Classification: LDA and QDA Approaches

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2019-02-28

# Classification and Categorization

General regression approaches we have taken so far have typically had the goal of modeling how a dependent variable (usually continuous, but in the case of logistic regression, binary, or with multinomial regression multiple levels) is predicted by a set of independent or predictor variables. The goals of this are often prediction, inference, forecasting, and understanding the underlying processes.

Many times, we might care less about making a numerical prediction, and more about classifying into categories to be able to take action, based on measureable properties. This is very similar to the goals of logistic regression, but probably with a preference for assessing the most likely categorization, rather than the probability of a category. Furthermore, logistic regression will typically ignore the prior likelihoods of the different outcomes. Classification methods would typically want to consider the overall base rate as well as the relative influence of different factors, in order to maximize the probability of getting the right prediction. Typically, for classification approaches, we need to select a criterion or cut-off of some sort to make a decision–hopefully one that will give the greatest chance of getting a new classification right.

There are many applications for classification, and recently the work on this has migrated from statisticians to computer scientists and specialists in machine classification, machine learning, and artificial intelligence.

Historically, there are many uses for classification, and the goals differ somewhat from approaches like logistic regression and MANOVA. For example, either approach could be used as a tool to help diagnose a disease. A logistic regression would produce the odds that a person has a disease, whereas the a machine classification might be used to determine whether a disease is present–perhaps with some margin of error. A regression might be used to determine the likelihood a company’s will declare bankruptcy, while a classification might be used to identify a set of ‘at-risk’ companies. Overall, these are very similar goals, and although there are many different approaches to achieve this, the solutions end up looking very similar.

Some of the terminology tends to differ across these two approaches. A regression approach might discuss variables, fitting, and inference; a classification might call these features, learning, and cross-validation. There are some aspects of applying regression models and classification models that have similar goals, but different methods. For example, in regression modeling, we often use anova, analysis of deviance, or a criterion such as AIC to compare models and determine whether a predictor should be used. In classification, the ultimate criterion is often classification accuracy, and a huge aspect of ‘machine learning’ is selecting and removing features to use in a model. In general, the attitude for regression models is typically interpretability, leading to small models with comprehensible measures. Oftentimes, classification approaches might start with hundreds or thousands of features, and combination rules and decision rules might be very abstract and difficult to comprehend. Furthermore, classification is more likely to use separate data sets or cross-validition to do variable selection (feature learning). Regression is more likely to use AIC or BIC methods to select variables on the whole data set.

# Classification as Logistic Regression

Logistic regression provides a method for making a prediction about the binary classification of something based on a set of predictors.

In the following data set, we asked participants to judge whether concepts were related. Some of the concepts were engineering terms; others were psychology terms. We also polled both engineeering and psychology students, and we wanted to know if we could tell them apart based on the speed and accuracy of their responses. Terms were classified into different bins, with e=engineering, p=psychology, r=related, and u=unrelated.

library(dplyr)  
  
data.raw <- read.csv("samediff-pooled.csv")  
data <- dplyr::filter(data.raw, cond %in% c("eerc", "eer", "eeu", "ep", "ppr",   
 "ppu"))  
  
  
stim <- c(as.character(data$word1), as.character(data$word2))  
acc <- c(data$corr, data$corr)  
rt <- c(data$rt, data$rt)  
sub <- c(data$subnum, data$subnum)  
data$pairs <- paste(data$word1, data$word2, sep = "-")  
  
dat.corr <- as.data.frame(tapply(data$corr, list(sub = data$sub, type = factor(data$cond)),   
 mean))  
dat.rt <- as.data.frame(tapply(data$rt, list(sub = data$sub, type = factor(data$cond)),   
 function(x) {  
 exp(mean(log(x), na.rm = T))  
 }))  
  
dat.joint <- cbind(dat.corr, dat.rt)  
  
survey <- read.csv("survey.csv")  
surv2 <- data.frame(sub = survey$subnum, eng = survey$engineering)  
  
joint <- data.frame(eng = surv2$eng, dat.joint)  
  
  
model1 <- glm(eng ~ ., data = joint, family = binomial)  
summary(model1)

Call:  
glm(formula = eng ~ ., family = binomial, data = joint)  
  
Deviance Residuals:   
 Min 1Q Median 3Q Max   
-1.65694 -1.07432 -0.08598 1.09348 1.76901   
  
Coefficients:  
 Estimate Std. Error z value Pr(>|z|)   
(Intercept) -3.322e+00 2.018e+00 -1.646 0.0997 .  
eer 2.279e-02 9.498e-01 0.024 0.9809   
eeu 6.761e-01 8.583e-01 0.788 0.4309   
ep 9.017e-01 1.916e+00 0.471 0.6380   
ppr 3.123e-01 9.844e-01 0.317 0.7510   
ppu 1.564e+00 1.043e+00 1.499 0.1337   
eer.1 -3.658e-05 2.387e-04 -0.153 0.8782   
eeu.1 -1.627e-04 2.979e-04 -0.546 0.5849   
ep.1 3.758e-04 3.804e-04 0.988 0.3232   
ppr.1 -1.092e-04 2.485e-04 -0.439 0.6605   
ppu.1 2.380e-04 2.761e-04 0.862 0.3886   
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
(Dispersion parameter for binomial family taken to be 1)  
  
