Scattering fundamentals

- Scattering can be broadly defined as the redirection of radiation out of the original direction of propagation, usually due to interactions with molecules and particles.
- Reflection, refraction, diffraction etc. are actually all just forms of scattering.
- Matter is composed of discrete electrical charges (atoms and molecules – dipoles).
- Light is an oscillating EM field – excites charges, which radiate EM waves.
- These radiated EM waves are scattered waves, excited by a source external to the scatterer.
- The superposition of incident and scattered EM waves is what is observed.
When does scattering matter?

- Scattering can be ignored whenever gains in intensity due to scattering along a line of sight are negligible compared to:
  - Losses due to extinction
  - Gains due to thermal emission
- Usually satisfied in the thermal IR band and for microwave radiation when no precipitation (rain, snow etc.) is present
- Also can be ignored when considering direct radiation from a point source, such as the sun
- In the UV, visible and near-IR bands, scattering is the dominant source of radiation along any line of sight, other than that looking directly at the sun
Radiative transfer with scattering

- Thermal IR and microwave bands:
  \[ dI = dI_{\text{abs}} + dI_{\text{emit}} = \beta_a (B - I) ds \]

- UV, visible and near-IR bands:
  \[ dI = dI_{\text{ext}} + dI_{\text{emit}} + dI_{\text{scat}} \]
  \[ dI = -\beta_e I ds + \beta_a B ds + \frac{\beta_s}{4\pi} \int_{4\pi} p(\cos \Theta) I(\hat{\Omega}') d\omega' ds \]
  \( \hat{\Omega}' \) = any direction

Types of scattering

- **Elastic scattering** – the wavelength (frequency) of the scattered light is the same as the incident light (*Rayleigh and Mie scattering*)

- **Inelastic scattering** – the scattered radiation has a wavelength different from that of the incident radiation (*Raman scattering, fluorescence*)

- **Quasi-elastic scattering** – the wavelength (frequency) of the scattered light shifts (e.g., in moving matter due to Doppler effects)
More types of scattering

• **Single scattering**: photons scattered only once
  - Prevails in optically thin media ($\tau << 1$), since photons have a high probability of exiting the medium (e.g., a thin cloud) before being scattered again
  - Also favored in strongly absorbing media ($\omega << 1$)

• **Multiple scattering**: prevails in optically thick, strongly scattering and non-absorbing media
  - Photons may be scattered hundreds of times before emerging

Parameters governing scattering

• (1) The wavelength ($\lambda$) of the incident radiation

• (2) The size of the scattering particle, usually expressed as the non-dimensional size parameter, $x$:

$$x = \frac{2\pi r}{\lambda}$$

• $r$ is the radius of a spherical particle, $\lambda$ is wavelength

• (3) The particle optical properties relative to the surrounding medium: the complex refractive index

• Scattering regimes:
  - $x << 1$: Rayleigh scattering
  - $x \sim 1$: Mie scattering
  - $x >> 1$: Geometric scattering
### Atmospheric particles

<table>
<thead>
<tr>
<th>Type</th>
<th>Size</th>
<th>Number concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas molecule</td>
<td>~10^{-4} \mu m</td>
<td>&lt; 3 \times 10^{19} \text{cm}^{-3}</td>
</tr>
<tr>
<td>Aerosol, Aitken</td>
<td>&lt; 0.1 \mu m</td>
<td>~10^4 \text{cm}^{-3}</td>
</tr>
<tr>
<td>Aerosol, Large</td>
<td>0.1-1 \mu m</td>
<td>~10^2 \text{cm}^{-3}</td>
</tr>
<tr>
<td>Aerosol, Giant</td>
<td>&gt; 1 \mu m</td>
<td>~10^{-1} \text{cm}^{-3}</td>
</tr>
<tr>
<td>Cloud droplet</td>
<td>5-50 \mu m</td>
<td>10^2-10^3 \text{cm}^{-3}</td>
</tr>
<tr>
<td>Drizzle drop</td>
<td>~100 \mu m</td>
<td>~10^3 \text{m}^{-3}</td>
</tr>
<tr>
<td>Ice crystal</td>
<td>10-10^2 \mu m</td>
<td>10^3-10^5 \text{m}^{-3}</td>
</tr>
<tr>
<td>Rain drop</td>
<td>0.1-3 mm</td>
<td>10-10^3 \text{m}^{-3}</td>
</tr>
<tr>
<td>Graupel</td>
<td>0.1-3 mm</td>
<td>1-10^2 \text{m}^{-3}</td>
</tr>
<tr>
<td>Hailstone</td>
<td>~1 cm</td>
<td>10^{-2-1} \text{m}^{-3}</td>
</tr>
<tr>
<td>Insect</td>
<td>~1 cm</td>
<td>&lt;1 \text{m}^{-3}</td>
</tr>
<tr>
<td>Bird</td>
<td>~10 cm</td>
<td>&lt;10^{-4} \text{m}^{-3}</td>
</tr>
<tr>
<td>Airplane</td>
<td>~10-100 m</td>
<td>&lt;1 \text{km}^{-3}</td>
</tr>
</tbody>
</table>

