text{max}}^{ij} \) is the maximum equivalent strain, and the equivalent strain \( \dot{\epsilon}_{ij} \) and strain rate \( \dot{\epsilon}_e \) are defined as

\[
\begin{align*}
\dot{\epsilon}_e &= \sqrt{\frac{3}{2}} \dot{\epsilon}_{ij} \\
\dot{\epsilon}_e &= \sqrt{\frac{3}{2}} \dot{\epsilon}_{ij}
\end{align*}
\]

(24)

The formulation of the deviatoric behaviour is obtained with constant deviatoric strain rate \( \dot{R}_{ij} = \Delta \dot{\epsilon}_{ij}/\Delta \xi \),

\[
\Delta \sigma_{ij} = 2 \left[ G_\infty h_t(e_{\text{max}}^{ij}) + \sum_{m=1}^N h_1(e_{\text{max}}^{ij}) h_2(R_e) \frac{G_m \rho_m}{\Delta \xi} (1 - e^{-\Delta \xi/\rho_m}) \right] \Delta \dot{\epsilon}_{ij} + \Delta \sigma_{ij}^R
\]

(25)

where

\[
R_e = \sqrt{\frac{3}{2}} \dot{R}_{ij} \dot{R}_{ij} \quad \text{and} \quad h_2(R_e) = \begin{cases} \left( \frac{R_e}{\dot{\epsilon}_C} \right)^{-x} & \text{if } R_e \geq \dot{\epsilon}_C \\ 1 & \text{if } R_e < \dot{\epsilon}_C \end{cases}
\]
and the residual part $\Delta \hat{\sigma}_R^R$ can be expressed in the recursive relation

$$\Delta \hat{\sigma}_R^R = \sum_{m=1}^{N} (1 - e^{-\Delta \zeta / \rho_m}) S_m(\zeta_n)$$

and

$$S_m(\zeta_n) = 2G_m h_1(e^e_m) h_2(R_c) \hat{R}_m(1 - e^{-\Delta \zeta / \rho_m}) + S_m(\zeta_{n-1}) e^{-\Delta \zeta / \rho_m}$$  \hspace{1cm} (26)

The incremental normal stresses can be then formulated by combining the volumetric and deviatoric behaviour, for example,

$$\Delta \sigma_{xx} = 1/3 \Delta \sigma_{kk} + \Delta \hat{\sigma}_{xx}$$

$$= \left[ K_\infty h_e(e^e_{\max}) + \sum_{m=1}^{N} h_1(e^e_{\max}) h_2(R_\max) \frac{K_m \rho_m}{\Delta \zeta} (1 - e^{-\Delta \zeta / \rho_m}) \right] \Delta \hat{\sigma}_{kk} + 1/3 \Delta \sigma_{ik}^R$$

$$+ 2 \left[ G_\infty h_e(e^e_{\max}) + \sum_{m=1}^{N} h_1(e^e_{\max}) h_2(R_c) \frac{G_m \rho_m}{\Delta \zeta} (1 - e^{-\Delta \zeta / \rho_m}) \right] \Delta \hat{\sigma}_{xx} + \Delta \hat{\sigma}_{xy}$$ \hspace{1cm} (27)

where $\Delta \hat{\sigma}_{kk}$ and $\Delta \sigma_{kk}$ are the incremental volumetric strain and stress, $\Delta \hat{\sigma}_{xx}$ and $\Delta \sigma_{xx}$ are the incremental deviatoric strain and stress components, and $\Delta \sigma_{ik}^R$ and $\Delta \hat{\sigma}_{ik}^R$ are the recursive part of the volumetric and deviatoric behaviour given in Equations (22) and (26). Incremental stresses $\Delta \sigma_{yy}$ and $\Delta \sigma_{xz}$ are determined in the same manner.

The incremental shear stress can be formulated by using only the deviatoric behaviour. For example,

$$\Delta \sigma_{xy} = \Delta \hat{\sigma}_{xy}$$

$$= 2 \left[ G_\infty h_e(e^e_{\max}) + \sum_{m=1}^{N} h_1(e^e_{\max}) h_2(R_c) \frac{G_m \rho_m}{\Delta \zeta} (1 - e^{-\Delta \zeta / \rho_m}) \right] \Delta \hat{\sigma}_{xy} + \Delta \hat{\sigma}_{xy}^R$$ \hspace{1cm} (28)

where $\Delta \hat{\sigma}_{xy}$ and $\Delta \sigma_{xy}$ are the incremental shear deviatoric strain and stress components, and the recursive term $\Delta \hat{\sigma}_{xy}^R$ is also given in Equation (26).