 Null deviance: 105.36 on 75 degrees of freedom  
Residual deviance: 97.54 on 65 degrees of freedom  
AIC: 119.54  
  
Number of Fisher Scoring iterations: 4

We can make a prediction about the log-odds of each person being an engineer, and compare it to the actual values

table(predict(model1, joint) > 0, joint$eng)

0 1  
 FALSE 26 14  
 TRUE 12 24

This is not bad–about 2/3 correct in each category. Maybe there is a better place to put the criterion. This could be true if we had mostly engineers or mostly psychologists.

sum(diag(table(predict(model1, joint) > -0.5, joint$eng)))

[1] 48

sum(diag(table(predict(model1, joint) > -0.25, joint$eng)))

[1] 50

sum(diag(table(predict(model1, joint) > -0.1, joint$eng)))

[1] 50

sum(diag(table(predict(model1, joint) > 0, joint$eng)))

[1] 50

sum(diag(table(predict(model1, joint) > 0.25, joint$eng)))

[1] 47

sum(diag(table(predict(model1, joint) > 0.5, joint$eng)))

[1] 40

If we looked at just the first 30 participants, the best criterion is different:

sum(diag(table(predict(model1, joint[1:50, ]) > -0.5, joint$eng[1:50])))

[1] 35

sum(diag(table(predict(model1, joint[1:50, ]) > -0.25, joint$eng[1:50])))

[1] 36

sum(diag(table(predict(model1, joint[1:50, ]) > -0.1, joint$eng[1:50])))

[1] 31

sum(diag(table(predict(model1, joint[1:50, ]) > 0, joint$eng[1:50])))

[1] 29

sum(diag(table(predict(model1, joint[1:50, ]) > 0.25, joint$eng[1:50])))

[1] 23

sum(diag(table(predict(model1, joint[1:50, ]) > 0.5, joint$eng[1:50])))

[1] 19

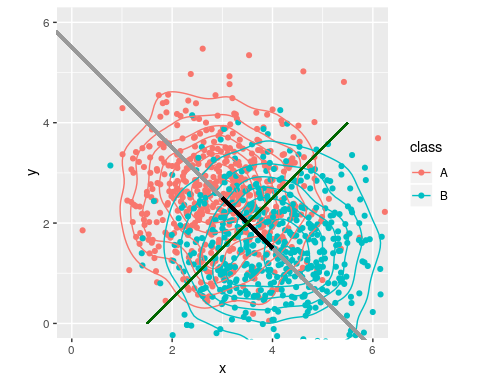
Here, a criterion of -.25 gives us better classification performance. This is just a consequence of base rate, because more engineers were stored in the first half of the data file.

This is OK, but a bit worrisome. None of the predictors were significant, so we might have gotten here just by chance. We might be over-fitting the data. In regression, we’d do inferential test or information criteria to identify which variables to use. Later, we’ll see how this is done for machine classificiation

## Basics of Machine Classification

We can see how logistic regression can be used as a classifier, as long as we can determine a resonable criterion. Prior to the widespread use of logistic regression (which requires maximum-likelihood estimation and computerized approaches), various approaches to classification were developed that use simplified models. One traditional model assumes that two groups have multivariate normal distributions with equal variance. In the figure below, we see two such 2-dimensional distributions, with a line drawn between the centers of each, and contours showing the basic data.

library(ggplot2)  
n <- 500  
class <- rep(c("A", "B"), each = n)  
x <- rnorm(n \* 2, mean = rep(c(3, 4), each = n))  
y <- rnorm(n \* 2, mean = rep(c(2.5, 1.5), each = n))  
ggplot(data.frame(class, y, x), aes(x = x, y = y, colour = class)) + geom\_point(size = 1.5) +   
 geom\_density\_2d() + geom\_segment(x = -1, y = 6.5, xend = 7, yend = -1.5,   
 col = "grey60", lwd = 1.2) + geom\_segment(x = 3, y = 2.5, xend = 4, yend = 1.5,   
 col = "black", lwd = 1.2) + geom\_segment(x = 1.5, y = 0, xend = 5.5, yend = 4,   
 col = "darkgreen") + coord\_fixed(ratio = 1, xlim = c(0, 6), ylim = c(0,   
 6))



Under our assumptions, if we draw a line between the centers and extend it out (the grey line), we can project any points onto this line (i.e., the closest point on the line to each point). We could use position along this line to predict category membership in a regression or logistic regression–and this is essentially what regression does. This mapping from input variables to a single function is called the discriminant function, and is equivalent to the weighted sum in regression. Now, if we want to classify any observation, we just need to determine which case is more likely. Given the assumptions of equal variance and normality, this can be shown to be a single criterion that maximizes our chances of being correct. In this case, that corresponds to where the green line intersect the block line, and if we move back to the original data, the entire green line is a good rule discriminating the two groups.