### Refractive indices of substances

(\lambda = 589 \text{nm} unless indicated)

<table>
<thead>
<tr>
<th>Substance</th>
<th>(n_r)</th>
<th>(n_i)</th>
<th>((n = n_r + i n_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.333</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Water (ice)</td>
<td>1.309</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>NaCl (salt)</td>
<td>1.544</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>H_2SO_4</td>
<td>1.426</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(NH_4)_2SO_4</td>
<td>1.521</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>SiO_2</td>
<td>1.55</td>
<td>0</td>
<td>(\lambda = 550 \text{nm})</td>
</tr>
<tr>
<td>Carbon</td>
<td>1.95</td>
<td>-0.79</td>
<td>(\lambda = 550 \text{nm})</td>
</tr>
<tr>
<td>Mineral dust</td>
<td>1.56</td>
<td>-0.006</td>
<td>(\lambda = 550 \text{nm})</td>
</tr>
</tbody>
</table>

The most significant absorbing component of atmospheric particles is *elemental carbon (soot)*; reflected in the large value of the imaginary part of the refractive index.

Other common atmospheric particles are purely scattering.
Light scattering regimes

There are many regimes of particle scattering, depending on the particle size, the light wave-length, and the refractive index.

This plot considers only single scattering by spheres. Multiple scattering and scattering by non-spherical objects can get really complex!

Scattering phase functions

Scattering phase functions derived from Mie theory (scattering by spherical particles)

The scattering phase function, or phase function, gives the angular distribution of light intensity scattered by a particle at a given wavelength

\[ p(\cos \Theta) = 1 \]
Asymmetry parameter

- Considering fluxes, rather than intensities, the important factor is the relative magnitude of scattering in the forward and backward directions.

- Asymmetry parameter ($g$) is the average value of $\cos \Theta$ for all scattered photons; i.e., $-1 \leq g \leq 1$

- $g > 0 = \text{forward scattering}; \quad g < 0 = \text{backscattering}; \quad g = 0 = \text{isotropic case}$

\[
\begin{align*}
\text{a) 1 photon} & & \text{b) 3 photons} \\
g = 0 & & g = 0.85 \\
\text{Albedo} & & \tau = 0 \\
\text{Diffuse} & & \tau^* = 10
\end{align*}
\]

Rayleigh scattering

- Atmospheric composition: $N_2$ (78%), $O_2$ (21%), Ar (1%)
- Size of $N_2$ molecule: 0.31 nm
- Size of $O_2$ molecule: 0.29 nm
- Size of Ar molecule: 0.3 nm
- Visible wavelengths ~400-700 nm

- Scattering of visible light off air molecules is Rayleigh Scattering
- Involves particles much smaller than the wavelength of incident light
- Responsible for the blue color of clear sky
Rayleigh scattering phase function

- $E$ is the orientation of the electric field vector in the incident wave

$$I = I_0 \frac{8 \pi^2 N \alpha^2}{\lambda^4 R^2} (1 + \cos^2 \theta)$$

Scattering at right angles is half the forward intensity for Rayleigh scattering

- $N$ = # of scatterers
- $\alpha$ = polarizability
- $R$ = distance from scatterer

- Recall that scattered skylight is 100% polarized when viewing the sky at a 90º angle from the sun
- Polarizability: ease with which electrons and nuclei can be displaced from their average positions

Vertically polarized
Horizontally polarized
Unpolarized

Variation in sky brightness

- The horizon sky is usually brighter than the zenith sky
- This is a result of single scattering (zenith) vs. multiple scattering (horizon)
Scattering from particles is much stronger than that from molecules.

They're bigger, so they scatter more.

For large particles, we must first consider the fine-scale scattering from the surface microstructure and then integrate over the larger scale structure.

If the surface isn’t smooth, the scattering is incoherent.

