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Subvolcanic plumbing systems imaged through crystal size distributions

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ABSTRACT
The configuration of subvolcanic magma storage regions exercises a fundamental control on eruptive style and hazard. Such regions can be imaged remotely, using seismic, geodetic, or magnetotelluric methods, although these are far from routine and rarely unambiguous. The textures of erupted volcanic rocks, as quantified through crystal size distributions (CSD), provide space- and time-integrated information on subvolcanic plumbing systems, although these data cannot be used readily for reconstruction of key parameters such as conduit geometry or magma chamber depth. Here we develop a numerical approach to interpretation of CSD in products of steady eruptions, based on crystallization kinetics and hydrodynamic flow simulation, to image subvolcanic plumbing systems. The method requires knowledge of magma properties, crystal growth kinetics (measured experimentally), and discharge rate (measured observationally). The method is applicable to steady-state eruptive regimes. Distributions of pressure, temperature, crystal content, and conduit cross-section area with depth are obtained from a CSD from a sample erupted from Mount St. Helens volcano, USA. Values of average conduit diameter (~30 m) and magma chamber depth (~14 km below the summit) are in good agreement with independent estimates.

INTRODUCTION
Erupted volcanic products are complex mixtures of quenched melt, bubbles, and crystals. The size and spatial arrangement of these components reflect, in different ways, magma plumbing systems and ascent conditions, such that the final texture of a volcanic rock provides space- and time-integrated information on the journey that magma has taken from the magma chamber through the conduit and onto the surface. Extracting the cryptic information contained within the textures of volcanic rocks has long been a goal of igneous petrology. For extrusive rocks the most complete and undisputed record can be obtained through the analysis of crystal populations, whereas for explosively erupted material, where crystallization is usually suppressed due to fast magma ascent rates, bubble size distributions are more informative (Toramaru, 2006).

In the crystal population of a volcanic rock, microlites and microphenocrysts grow during ascent through the conduit, whereas phenocrysts reflect (often protracted) storage conditions in magma chambers. Most studies (Gutierrez and Parada, 2010; Magee et al., 2010; Simakin and Bindeman, 2008) use crystal size distributions (CSD) to infer magma storage conditions in magma chambers and plutons, while few studies have explored the relationship between micro- to macroscopic CSD and magma ascent conditions (e.g., Cashman, 1992; Noguchi et al., 2008; Toramaru et al., 2008). These studies assume a simple analytical form of the CSD (Marsh, 1998), \( n(L) = \frac{n_0}{L} \exp(-L/L_0) \), where \( n \) is the population density of crystals of size \( L \) defined as:

\[
n(L) = \frac{dN}{dL} \approx \frac{\Delta N}{\Delta L},
\]

where \( \Delta N \) is the number of crystals in the interval \( \Delta L \), \( n_0 \) is the CSD intercept, and \( 1/L_0 \) is the slope in the standard CSD plot of ln(\( n \)) versus length \( L \). The parameters \( n_0 \) and \( L_0 \) are obtained from a best fit to the data and are used to explore characteristic time scales for magma crystallization. Here we show that different crystal sizes reflect ascent conditions at different depths, such that novel insights into magma plumbing systems can be obtained from the shape of a CSD. We restrict our study to steady-state regimes of magma extrusion, characterized by viscous, highly crystallized magma that forms lava domes at the surface (Harris et al., 2003; Melnik and Sparks, 1999; Sparks et al., 1998; Swanson and Holcomb, 1990).

MODEL
If magma ascends steadily in a conduit with cross-section area \( S(z) \) (\( z \) increases downward from the Earth’s surface at \( z = 0 \)), the evolution of population density is defined by the following equation with boundary conditions (Marsh, 1998):

\[
-Q \frac{\partial n(z,L)}{\partial z} + U(z) S(z) \frac{\partial n(z,L)}{\partial L} = 0;
\]

\[
n(z,0) = \frac{J(z)}{U(z)}; \quad n(0,L) = n_{op}(L),
\]

where \( Q \) (m/s) is the discharge rate, \( U \) (m/s) is a linear crystal growth rate, assumed to be independent of crystal size, \( J \) (1/m²/s) is a nucleation rate, and \( n_{op} \) is a measured population density in an erupted sample. Crystal growth and nucleation rates are functions of undercooling and can be parameterized in terms of magma temperature \( T \), liquidus temperature \( T_L \) (that depends on pressure \( P \) or water content in water-saturated conditions) for the crystal phase of interest, and mass fraction of crystallized magma, X. Melnik and Sparks (2005; following Hort and Spohn, 1991) were able to obtain good agreement with experimental studies of degassing-induced crystallization (Couch et al., 2003b).