Under these assumptions, there are a number of approaches that can be used. In fact, MANOVA is essentially equivalent, but framing the model backwards. The most common approach used is referred to as linear discriminant analysis (LDA), or sometimes multivariate discrimination analysis (MDA). The assumptions of of this approach bit stronger than regression (requiring normal input predictors and equal variances). If these assumptions hold or we can transform them so they work, we can get improved classification results over other methods. In practice, the methods are likely to be almost equivalant to logistic regression.

#Linear discriminant analysis

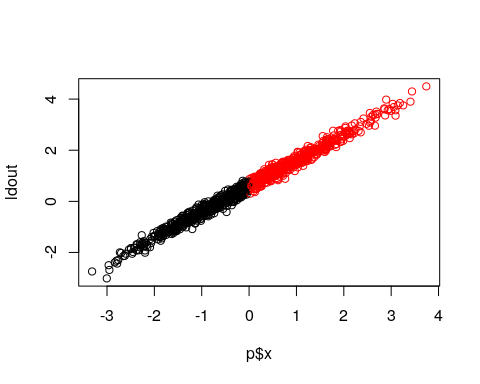
Using the fake data from the figure, we can fit an lda model, using the MASS library:

library(MASS)  
model0 <- lda(as.factor(class) ~ x + y)  
model0

Call:  
lda(as.factor(class) ~ x + y)  
  
Prior probabilities of groups:  
 A B   
0.5 0.5   
  
Group means:  
 x y  
A 2.960500 2.526817  
B 4.027253 1.487700  
  
Coefficients of linear discriminants:  
 LD1  
x 0.7221432  
y -0.6661971

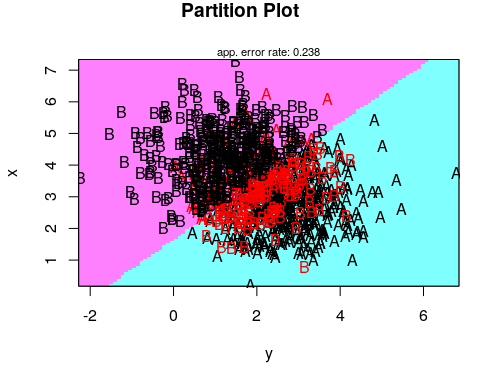
We can see that the simple LDA finds the mean of group A and B along the two measured dimensions, and then reports ‘coefficients of linear discriminants’. This is the direction in XY space that best discriminates the two groups. If we could map each observation onto this line, we can easily make a decision that optimally classifies the two groups. We can simply multiply each observed variable by the coefficient to get a sum value, which is the value used to discriminate the two classes, once an optimal threshold is chosen. You can see that if you do predict() on a model, the $class tells you predicted class, and $x tells you the exact values we calcule in ldout.

ldout <- x \* 0.62 - y \* 0.7857  
p <- predict(model0)  
  
plot(p$x, ldout, col = p$class)



This is a bit clearer if we visualize, which we can do via the klaR library, which has a number of classification schemes available, including a number of visualization methods. Let’s look at a ‘partimat’ plot:

library(klaR)  
partimat(as.factor(class) ~ x + y, method = "lda")



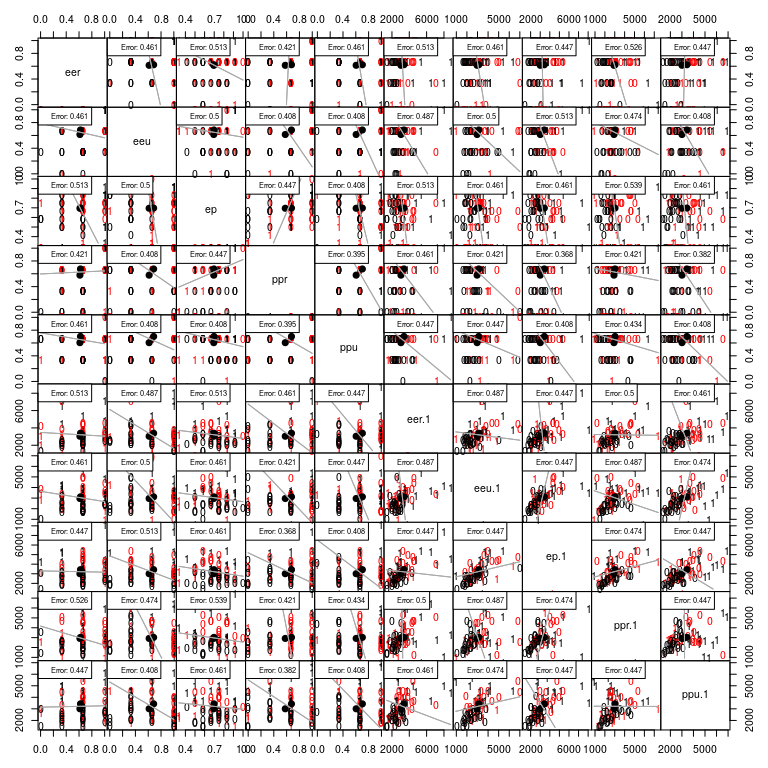
partimat(as.factor(class) ~ x + y, method = "lda", plot.matrix = TRUE, imageplot = FALSE) # takes some time ...