If the surfaces are smooth, then we use Snell’s Law and angle-of-incidence-equals-angle-of-reflection.

Then we add up all the waves resulting from all the input waves, taking into account their coherence, too (Mie theory).

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Scattering by a dipole array

• Explains forward scattering by particles of similar size or larger than the wavelength of incident light. The larger the particle, the more it scatters in the forward direction relative to the backward direction.

• For particles (or molecules) much smaller than the wavelength, dipole separation is much smaller than wavelength, so phase differences are small, and scattering is roughly the same in all directions.
Rayleigh and Mie scattering

Rayleigh Scattering

Mie Scattering

When there is large particulate matter in the air, the forward lobe of Mie scattering is dominant. Since it is not very wavelength dependent, we see a white glare around the sun.

- Scattering determines the brightness and color of the sky

Explain the colors...

http://www.flickriver.com/photos/tags/scattering/interesting/
Optical phenomena

- **Rainbow**: for large particles (x = 10,000), the forward and backward peaks in the scattering phase function become very narrow (almost non-existent). Light paths are best predicted using geometric optics and ray tracing.
- **Primary rainbow**: single internal reflection
- **Secondary rainbow**: double internal reflection

Rainbow Light Paths

Rainbows

- **Rainbows**: angular relationships predicted from geometric optics and ray tracing (using Snell’s Law)
- **Focusing of energy at a particular scattering angle gives the rainbow**
Rainbows

- Rainbows are seen at an angle of 42º above the antisolar point
- So if the sun is too high in the sky (higher than 42º), you don’t see them

Optical phenomena

- **Fogbow**: spikes in scattering phase function present but not sharp as for rainbows. Hence the separation of colors (due to varying refractive index) is not as vivid as a normal rainbow. A whitish ring centered on one’s shadow (i.e. opposite the sun) is seen.
- Arises when water droplets have a size characteristic of fog and clouds rather than rain
Optical phenomena

- **Glory**: opposite end of the phase function from the corona. Seen as a 'halo' around one's shadow when looking at a fog bank with the sun at your back. Also seen from aircraft.
- Glories have vivid colors if the range of drop sizes in the fog is relatively narrow, otherwise they are whitish.

Optical phenomena

- **Corona**: for intermediate values of the size parameter (x), the forward scattering peak is accompanied by weaker *sidelobes*. If you were to view the sun through a thin cloud composed of identical spherical droplets (with x = 100 or less), you would see closely spaced rings around the light source. The angular position of the rings depends on wavelength, so the rings would be colored. This is a *corona*.
- Because few real clouds have a sufficiently narrow distribution of drop sizes, coronas are usually more diffuse and less brightly colored.
- Also not a good idea to look directly at the sun....
Reddening/Blueing

\[ Q_e = \frac{\sigma_e}{\pi r^2} \]

\[ \beta_e = \sigma_e N \]

- Assume \( r \) is constant, so variations in \( x \) are due to variations in \( \lambda \)
- Hence increasing \( x \) implies decreasing \( \lambda \), and vice versa.
- For \( 0 < x < 6 \), shorter wavelengths attenuated more: reddening (e.g., setting sun)
- For \( 6 < x < 11 \), longer wavelengths attenuated more: blueing

Reddening/Blueing

- Extinction efficiency against wavelength for selected water droplet radii
- Haze: 0.1-0.3 \( \mu m \) – classic reddening behavior observed on a hazy day
- Intermediate radius (1 \( \mu m \)) – complex behavior, blue and red light attenuated, with attenuation minimum at 0.5-0.6 \( \mu m \) – would give a green sun at sunset
- For larger radii (10 \( \mu m \) – typical cloud droplet) – no strong wavelength dependence
Once in a blue moon…

Blueing of sunlight or moonlight is only rarely observed as it requires an unusual distribution of aerosol sizes for the blueing to dominate over the reddening by air molecules.

Blue moons have been observed after large volcanic eruptions and forest fires. Blue moons and blue-green suns were seen after the 1883 eruption of Krakatoa (Indonesia).

Blue Ridge Mountains

Trees emit volatile organic compounds that oxidize in the air to form tiny oil droplets. These droplets scatter light to produce a blue hue.

Note that in this case the background is dark – so the color arises from light that has been scattered.