Once the incremental stress components are developed, the incremental stiffness terms can be calculated as

$$\frac{\partial \Delta \sigma_{xx}}{\partial \Delta e_{xy}} = \left[ K_\infty h_e(e^e_{\max}) + \sum_{m=1}^{N} h_1(e^e_{\max}) h_2(R_\max) \frac{K_m \rho_m}{\Delta \zeta} (1 - e^{-\Delta \zeta / \rho_m}) \right]$$

$$+ 4 \left[ G_\infty h_e(e^e_{\max}) + \sum_{m=1}^{N} h_1(e^e_{\max}) h_2(R_c) \frac{G_m \rho_m}{\Delta \zeta} (1 - e^{-\Delta \zeta / \rho_m}) \right] = K_{d1}$$

$$\frac{\partial \Delta \sigma_{xx}}{\partial \Delta e_{yy}} = \left[ K_\infty h_e(e^e_{\max}) + \sum_{m=1}^{N} h_1(e^e_{\max}) h_2(R_\max) \frac{K_m \rho_m}{\Delta \zeta} (1 - e^{-\Delta \zeta / \rho_m}) \right]$$

$$- 2 \left[ G_\infty h_e(e^e_{\max}) + \sum_{m=1}^{N} h_1(e^e_{\max}) h_2(R_c) \frac{G_m \rho_m}{\Delta \zeta} (1 - e^{-\Delta \zeta / \rho_m}) \right] = K_{d2}$$

$$\frac{\partial \Delta \sigma_{xy}}{\partial \Delta e_{xy}} = 2 \left[ G_\infty h_e(e^e_{\max}) + \sum_{m=1}^{N} h_1(e^e_{\max}) h_2(R_c) \frac{G_m \rho_m}{\Delta \zeta} (1 - e^{-\Delta \zeta / \rho_m}) \right] = K_{d3}$$

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With these terms, the incremental 3D damage-coupled viscoelastic behaviour can be formulated as

\[
\begin{bmatrix}
\Delta \sigma_{xx} \\
\Delta \sigma_{yy} \\
\Delta \sigma_{zz} \\
\Delta \sigma_{xy} \\
\Delta \sigma_{yz} \\
\Delta \sigma_{xz}
\end{bmatrix} = \begin{bmatrix}
K_{d1} & K_{d2} & K_{d1} & 0 & 0 & 0 \\
. & K_{d1} & K_{d2} & 0 & 0 & 0 \\
. & . & K_{d1} & 0 & 0 & 0 \\
. & . & . & K_{d3} & 0 & 0 \\
. & . & . & . & K_{d3} & 0 \\
. & . & . & . & . & K_{d3}
\end{bmatrix} \begin{bmatrix}
\Delta e_{xx} \\
\Delta e_{yy} \\
\Delta e_{zz} \\
\Delta e_{xy} \\
\Delta e_{yz} \\
\Delta e_{xz}
\end{bmatrix} + \begin{bmatrix}
\Delta \sigma_{kk}^R + \Delta \sigma_{ks}^R \\
\Delta \sigma_{kk}^R + \Delta \sigma_{ks}^R \\
\Delta \sigma_{kk}^R + \Delta \sigma_{ks}^R \\
\Delta \sigma_{kk}^R + \Delta \sigma_{ks}^R \\
\Delta \sigma_{kk}^R + \Delta \sigma_{ks}^R \\
\Delta \sigma_{kk}^R + \Delta \sigma_{ks}^R
\end{bmatrix}
\]

(30)

This damage-coupled viscoelastic model can then be defined in the ABAQUS user material subroutine, and then combined with particular ABAQUS elements to implement a displacement-based non-linear finite element analysis.