If you think about the line connecting the centers of the two groups, it goes from the upper left to lower right. This direction can be defined by a vector: (.62, -.78).

Let’s look at the engineering data set, which has more than two predictors

partimat(as.factor(eng) ~ ., data = joint, method = "lda", plot.matrix = TRUE,   
 imageplot = FALSE)



This shows the classification along each pair of dimensions.

library(MASS)  
library(DAAG)  
ll <- lda(eng ~ ., data = joint)  
ll

Call:  
lda(eng ~ ., data = joint)  
  
Prior probabilities of groups:  
 0 1   
0.5 0.5   
  
Group means:  
 eer eeu ep ppr ppu eer.1 eeu.1  
0 0.6140351 0.6140351 0.6951754 0.5789474 0.6052632 3038.008 2962.742  
1 0.6228070 0.6842105 0.6973684 0.6754386 0.7017544 3387.195 3120.454  
 ep.1 ppr.1 ppu.1  
0 3005.707 2921.188 3007.688  
1 3428.785 3024.660 3432.817  
  
Coefficients of linear discriminants:  
 LD1  
eer 3.672284e-02  
eeu 9.989394e-01  
ep 1.295582e+00  
ppr 4.695055e-01  
ppu 2.385932e+00  
eer.1 -5.396259e-05  
eeu.1 -2.316937e-04  
ep.1 4.827826e-04  
ppr.1 -1.456996e-04  
ppu.1 3.812870e-04

IF we look at group means and the coefficients, we can see that a few of the measures differ substantially between the two groups, but not all. The largest coefficients typically map onto the dimensions with the greatest difference between groups.

## Predicting class from an LDA model

predict(ll)

$class  
 [1] 0 1 0 1 1 0 1 0 1 1 0 0 1 0 1 0 1 1 1 0 0 1 1 1 0 1 0 1 0 1 0 0 0 0 1  
[36] 1 1 0 1 0 0 1 0 1 0 0 1 1 0 1 0 0 1 1 1 1 1 0 1 0 0 0 0 1 0 0 0 0 0 1  
[71] 1 0 0 0 0 0  
Levels: 0 1  
  
$posterior  
 0 1  
11 0.5512784 0.4487216  
12 0.4907813 0.5092187  
13 0.5540551 0.4459449  
22 0.1502546 0.8497454  
31 0.1237940 0.8762060  
32 0.6055351 0.3944649  
33 0.4234531 0.5765469  
41 0.6275495 0.3724505  
51 0.1367011 0.8632989  
52 0.4797138 0.5202862  
61 0.5544694 0.4455306  
62 0.6745057 0.3254943  
63 0.4469692 0.5530308  
64 0.6752190 0.3247810  
71 0.3398060 0.6601940  
81 0.5468117 0.4531883  
101 0.4014884 0.5985116  
102 0.3694719 0.6305281  
104 0.3822059 0.6177941  
201 0.8183543 0.1816457  
202 0.5981471 0.4018529  
203 0.3306295 0.6693705  
301 0.4570754 0.5429246  
302 0.4452991 0.5547009  
303 0.6392452 0.3607548  
304 0.2763718 0.7236282  
401 0.5196067 0.4803933  
402 0.4912082 0.5087918  
501 0.5067966 0.4932034  
502 0.4105879 0.5894121  
503 0.6803426 0.3196574  
504 0.5564842 0.4435158  
505 0.5814196 0.4185804  
507 0.6919807 0.3080193  
508 0.3626906 0.6373094  
509 0.4488615 0.5511385  
510 0.4027089 0.5972911  
 [ reached getOption("max.print") -- omitted 39 rows ]  
  