Scattering in the Rayleigh regime

\[ Q_s = 4x \text{Im} \left[ \frac{m^2 - 1}{m^2 + 2} \right] + \frac{8}{3} x^3 \text{Re} \left( \frac{m^2 - 1}{m^2 + 2} \right) \]

\[ Q_a = \frac{8}{3} x^3 \left| \frac{m^2 - 1}{m^2 + 2} \right| \]

Hence, for \( x \ll 1 \), and provided \( \text{Im}(m) \neq 0 \):

\[ Q_s \ll Q_a \approx Q_e \]

\[ \tilde{\omega} \equiv \frac{Q_s}{Q_e} \propto x^3 \]

Implications for absorption of thermal IR radiation by atmospheric gases?
**Scattering cross-section**

\[ Q_s = \frac{\sigma_s}{\pi r^2} \quad \beta_s = \sigma_s N \]

- According to Mie theory in the limit of \( x << 1 \) (i.e., small particles), the scattering efficiency \( Q_s \) of a particle in the Rayleigh regime is proportional to \( x^4 \).
- Hence, \( Q_s \) is proportional to \( \left( \frac{r}{\lambda} \right)^4 \).
- Using the above definition of the scattering efficiency, this implies that the scattering cross-section \((\sigma_s)\), which is what determines how much radiation is scattered, we have:
  \[ \sigma_s \propto \frac{r^6}{\lambda^4} \]
- Note that this only applies in the Rayleigh regime, i.e. for \( x << 1 \).

**Radar observations of precipitation**

- Allows tracking of severe weather systems in near real-time
- Relies on scattering of microwave radiation (active system) by hydrometeors
Radar observations of precipitation

- Radar transmitter sends out a series of short pulses of microwave radiation, and a receiver measures the backscattered intensity as a function of the time elapsed following each transmitted pulse ($\Delta t$).
- The one-way distance $d$ to the target is then: 
  $$d = \frac{c\Delta t}{2}$$
  where $c$ is the speed of light.
- The backscattered power $P$ received by the radar antenna is given by the following proportionality:
  $$P \propto \frac{\eta}{d^2}$$
  Where $\eta$ (eta) is the backscatter cross-section per unit volume of air. This is the sum of the backscatter cross-sections ($\sigma_{b,i}$) of all the particles in the sampled volume of air $V$, divided by $V$:
  $$\eta = \frac{1}{V} \sum_i \sigma_{b,i}$$
  $\sigma_b$ is closely related to $\sigma_s$, but only accounts for the radiation scattered backwards toward the radar antenna.

Rayleigh regime for raindrops

The Rayleigh regime for raindrops corresponds to wavelengths of ~10 cm
US Operational Weather Radar network: $\lambda = 10.71$ cm
Radar observations of precipitation

- Radar backscatter efficiency ($Q_{\text{back}}$) for water and ice spheres at the wavelength of the WSR-88D operational weather radar (wavelength = 10.71 cm)
- Up to Diameters of ~6 mm, the Rayleigh relationship ($Q_{\text{back}} \propto r^4$) holds
- 6 mm is the rough upper limit of the size of raindrops observed in heavy rain

Note that Rayleigh relationship holds up to $D = \sim 3$ cm for ice (i.e. hailstones)

- Because of these relationships:
  
  $$Q_{s} = \frac{\sigma_{s}}{\pi r^{2}}$$

  The backscattered power measured by the radar receiver is actually proportional to a reflectivity factor, $Z$:

  $$Z = \int_{0}^{\infty} n(D) D^6 dD$$

  where $D$ is droplet diameter and $n(D)$ is the droplet size distribution function

  - Hence the reflectivity factor is equal to the sum of the sixth powers of the diameters of all the drops in a unit volume of air.

  - Most weather radars record and display estimates of $Z$ at each range $d$.

  - Standard units of $Z$ are mm$^6$ m$^{-3}$ (D in mm), but due to the enormous range of observed values of $Z$, a non-dimensional logarithmic unit dBZ is used:

  $$Z \ [\text{dBZ}] = 10 \log (Z)$$
Radar observations of precipitation

A typical weather radar measures reflectivities ranging from -20 to 70 dBZ.

Because of the $D^6$ dependence in $Z$, reflectivity is strongly influenced by the few largest drops in a volume of air – a single drop of diameter 5 mm reflects more microwave radiation than 15,000 drops of 1 mm diameter.

Clouds (D of ~20 μm) are invisible to most radars, despite large droplet concentrations.

Radar reflectivity example

Take a cloud containing 100 cloud droplets per cm$^3$ with diameter 20 μm.