3. MICROSTRUCTURAL MODELLING

Asphalt mixture can be described as a multi-phase material containing coarse aggregates, mastic cement (including asphalt binder and fine aggregates) and air voids (see Figure 1). The load transfer between the aggregates plays a primary role in determining the load-carrying capacity and failure of such complex materials. In order to develop a micromechanical model of this behaviour, proper simulation of the load transfer between the aggregates must be accomplished. The aggregate material is normally much stiffer than the mastic, and thus aggregates are taken as rigid particles. On the other hand, the mastic cement is a compliant material with elastic, inelastic, and time-dependent behaviours. Additionally, in most cases, when asphalt mixture is subject to mechanical loading, it generates distributed microstructure damage in the form of microcrack nucleation and growth due to higher stress concentration along the aggregate—mastic interface within the material. This microstructural modelling effort is to account for viscoelasticity and damage evolution properties of asphalt mixture.

In order to properly account for the load transfer between aggregates, it is assumed that there is an effective mastic zone between neighbouring particles. It is through this zone that the micromechanical load transfer occurs between each aggregate pair. This loading can be reduced to resultant normal and tangential forces and a moment as shown in Figure 1.

In general, asphalt mixture contains aggregate of highly irregular geometry as shown in Figure 2(a). The approach used in this study is to allow variable size and shape using an elliptical aggregate model as represented in Figure 2(b). The finite element model shown in Figure 2(c) uses a rectangular strip to simulate the effective asphalt mastic zone, and incorporates the ABAQUS user material subroutine with continuum elements to model the effective asphalt mastic behaviour.

As shown in Figure 3, the general modelling scheme employed four-node quadrilateral elements to mesh the effective cement material, and a rigid body defined with two-node rigid elements sharing the particle centre to model each aggregate. The particle centre is referred to as the master node of the aggregate rigid body, and thus each rigid body element would have identical translation and rotation as the represented aggregate. The rigid elements act to link the
mastic deformation with the aggregate rigid body motion, thereby establishing the micromechanical deformation behaviour. Properties of the quadrilateral elements are specified through a user-defined material subroutine within the ABAQUS code and this allows incorporation of the developed linear and damage-coupled viscoelastic mastic cement models.

A MATLAB code was developed to generate 2D numerical samples of asphalt mixture, and the element geometry properties and connectivity were created and saved in input files for ABAQUS modelling and analysis. A 2D indirect tension sample has been generated with
MATLAB code as shown in Figure 4(a), and its mesh figure was modelled by using ABAQUS elements as shown in Figure 4(b). This particular model has 65 particles (in four particle size groupings), 195 effective mastic zones, 7.6% porosity and an approximate overall diameter of 102 mm (4 in). This microgeometry results in a total of 780 deforming mastic elements and 1170 rigid aggregate elements with connectivity as shown.

4. COMPARISON BETWEEN LINEAR VISCOELASTIC PREDICTION OF MIXTURE CREEP STIFFNESS AND TEST DATA

The purpose of this study is to compare the mixture creep stiffness of a viscoelastic simulation on a computer-generated image model with test results from laboratory asphalt specimens. In order to capture real microstructure of asphalt mixture, simulation material models were generated using image analysis procedures from sectioned surface photographic data of actual asphalt specimens. These imaging techniques and simulation model generation were described in the previous work [36].

The image model generation was conducted on a uniaxial compression asphalt specimen shown in Figure 5. The specimen is a 19 mm (nominal maximum aggregate size) mixture produced in the work done by You [37], and You and Buttler [38]. Its image was obtained by high-resolution scanner and preliminarily processed with imaging technology. Figure 5(a) shows a binary image of the sectioned specimen, where the sample aggregates have passed through a 2.36 mm sieve and its asphalt content is about 14%. In this specimen, the mastic portion includes aggregate finer than the 2.36 mm (passing the sieve) and asphalt binder. For the image processing, each sieved aggregate was labelled and selected as a separate image. The boundary
pixels of each aggregate were extracted, and the co-ordinates of these pixels were stored in an array for the ellipse fitting. A least-squares, ellipse-fitting algorithm was incorporated to determine the ‘best’ ellipse to represent each irregular aggregate geometry. The fitted ellipse was then generated for each sieved aggregate with the developed MATLAB Code as shown in Figure 5(b). Based on these fitted ellipse parameters (centre co-ordinates, size and orientation), a computation model is generated with maximally filled cementation between neighbouring particles using MATLAB as shown in Figure 5(c). For this compressive computational model, the porosity was approximately 2% and the aggregate percentage was about 62%.