$x  
 LD1  
11 -0.319286006  
12 0.057204773  
13 -0.336707250  
22 2.687541519  
31 3.035585511  
32 -0.664798373  
33 0.478708503  
41 -0.809266716  
51 2.858725700  
52 0.125937361  
61 -0.339308577  
62 -1.130227198  
63 0.330278399  
64 -1.135269580  
71 1.030214376  
81 -0.291302314  
101 0.619325999  
102 0.829066967  
104 0.744858679  
201 -2.334858402  
202 -0.616973413  
203 1.094091479  
301 0.266988525  
302 0.340762460  
303 -0.887400074  
304 1.493035179  
401 -0.121714764  
402 0.054555562  
501 -0.042172729  
502 0.560798315  
503 -1.171660463  
504 -0.351965570  
505 -0.509715466  
507 -1.255499384  
508 0.874394450  
509 0.318408496  
510 0.611451229  
602 -1.123001743  
603 1.264762328  
605 -1.974224117  
606 -1.222998498  
607 0.924897502  
608 -0.735353258  
609 0.548135292  
611 -0.063963944  
701 -1.050595289  
702 0.109848246  
703 1.802930456  
704 -0.836690773  
705 0.046500471  
706 -1.153642814  
707 -0.238165272  
708 0.510908846  
709 0.283663800  
801 0.590030309  
802 1.517968113  
803 0.503132773  
804 -0.350171265  
805 1.234031870  
806 -0.169898973  
807 -1.018771624  
808 -0.060496899  
809 -0.336577470  
901 1.310707947  
903 -0.835666615  
905 -1.652252375  
906 -0.008369771  
907 -0.003339867  
908 -1.266912360  
909 1.161982264  
910 0.284245408  
911 -1.372632710  
913 -0.295670105  
914 -0.355635080  
916 -1.285565045  
 [ reached getOption("max.print") -- omitted 1 row ]

table(predict(ll)$class, joint$eng)

0 1  
 0 27 14  
 1 11 24

confusion(joint$eng, predict(ll)$class)

Overall accuracy = 0.671   
  
Confusion matrix   
 Predicted (cv)  
Actual [,1] [,2]  
 [1,] 0.711 0.289  
 [2,] 0.368 0.632

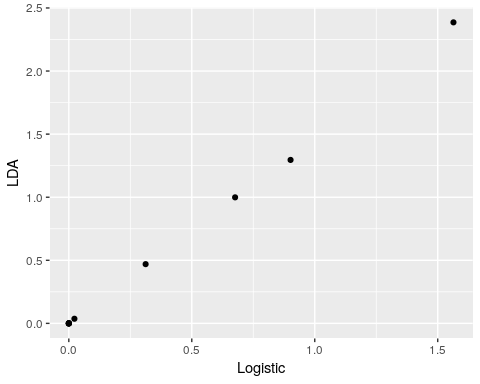
sum(diag(table(predict(ll)$class, joint$eng)))

[1] 51

Let’s examine the different aspects of the results.

First, the model reports the prior probabliities of groups. This will be the number of each type of input value. Note that if the true base rate differs from the training set (something that might be very likely), we might want to set this explicitly when we predict the data. Next, we see the mean values across each of the variables measured. This is the center of the two normal distributions. Then, we see coefficients of lienar discriminants–these are equivalant to the beta weight used to create the discriminant function. It shows the best guess classifications, and finally posterior likelihoods of each–this should be very similar to estimated probabilities from the logistic model. Finally, $x shows the discriminant function value, which we could use to choose a different decision criterion. There are several methods for determining the best decision rule.

library(GGally)  
ggplot(data.frame(Logistic = model1$coefficients[-1], LDA = (ll$scaling)[, 1]),   
 aes(x = Logistic, y = LDA)) + geom\_point()

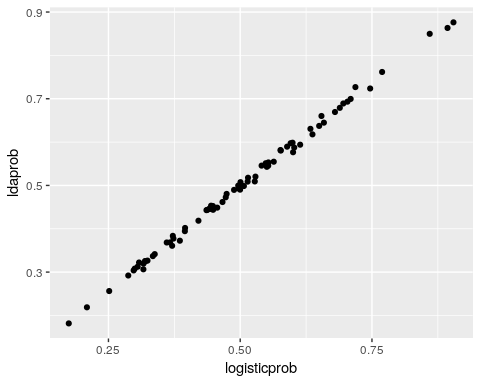


Here, the coefficients from the logistic and lda models are not identical, but their correlation is 1.0! If we compare the predicted probability vs. the lda likelihood, we see that they are highly correlated.

logit <- function(lo) {  
 1/(1 + exp(-lo))  
} ##This is the inverse of the logodds function. The   
df <- data.frame(logistic = predict(model1), lda = (predict(ll)$x)[, 1], logisticprob = logit(predict(model1)),   
 ldaprob = predict(ll)$posterior[, 2])  
  
ggplot(df, aes(x = logistic, y = lda)) + geom\_point()



ggplot(df, aes(x = logisticprob, y = ldaprob)) + geom\_point()

 Notice how the discriminant value is almost the same as the log-odds predicted value in logistic regression, and transforming these to probabilities also produces almost identical values.