Figure 1 illustrates the main concept of the algorithm that allows us to reconstruct parameter
distributions along the conduit for steady-state magma ascent. For the sake of simplicity in Figure 1, we assume that crystal growth rate is constant, the conduit has a constant cross-section area (which means that velocity $V$ is also constant), and the ascent is isothermal. In this case Equation 2 has a simple analytical solution: $n(Cz - L) = n_{\text{top}}(L,C = U/V$, where $C$ is the characteristic velocity. Thus the number density of crystals, $n$, in an ascending magma remains constant along the straight lines labeled $C = L = A$, where $A$ is a constant. For a given crystal growth rate, $U$, this allows us to obtain the depth distribution of nucleation rate, $J$, using the boundary condition at $L = 0$. As nucleation rate depends on undercooling $\Delta T = T - T_p(P)$, the pressure distribution with depth can be calculated. The volume fraction of crystals ($\beta$) (e.g., plagioclase) at any depth can be found as a moment of the CSD:

$$\beta = \int_0^L n(l) l^2 dl; \sigma = \left[1 - \frac{\pi}{6} \frac{I S}{L_c} \right]$$

Here $\sigma$ is the real intermediate dimension (three-dimensional, 3-D, intermediate length), smallest dimension (3-D width), and longest dimension (3-D length), respectively (Higgins, 2000). Because CSDs are calculated for a particular mineral, the total volume fraction of crystals can be calculated assuming the proportions of all minerals remain constant, $\beta = \beta_0$, where $w$ is the proportion of plagioclase in total crystal population. The total mass fraction of crystals, $X$, can be calculated if densities of crystals and melt are known (Brennen, 2005).

We can relax the simplifying assumptions made above and use a similar method to obtain the distribution of parameters with depth from a measured CSD. For viscous laminar flows of incompressible magma with release of latent heat of crystallization, the system of equations has the following form:

$$V(z)S(z) = Q = \text{const}$$

$$\frac{dp}{dz} = \rho g + \lambda \mu (P(T, \beta_{\text{eq}}) V(z) \frac{S(z)}{S(z)}$$

$$\frac{dT}{dz} = \frac{L_c \alpha X}{C_p} \frac{dX}{dz}$$

$$U = U(p,T,X), J = J(P,T,X)$$

where $\rho$ is the magma density, $\mu$ is the viscosity that depends on $P$ (or volatile content assuming equilibrium degassing), $T$, and $\beta_{\text{eq}}$ (Costa et al., 2007a), $g$ is gravitational acceleration, $L_c$ is the latent heat of crystallization, and $C_p$ is the heat capacity of magma (Blundy et al., 2006). For a particular crystal phase, $U$ and $J$ can be generic functions of their argument, obtained from experimental studies of crystallization kinetics (Couch et al., 2003b; Hammer et al., 1999). In the generalized case characteristics are no longer linear and the solution must be found by integration of Equations 2–4 downward from the known conditions at the surface (i.e., the erupted rock).

The conduit flow model provides derivatives of temperature and pressure for a given discharge rate. Crystal growth and nucleation rates depend on the calculated pressure, temperature, and amount of crystallized material. The conduit flow model specified here is simpler than some existing models (e.g., Costa et al., 2007a). However, the flow and crystallization components are separated, and so the method could easily be adapted to a more advanced flow model. For example, it is possible to take into account magma compressibility (due to volatile exsolution and escape), heat loss to surrounding rocks, and shear heating. Costa et al. (2007b) showed that, for intermediate discharge rates, viscous dissipation balances heat loss and temperature remains close to a constant value. In this case the main mechanism for temperature change is the release of latent heat of crystallization, which is the only mechanism included in Equation 4.

The steady-state assumption is more restrictive. If magma ascends in pulses, the applicability of the model depends on the degree of transience. If repose periods are small compared to overall magma ascent or characteristic crystal growth times, then the model is still valid. If time-averaged discharge rate is more or less constant during an episode of eruption, the model will give time-averaged distributions of the properties in the conduit.