In the laboratory, the creep stiffnesses of mastic and asphalt mixture were measured at three temperatures and different loading time in the study by You [37], and You and Buttlar [39]. As mentioned previously, the mastic comprised of the portion with the aggregate gradation finer than 2.36 mm sieve and the volume of binder normally used in the entire asphalt concrete mixture. Thus, the mastic had around 14% asphalt binder by weight. A regression-fitting method was employed to calibrate a generalized Maxwell models with Prony series, and generate master stiffness curves for mastic and asphalt mixture from test data. Due to length limitations, shift factor computations and master curve generation procedures are not given here but can be found in References [40,41].

The shifted creep stiffness at three temperatures and the master stiffness curve (reference temperature \(-20^\circ\text{C}\)) for mastic are shown in Figure 6. The time–temperature shift factors \(z_T(T)\) were calculated as 0.1 and 0.008 for the temperatures \(-10\) and \(0^\circ\text{C}\). The calibrated Maxwell model parameters with Prony series for mastic materials are given in Table I, and its corresponding master stiffness curve is plotted in Figure 6. Similarly, the shifted creep stiffness at three temperatures and master stiffness curve (reference temperature \(-20^\circ\text{C}\)) for mixture are shown in Figure 7. The time–temperature shift factors \(z_T(T)\) were calculated as 0.159 and 0.002 at the temperatures \(-10\) and \(0^\circ\text{C}\).

After calibrating mastic material properties and generating mastic and mixture stiffness master curves, a comparison study was conducted between viscoelastic simulation using calibrated mastic properties and the image model (shown in Figure 5(c)), and test data of laboratory mixture specimen (shown in Figure 7). For the compression creep simulation, the \(x\)- and

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Figure 5. Uniaxial compression image model generation: (a) smooth surface of asphalt specimen; (b) elliptical fitted aggregates; and (c) image model with aggregate and mastic.
The constant force loading was evenly divided and imposed on particles of the top layer. In the simulations, axial strain was calculated by dividing the average vertical displacement of top particles with the initial height of the undeformed specimen. Axial stress was obtained by dividing the constant loading force on the top layer with the specimen initial cross-section area.

Table I. Fitted generalized Maxwell model parameters for mastic behaviour.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_0)</td>
<td>59.66 MPa</td>
</tr>
<tr>
<td>(E_1)</td>
<td>5710.63 MPa</td>
</tr>
<tr>
<td>(E_2)</td>
<td>2075.07 MPa</td>
</tr>
<tr>
<td>(E_3)</td>
<td>1449.00 MPa</td>
</tr>
<tr>
<td>(E_4)</td>
<td>734.90 MPa</td>
</tr>
<tr>
<td>(\rho_1)</td>
<td>26.24 s</td>
</tr>
<tr>
<td>(\rho_2)</td>
<td>311.89 s</td>
</tr>
<tr>
<td>(\rho_3)</td>
<td>1678.80 s</td>
</tr>
<tr>
<td>(\rho_4)</td>
<td>19952.62 s</td>
</tr>
</tbody>
</table>

Figure 6. Creep stiffness master curve for mastic.

Figure 7. Creep stiffness master curve for mixture.

\(y\)-displacements of the particles on the bottom layer and the \(x\)-displacements of the particles on the top layer were constrained. The constant force loading was evenly divided and imposed on particles of the top layer. In the simulations, axial strain was calculated by dividing the average vertical displacement of top particles with the initial height of the undeformed specimen. Axial stress was obtained by dividing the constant loading force on the top layer with the specimen initial cross-section area. The finite element model simulation results for creep stiffness of...
asphalt mixture were compared with mixture stiffness master curve over a reduced time period ($10^4$ s) as shown in Figure 8. The average relative error between model prediction and test data was approximately 11.7%. These errors are caused by the microstructural model assumption of rigid aggregates with infinite stiffness and elliptical shape, and also due to the approximation of master curves for mastic and mixture. In spite of some differences in the beginning of the curve, these comparison curves of mixture creep stiffness are very close after some reduced time. Therefore, this proposed micromechanical finite element model with user material subroutine is applicable to predict the global viscoelastic behaviour of asphalt mixture.