In terms of classification performance, the prediction for LDA was 1 error less than logistic regression, but the two models are really essentially identical, with the main difference being how the parameters are fit and the assumptions being made.

##Cross-validation

It is easy to overfit classification data, and so we must be careful to avoid this. Generally, just as with variable selection in regression models, we are worried about determining the best subset of predictors to use. Since using more variables will never make the model worse at fitting its own data, it is useful to hold some data out and test the model on the held-out data. A common approach is to use leave-one-out cross-validation. This approach will fit the model N times for N observations, fitting each left-out case in each model.

The LDA model allows you to do this automatically using the CV=TRUE option. This will do the classification automatically, instead of embedding it within the predict function and doing it manually.

ll2 <- lda(eng ~ ., data = joint, CV = TRUE)  
## Original model  
confusion(joint$eng, predict(ll)$class)

Overall accuracy = 0.671   
  
Confusion matrix   
 Predicted (cv)  
Actual [,1] [,2]  
 [1,] 0.711 0.289  
 [2,] 0.368 0.632

sum(diag(table(joint$eng, predict(ll)$class)))

[1] 51

## cross-validation:  
confusion(ll2$class, joint$eng)

Overall accuracy = 0.461   
  
Confusion matrix   
 Predicted (cv)  
Actual 0 1  
 0 0.465 0.535  
 1 0.545 0.455

sum(diag(table(ll2$class, joint$eng)))

[1] 35

## correspondence between predictions  
confusion(ll2$class, predict(ll)$class)

Overall accuracy = 0.789   
  
Confusion matrix   
 Predicted (cv)  
Actual 0 1  
 0 0.791 0.209  
 1 0.212 0.788

sum(diag(table(ll2$class, predict(ll)$class)))

[1] 60

In contrast to the 51 cases we got correct before, the cross-validation gets just 35 correct (out of 76)–this is actually worse than chance! This is in spite of the fact that there is a lot of agreement between the two models.

Notice that we no longer have a single lda model to look at. the output of the cross-validation is simpler. We can’t look at the linear discriminant values or means because there is no longer one model–we tested N models.

ll2

$class  
 [1] 0 0 0 1 1 0 0 0 1 0 0 0 1 0 1 0 1 1 1 0 0 1 0 0 0 1 0 0 0 1 0 0 1 0 0  
[36] 1 1 0 1 0 0 1 0 1 0 0 0 1 0 1 0 0 1 1 0 1 1 1 1 1 0 1 0 1 0 0 1 1 0 1  
[71] 1 0 0 1 0 0  
Levels: 0 1  
  
$posterior  
 0 1  
11 0.7932692 0.2067308  
12 0.5469743 0.4530257  
13 0.6111616 0.3888384  
22 0.1626239 0.8373761  
31 0.1460898 0.8539102  
32 0.6590130 0.3409870  
33 0.5343306 0.4656694  
41 0.6910390 0.3089610  
51 0.1772707 0.8227293  
52 0.5495802 0.4504198  
61 0.5799697 0.4200303  
62 0.7598981 0.2401019  
63 0.4867715 0.5132285  
64 0.7752926 0.2247074  
71 0.4180372 0.5819628  
81 0.5583747 0.4416253  
101 0.4326410 0.5673590  
102 0.3218178 0.6781822  
104 0.2705828 0.7294172  
201 0.7983831 0.2016169  
202 0.5647722 0.4352278  
203 0.2553252 0.7446748  
301 0.5082852 0.4917148  
302 0.6851039 0.3148961  
303 0.5797293 0.4202707  
304 0.2312416 0.7687584  
401 0.5506530 0.4493470  
402 0.5486566 0.4513434  
501 0.5358578 0.4641422  
502 0.3694875 0.6305125  
503 0.8212536 0.1787464  
504 0.5295038 0.4704962  
505 0.4794108 0.5205892  
507 0.6428161 0.3571839  
508 0.5490367 0.4509633  
509 0.4697326 0.5302674  
510 0.4345844 0.5654156  
 [ reached getOption("max.print") -- omitted 39 rows ]  
  