What is the radar reflectivity factor?

$$Z = \int_{0}^{\infty} n(D)D^6 dD = \sum_i N_i D_i^6$$

Hence we have $N = 100 \times 10^6$ m$^{-3}$ and $D = 20 \times 10^{-3}$ mm, so $Z = 0.0064$ mm$^6$m$^{-3}$.

So $Z [\text{dBZ}] = -22$ dBZ (i.e. very low reflectivity).
The lidar equation

\[ P_r(R) = \frac{C \ h \ \beta}{R^2} \frac{1}{2 \ 4\pi} \exp\left(-2 \int_{0}^{R} \sigma_{\text{ext}}(r') \ dr'\right) \]

- \( P_r \) = received power
- \( R \) = range
- \( C \) = lidar ‘constant’ (transmitted power, receiver cross-section, etc.)
- \( h \) = pulse duration
- \( \beta/4\pi \) = backscatter factor (km\(^{-1}\) sr\(^{-1}\))
- \( \sigma_{\text{ext}} \) = extinction coefficient

**CALIPSO space-borne lidar**

**Direct and diffuse radiation**

- Random paths of 100 photons in a plane-parallel, isotropically scattering layer for variable single scatter albedo (\( \omega \))
- Some photons are transmitted directly through the cloud
- Radiation re-emerging from the top of the layer (e.g., a cloud) determines the albedo
- Radiation that emerges from the cloud base after scattering is diffuse radiation
- The remaining photons are absorbed and their energy goes into warming the cloud
Visibility

Determined by the visual contrast between the brightness of an object and its surroundings.

Atmospheric scattering reduces contrast by adding a source of radiation to the line-of-sight that is independent of the brightness of the target. This source is integrated along the line-of-sight, and so is greater for a longer path.

The distance at which the contrast of an object is reduced to the minimum required for visual detection defines the visibility.

Visibility

• Visibility depends on the relative difference (or contrast) between the light intensity from an object and from the intervening atmosphere.
• A simple analysis expresses visibility as a Beer’s Law problem:

\[ C(x) = \exp(-\beta_e x) \]

• Where \( C(x) \) is the contrast, decreasing exponentially with distance from the object. \( \beta_e \) is the extinction coefficient of the intervening atmosphere.

• The lowest visually perceptible brightness contrast is called the threshold contrast, and is typically about 2% (\( C(x) = 0.02 \)). Hence, at the threshold contrast:

\[ x_v = \frac{3.912}{\beta_e} \]

Koschmeider equation

• Where \( \beta_e \) and \( x \) have similar units (m\(^{-1}\) and m)

Optical thickness?
Visibility

• In the absence of aerosols, extinction is due purely to Rayleigh scattering.
• At sea level the Rayleigh atmosphere has an extinction coefficient $\beta_e$ of $\sim 13.2 \times 10^{-6} \text{ m}^{-1}$ at a wavelength of 520 nm.
• This gives a visual range in the cleanest possible atmosphere of $\sim 296$ km.
• Note that Rayleigh scattering is proportional to air density and decreases with altitude.
• Mie scattering by aerosol particles comparable in size to visible wavelengths is responsible for most visibility reduction, and dominates in urban areas.
• Note that this simple analysis of visibility neglects the reflective properties of the object, the direction of incident sunlight, the scattering phase function (which varies with aerosol type), etc.

$$x_v \approx \frac{1}{\beta_e} \ln \left[ \frac{200\mu}{\omega \ p(\cos \theta)} + 1 \right]$$

$\mu = \cos \text{ (solar zenith angle)}$
$\omega = \text{ single scattering albedo}$
$p(\cos \theta) = \text{ phase function}$

USA visibility

Annual Average Standard Visual Range in the Contiguous United States, 2004

- Sulfate and nitrate particles
- Humidity
- Organic carbon
- Soil dust
Visibility

- A general term for light scattered by molecules and particles along a line of sight is **airlight**.
- Airlight initially increases linearly with optical thickness (more scattering), but the increase slows down as *multiple scattering* comes into play.
- A threshold contrast of 2% (0.02) corresponds to an optical thickness of ~3.9.
- Mie scattering by aerosol particles comparable in size to visible wavelengths (0.1-1 µm) is responsible for most visibility reduction, and dominates in urban areas.
- Scattering by air molecules usually has a minor influence on urban visibility.
- Particle absorption is ~5-10% of extinction in remote areas and up to 50% in urban areas (*carbon*).
- **Nitrogen dioxide (NO₂)** is the only light absorbing gas present in significant quantities in the troposphere.
- NO₂ is strongly blue-absorbing, and hence colors plumes red, brown or yellow.