5. CYCLIC LOADING RESPONSES WITH LINEAR AND RATE-INDEPENDENT DAMAGE-COUPL ED VISCOELASTIC MODELS

The cyclic loading responses of linear and rate-independent damage-coupled viscoelastic models were investigated with indirect tension simulations on the numerical sample (shown in Figure 4). The mastic elements had 3-parameter viscoelastic constitutive properties with relaxed elastic moduli $E_\infty = 412.8$ MPa and one Maxwell element of spring constant $E_1 = 1232$ MPa and relaxation time $\rho_1 = 6.5$ s. These constitutive properties were selected from asphalt mixture characterization testing by Gibson et al. [42]. It was assumed that Poisson’s ratio $v$ did not change with time and was given as 0.3. Model rate-independent damage parameters were chosen as $b = 1$, $\beta = 0.3$, and softening strain $\varepsilon_0 = 0.2$. The rate-dependent damage parameter $\alpha$ was taken as zero, and thus this initial study compares only the rate-independent damage with linear viscoelastic behaviour.

For the following cyclic loading simulations, both displacement- and force-controlled boundary conditions were used on the numerical sample. For the displacement-controlled boundary conditions, both horizontal and vertical displacements of the bottom pair of aggregates are
constrained, while the top particle pair accepts the applied vertical displacement loading. Under force-control, the vertical force is imposed to the top particle pair, and the displacements of the bottom pair of aggregates are again constrained.

Linear and damage-coupled viscoelastic simulations were conducted and compared for both displacement- and force-controlled boundary conditions. Viscoelastic behaviour is demonstrated by the decreasing relaxation force and increasing creep displacement with the unreversed loading cycles. The results also show that viscoelastic damage behaviour reduces the sample's load-carrying ability for the displacement-controlled boundary conditions, and increases the sample creep displacement for the force loading.

Numerical simulations were also conducted with an incrementally increasing reversed cyclic loading as shown in Figure 9. Figure 9(a) shows the linear viscoelastic responses of the numerical sample under the displacement and force loading. The maximum points in each loop are approximately located along a straight line in these two figures. Figure 9(b) gives the damage-coupled viscoelastic responses. Since the rate-independent damage variables in the
model are all defined as exponential functions, the maximum points are distributed along an exponential curve. These damage-coupled cyclic responses illustrate that rate-independent damage increases with the maximum deformation. This occurs since the rate-independent damage parameters are a function of maximum strain. These results also demonstrate that most of the loss in stiffness takes place during the first cyclic deformation, and the steady-state response is obtained after very few cycles with the same loading amplitude.

6. MICROSTRUCTURAL EFFECTS ON RATE-INDEPENDENT VISCOELASTIC DAMAGE BEHAVIOUR OF ASPHALT NUMERICAL SAMPLES

Micro-parameter effects on inelastic damage behaviour of asphalt numerical samples were investigated in detail by Dai and Sadd [43]. Following this previous work, the relationship between microstructure parameters and damage-coupled viscoelastic behaviour of particular asphalt model samples is studied. The microstructure parameters of asphalt mixture can be categorized by: proportions of asphalt mixture components such as model porosity (area ratio of aid voids to entire sample) and aggregate percentage (area percentage of aggregate over the sample model); particle/aggregate measures including particle orientation, shape and size; and packing fabric descriptions such as aggregate gradation (aggregate area percentage passing different sieving sizes) and branch vector distribution. Particle orientation is commonly represented by an angular measure (with respect to a reference direction) of the particle’s longest axis. Particle shape ratio (aspect ratio) is defined as the ratio of the longest axis dimension to the shortest axis dimension. Particle size is the longest axis dimension while the branch vector runs from one particle centre to another neighbouring particle centre. Packing fabric can be categorized as the branch vector distribution and aggregate gradation. Figure 10 illustrates particle orientation and branch vectors for a model sample of cemented particulate material. In order to quantify the distributions of these vectors for an entire sample, an angular distribution plot (Rose diagram) is normally constructed. Such a plot indicates the frequency of the vectors lying in a particular direction as a function of angular measure. These will be used to correlate various numerically generated asphalt samples for finite element simulation.