$terms  
eng ~ eer + eeu + ep + ppr + ppu + eer.1 + eeu.1 + ep.1 + ppr.1 +   
 ppu.1  
attr(,"variables")  
list(eng, eer, eeu, ep, ppr, ppu, eer.1, eeu.1, ep.1, ppr.1,   
 ppu.1)  
attr(,"factors")  
 eer eeu ep ppr ppu eer.1 eeu.1 ep.1 ppr.1 ppu.1  
eng 0 0 0 0 0 0 0 0 0 0  
eer 1 0 0 0 0 0 0 0 0 0  
eeu 0 1 0 0 0 0 0 0 0 0  
ep 0 0 1 0 0 0 0 0 0 0  
ppr 0 0 0 1 0 0 0 0 0 0  
ppu 0 0 0 0 1 0 0 0 0 0  
eer.1 0 0 0 0 0 1 0 0 0 0  
 [ reached getOption("max.print") -- omitted 4 rows ]  
attr(,"term.labels")  
 [1] "eer" "eeu" "ep" "ppr" "ppu" "eer.1" "eeu.1" "ep.1"   
 [9] "ppr.1" "ppu.1"  
attr(,"order")  
 [1] 1 1 1 1 1 1 1 1 1 1  
attr(,"intercept")  
[1] 1  
attr(,"response")  
[1] 1  
attr(,".Environment")  
<environment: R\_GlobalEnv>  
attr(,"predvars")  
list(eng, eer, eeu, ep, ppr, ppu, eer.1, eeu.1, ep.1, ppr.1,   
 ppu.1)  
attr(,"dataClasses")  
 eng eer eeu ep ppr ppu eer.1   
"numeric" "numeric" "numeric" "numeric" "numeric" "numeric" "numeric"   
 eeu.1 ep.1 ppr.1 ppu.1   
"numeric" "numeric" "numeric" "numeric"   
  
$call  
lda(formula = eng ~ ., data = joint, CV = TRUE)  
  
$xlevels  
named list()

The best practice in a situation like this might be to use cross-validation accuracy to help guide variable selection. You might use a stepwise procedure, and only include a variable if it improves cross-validation accuracy. You might use the single best model at the end, but still acknowledge cross-validation performance. In this case, results such as this led our research lab to conclude that there was no substantial difference between groups, and we developed new behavioral methods that were more powerful.

##LDA with multiple categories

The other advantage of LDA over regression is that it handles multiple categories directly. Here, just as with the multinom() model, it creates discriminant functions for classes, each compared against a baseline. Let’s look at the iris data for which we examined previously under the multinomial model.

m.iris <- lda(Species ~ ., data = iris)  
m.iris

Call:  
lda(Species ~ ., data = iris)  
  
Prior probabilities of groups:  
 setosa versicolor virginica   
 0.3333333 0.3333333 0.3333333   
  
Group means:  
 Sepal.Length Sepal.Width Petal.Length Petal.Width  
setosa 5.006 3.428 1.462 0.246  
versicolor 5.936 2.770 4.260 1.326  
virginica 6.588 2.974 5.552 2.026  
  
Coefficients of linear discriminants:  
 LD1 LD2  
Sepal.Length 0.8293776 0.02410215  
Sepal.Width 1.5344731 2.16452123  
Petal.Length -2.2012117 -0.93192121  
Petal.Width -2.8104603 2.83918785  
  
Proportion of trace:  
 LD1 LD2   
0.9912 0.0088

m.irisCV <- lda(Species ~ ., data = iris, CV = TRUE)  
  
table(iris$Species, m.irisCV$class)

setosa versicolor virginica  
 setosa 50 0 0  
 versicolor 0 48 2  
 virginica 0 1 49

Now, the classification is very good, even with cross-validation. We can see two sets of coefficients–the two discriminant functions distinguishing each pairing of outcomes.

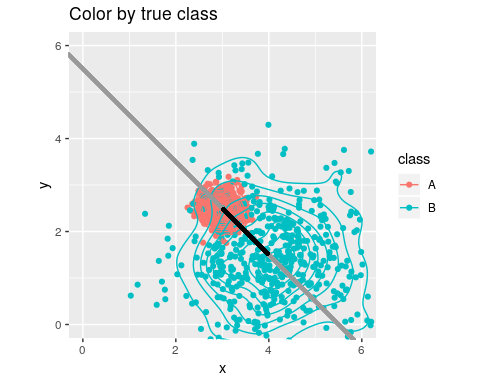
# Quadratic Discriminant Analysis

One of the assumptions of LDA is that the two distributions have equal variance. If we relax this assumption, the best classification no longer has to be a line separating the space. We can get curved boundaries, or even a small region within a larger region. For exmaple:

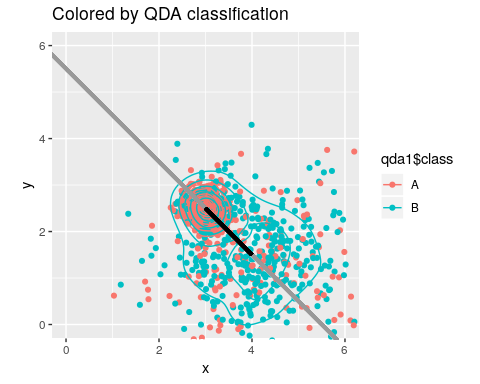
qda1 <- qda(class ~ x + y, CV = TRUE)  
table(class, qda1$class)