CASE STUDY

We apply our method to the plagioclase CSD obtained from sample SH156–2 (Muir, 2009) from the 17 June 1984 lava dome building eruption of Mount St. Helens volcano, USA (Fig. 1A). An important feature of this CSD is that it provides information down to the very small crystals (<10 µm) that grow in the uppermost parts of the conduit, thereby extending the depth range that we can image. The CSD $\ln(n)$ versus $L$ plot (Fig. 1A) is concave up, and although microlites <50 µm could be approximated by a straight line, the data overall would be poorly described in this way. The nonlinearity of the CSD provides resolution on parameter variation with depth. Figure 2A shows the distributions of pressure $P$, and cross-section area of the conduit $S$ calculated from the CSD using our approach, with the input parameters in Table DR1 in the GSA Data Repository1. At the surface $S$ is ~840 m$^2$ (equivalent circular conduit diameter is ~32 m), in good agreement with independent estimates (Barmin et al., 2002; Rutherford and Hill, 1993). After a decrease to 350 m$^2$ (~22 m diameter) at 1.2 km depth, $S$ reaches a nearly constant value similar to the surface value between 4 and 8 km, below which it increases sharply, corresponding to a transition to the magma chamber. This chamber depth agrees with tomographic (Waite and Moran, 2009) and magnetotelluric images (Hill et al., 2009) and dissolved volatile contents of melt inclusions (Blundy et al., 2008). Our calculated conduit cross sections are less than those from tomography and magnetotellurics because these methods are sensitive to the presence of partially molten rock, including sidewall mushes, whereas we are imaging only the effective area through which magma ascends, which is inevitably narrower.

1GSA Data Repository item 20111128, crystal size distribution (CSD) acquisition procedure, parameters used for the simulations, and sensitivity analysis, is available online at www.geosociety.org/pubs/ft2011.htm, or on request from editing@geosociety.org or Documents Secretary, GSA, P.O. Box 9140, Boulder, CO 80301, USA.
Magma undercooling (Fig. 2B), relative to $T_\text{eq}$ decreases progressively with depth because crystal growth rate increases from $10^{-11.4} \text{ m/s}$ at the surface to $10^{-10.4} \text{ m/s}$ at the chamber depth, leading to faster system equilibration. At low undercooling, the nucleation rate drops by 10 orders of magnitude. The transition downward from the conduit to the magma chamber leads to long residence times that permit the growth of the large microphenocrysts (the right side of the CSD plot).

Figure 3 shows the evolution of CSD with depth calculated from Equation 2. In the lower part of the conduit ($z = 8 \text{ km}$), growth of existing crystals is the dominant mechanism, resulting in a relatively flat CSD. Maximum crystal size increases progressively during magma ascent because crystals that were nucleated in the magma chamber continue to grow, producing overgrowth rims. In the shallower part of the system the nucleation of new crystals starts to dominate over growth, leading to steep (negative) slopes of CSD for small crystals.

**DISCUSSION**

Our approach allows us to obtain a complete self-consistent description of the subvolcanic plumbing system, including conduit flow properties and magma chamber depth, from a single rock sample. Traditional methods (Cashman, 1990) based on the CSD slope and intercept can provide only depth-averaged properties. All model input parameters, including crystal growth, nucleation kinetics, magma density, viscosity, and thermal properties can be measured in laboratory experiments for a particular magma composition. In the absence of volcano-specific experiments it is possible to use experimental data from compositionally similar systems (Table DR1). Sensitivity analysis (see the Data Repository) indicates that the key geometric parameters are readily scalable. Calculated chamber depth is particularly insensitive and hence robust. An important limitation of the model comes from assuming steady-state magma ascent conditions requiring that, during ascent of an individual particle, discharge rate remains approximately constant. After the cross-section area of the conduit is determined by simulations, ascent time can be calculated and the validity of the steady-state assumptions assessed. It is clear that system properties obtained depend on a conduit flow model. Further sensitivity analysis is necessary to see the influence of the processes that are not currently considered in the model. For example, Simakin and Salova (2004) suggested that nucleation rate is a strong function of the thermal prehistory of the melt.

There are several fruitful developments of the current model. First, a more advanced conduit flow model accounting for volatile exsolution, bubble growth, and gas escape can be used instead of Equation 4. Several models of this type already exist (Costa et al., 2007a; Diller et al., 2006; Melnik and Sparks, 2005; de’ Michieli Vitturi et al., 2008). Second, assessment of CSDs from multiple minerals could test predictions of the model or solve for parameters that are poorly known. For example, using plagioclase and orthopyroxene CSDs from the same sample will allow estimation of discharge rate for prehistoric or unmonitored eruptions. Kinetics of crystal growth for these two minerals can be studied simultaneously by analysis of decompression-induced crystallization experiments (Couch et al., 2003a; Hammer et al., 1999). Analyses of CSDs from experimental samples with known decompression history provide a basis for model validation (Brugger and Hammer, 2010).

Our method provides a new and powerful quantitative tool for the interpretation of CSDs in the products of effusive eruptions. This quick, simple, and relatively low cost method can be used to extract key parameters of the subvolcanic plumbing system from individual samples. Where multiple samples are available, the temporal evolution of these parameters can be established for comparison to indirect measurements of subvolcanic magma movement and storage, such as seismic tomography, geodesy, and gas chemistry.

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