![Figure 10. Vector microstructure measures in particulate materials.](image-url)
Microstructure effects were investigated through finite element simulation of a series of asphalt numerical samples. Using a specially developed MATLAB code, numerical samples were generated with controllable microstructure variation in an effort to determine the effect of a particular microstructural variable on the material damage viscoelastic response. A series of the finite element simulations were conducted on indirect tension samples, and displacement-controlled boundary conditions were used (as shown in Figure 4). The viscoelastic material constants include $E_\infty = 412.8$ MPa, $v = 0.3$, and six Maxwell elements from asphalt mixture characterization testing by Gibson et al. [42]. Damage parameters are same as previous section, and rate-independent damage behaviour was also considered in this microstructure study.

6.1. Model porosity

Air voids provide an important indication of the mixture's pavement service performances. Model porosity, which is defined as area ratio of air voids over entire sample, affects the

![Figure 11. Damage-coupled viscoelastic simulations with different model porosity.](image)
viscoelastic damage response of the model. Figure 11 illustrates three indirect tension damage-coupled viscoelastic simulations on models with porosities of 1.1, 4.6 and 6.7%. Each numerical sample had the same number of particles (181) and elements (574), and had identical particle locations, shape (aspect ratio), and particle area percentage. Particle size and number percentage are also same as: 2 mm (44.8%), 3 mm (14.9%), 4 mm (8.3%), 7 mm (16%), 9 mm (10.5%) and

![Figure 11](image1.png)

![Figure 12](image2.png)

Figure 12. Damage-coupled viscoelastic simulations with added small particles.
11 mm (5.5%). All samples had the same branch and orientation vector distributions as shown in the figure. The model porosity was modified by only changing the effective mastic zone width while other model parameters were kept the same. The damage-coupled viscoelastic simulations of these numerical samples were conducted under the same unreversed saw-toothed displacement loading and the results (shown in Figure 11) indicated that lower-porosity models had stiffer behaviour.

6.2. Aggregate gradation

Aggregate gradation curves are very helpful in making necessary adjustments to asphalt mixture design. In order to investigate the effect of aggregate gradation, a pair of numerical models was generated with variation in the smaller-sized particles. The two indirect tension models, shown in Figure 12, were created from five different particle size groupings: 2, 4, 7, 11 and 14 mm. Model B-1 was composed of a mix of 96 particles, and the particle distribution for different sizes are 2 mm (area percentage of 2% for the 23 particles), 4 mm (area percentage of 4% for the 15 particles), 7 mm (area percentage of 24% for the 29 particles), 11 mm (area percentage of 38% for the 19 particles) and 14 mm (area percentage of 33% for the 10 particles). This resulted in a model with 292 finite elements and a particle percentage of 54.6%. Model B-2 included additional small aggregates of 2 and 4 mm size, and the particle distribution changed to 2 mm (area percentage of 7% for the 108 particles), 4 mm (area percentage of 4% for the 15 particles), 7 mm (area percentage of 22% for the 29 particles), 11 mm (area percentage of 36% for the 19 particles) and 14 mm (area percentage of 31% for the 10 particles). The added small aggregates lead to a model with 181 total particles, 574 finite elements and a particle percentage of 64.1%. The gradation curves for each of the models are plotted as particle area passing percentage with different particle sizes shown in Figure 12. These curves are compared with typical gradations of some actual asphalt mixture in the same plot with 0.45 power of aggregate sieving sizes. All other micro-parameters are identical in the two models. The damage-coupled viscoelastic simulation results under the unreversed saw-toothed displacement loading illustrate that the added small aggregates increase the model stiffness and load-carrying ability.

7. LOADING RATE EFFECT ON RATE-DEPENDENT VISCOELASTIC DAMAGE BEHAVIOUR

The purpose of this section is to describe rate-dependent damage behaviour and loading rate effects on sample damage by assuming the strain rate $\dot{\varepsilon}$ is larger than the threshold strain rate $\dot{\varepsilon}_C$ and the parameter $z$ has a positive value. For this study, indirect tension simulations were again conducted on the same numerical sample (shown in Figure 4) with displacement-controlled boundary conditions. These simulations used the same 3-parameter viscoelastic constitutive properties as previous cyclic loading study, and the rate-independent damage parameters were chosen as $b = 1$, $\beta = 0.5$, and softening strain $\varepsilon_0 = 0.1$.

Effects of the two model parameters $\dot{\varepsilon}_C$ and $z$ used in rate-dependent damage variable $h_2$ are illustrated in Figure 13. The loading displacement increases linearly from zero to 6 mm with a loading rate 0.6 mm/s. Figure 13(a) shows the simulation results for three different threshold strain rates $\dot{\varepsilon}_C$ with constant $z$. These results indicate that more rate-dependent damage is generated as the threshold strain rate decreases. Figure 13(b) shows simulation results for the
cases of $\alpha = 0.1$, 0.3 and 0.5 with constant threshold strain rate. Note that the sample has more rate-dependent damage behaviour as $\alpha$ is increased.

Results of loading rate effects on asphalt viscoelastic damage behaviour are demonstrated in Figure 14. For these indirect tension simulations, rate-dependent damage parameters were chosen as $\dot{c}_e = 0.001$ and $\alpha = 0.1$. Figure 14(a) shows the simulation results under the monotonic linear loading of 5, 1, 0.5 and 0.25 mm/s. Solid lines indicate the linear viscoelastic behaviour, dashed lines indicate the damage-coupled viscoelastic behaviour. Higher loading rates generate larger sample reaction forces since the sample has less relaxation time. Comparing the linear and damage-coupled viscoelastic behaviour, the higher loading rate also leads to more rate-dependent

Figure 13. Parameter study of rate-dependent damage variable $h_2$: (a) simulation with different $\dot{c}_e$; and (b) simulation with different $\alpha$.

Figure 14. Loading rate effect on the asphalt viscoelastic damage behaviour: (a) simulations under monotonic linear loading; and (b) simulations under reversed saw-toothed loading.
8. CONCLUSIONS

A micromechanical constitutive model was formulated and used to simulate the 2D linear and damage-coupled viscoelastic behaviour of asphalt mixture. The aggregate-mastic microstructure was simulated by incorporating the user material subroutine with continuum elements for asphalt mastic and rigid body elements for each aggregate. Rate-independent and rate-dependent damage mechanisms are defined and combined within the Schapery non-linear viscoelastic model. A finite element incremental algorithm with recursive relationships for the linear and damage-coupled viscoelastic behaviour was developed. The numerical implementation of this...
method was also addressed. This behaviour was then defined in the ABAQUS user material subroutine for the asphalt mastic to predict global linear and damage-coupled viscoelastic behaviour of asphalt mixture.

The linear viscoelastic predictions of mixture creep stiffness using calibrated mastic properties and an image computational model were compared with specimen test data. The average relative error between model prediction and mixture master curve is approximately 11.7%. The errors are caused by the model assumption of rigid aggregates with infinite stiffness and idealized elliptical shape, and the approximate material master curves. In spite of some difference in the beginning, the comparison curves showed very close response of mixture creep stiffness after some reduced time. Therefore, this proposed micromechanical finite element model is applicable to predict the global viscoelastic behaviour of asphalt mixture.

The cyclic loading response of linear and rate-independent damage viscoelastic asphalt mixture was compared for both displacement- and force-controlled boundary conditions. These cyclic loading responses show rate-independent damage increases with the maximum deformation. The results also demonstrate that most of the loss in material stiffness occurs during the first cycle of deformation, and both linear and damage viscoelastic behaviour reaches steady state after very few cycles.

Specific microstructure parameters of asphalt samples were identified and categorized for numerical analysis. A series of indirect tension asphalt samples were then generated with systematic variation of particular microstructures. Numerical experiments included investigations on sample porosity and aggregate gradation. Model porosity studies indicated that the samples with lower porosity resulted in higher load-carrying behaviour. Simulation investigations on aggregate gradation showed the sample had higher stiffness with added small aggregates.

The study of loading rate effects showed that higher loading rate leads to a stiffer asphalt sample response and also generates more rate-dependent damage behaviour. By using the unified damage model, this study also compared the viscoelastic response of linear, rate-independent damage, and rate-dependent damage material properties under different loading rate and amplitude.

The linear viscoelastic model verification study was conducted by comparing the mixture creep stiffness with test data, and the simulation results on 2D models had satisfactory prediction. As future modelling efforts are extended to 3D, it is expected that model predictions will be closer to test data. The damage-coupled viscoelastic simulations were in qualitative agreement with observed and expected behaviour of asphalt mixture. More detailed experimental test data are needed to further verify the quantitative results of such viscoelastic damage modelling effort.

REFERENCES