class A B  
 A 393 107  
 B 131 369

n <- 500  
class <- rep(c("A", "B"), each = n)  
x <- rnorm(n \* 2, mean = rep(c(3, 4), each = n), sd = rep(c(0.25, 1), each = n))  
y <- rnorm(n \* 2, mean = rep(c(2.5, 1.5), each = n), sd = rep(c(0.25, 1), each = n))  
  
ggplot(data.frame(class, y, x), aes(x = x, y = y, colour = class)) + geom\_point(size = 1.5) +   
 geom\_density\_2d() + geom\_segment(x = -1, y = 6.5, xend = 7, yend = -1.5,   
 col = "grey60", lwd = 1.2) + geom\_segment(x = 3, y = 2.5, xend = 4, yend = 1.5,   
 col = "black", lwd = 1.2) + coord\_fixed(ratio = 1, xlim = c(0, 6), ylim = c(0,   
 6)) + ggtitle("Color by true class")



ggplot(data.frame(class, y, x), aes(x = x, y = y, colour = qda1$class)) + geom\_point(size = 1.5) +   
 geom\_density\_2d() + geom\_segment(x = -1, y = 6.5, xend = 7, yend = -1.5,   
 col = "grey60", lwd = 1.2) + geom\_segment(x = 3, y = 2.5, xend = 4, yend = 1.5,   
 col = "black", lwd = 1.2) + coord\_fixed(ratio = 1, xlim = c(0, 6), ylim = c(0,   
 6)) + ggtitle("Colored by QDA classification")



We can see how an LDA model will suffer. If all points are projected onto the discriminant line, the single boundary on that line will not be ideal. If we were able to make a curved boundary in this xy space, we could capture correct classifications. Quadratic Discriminant Analysis (QDA) permits this. It provides a more powerful classifier that can capture non-linear boundaries in the feature space. Thus, it is also less constrained, so requires more careful analysis to ensure we don’t overfit the model. How does it work with our real data set?

library(MASS)  
q <- qda(eng ~ ., data = joint, CV = TRUE)  
q

$class  
 [1] 0 1 0 1 1 0 0 0 0 1 0 0 1 1 1 0 0 0 1 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 1  
[36] 0 0 0 1 1 0 0 0 1 0 0 0 1 1 0 1 0 1 0 1 1 0 1 1 1 0 0 0 1 0 0 0 0 1 1  
[71] 1 0 0 1 0 0  
Levels: 0 1  
  
$posterior  
 0 1  
11 9.974693e-01 2.530667e-03  
12 8.815744e-02 9.118426e-01  
13 9.472289e-01 5.277114e-02  
22 1.370973e-01 8.629027e-01  
31 1.379738e-16 1.000000e+00  
32 9.478123e-01 5.218767e-02  
33 7.036601e-01 2.963399e-01  
41 7.048625e-01 2.951375e-01  
51 1.000000e+00 1.103873e-08  
52 3.288272e-01 6.711728e-01  
61 8.394071e-01 1.605929e-01  
62 9.297141e-01 7.028591e-02  
63 4.762053e-01 5.237947e-01  
64 5.772267e-02 9.422773e-01  
71 1.996504e-03 9.980035e-01  
81 7.733766e-01 2.266234e-01  
101 8.181787e-01 1.818213e-01  
102 7.929339e-01 2.070661e-01  
104 1.165120e-03 9.988349e-01  
201 9.983142e-01 1.685776e-03  
202 6.645369e-01 3.354631e-01  
203 6.743729e-01 3.256271e-01  
301 9.814788e-01 1.852123e-02  
302 1.627353e-05 9.999837e-01  
303 2.929281e-01 7.070719e-01  
304 3.462853e-02 9.653715e-01  
401 6.924185e-01 3.075815e-01  
402 9.115441e-01 8.845587e-02  
501 4.643224e-01 5.356776e-01  
502 4.199688e-01 5.800312e-01  
503 9.999999e-01 1.320222e-07  
504 8.343022e-01 1.656978e-01  
505 8.036457e-01 1.963543e-01  
507 8.672052e-01 1.327948e-01  
508 7.843539e-05 9.999216e-01  
509 6.728246e-01 3.271754e-01  
510 8.772416e-01 1.227584e-01  
 [ reached getOption("max.print") -- omitted 39 rows ]  
  
$terms  
eng ~ eer + eeu + ep + ppr + ppu + eer.1 + eeu.1 + ep.1 + ppr.1 +   
 ppu.1  
attr(,"variables")  
list(eng, eer, eeu, ep, ppr, ppu, eer.1, eeu.1, ep.1, ppr.1,   
 ppu.1)  
attr(,"factors")  
 eer eeu ep ppr ppu eer.1 eeu.1 ep.1 ppr.1 ppu.1  
eng 0 0 0 0 0 0 0 0 0 0  
eer 1 0 0 0 0 0 0 0 0 0  
eeu 0 1 0 0 0 0 0 0 0 0  
ep 0 0 1 0 0 0 0 0 0 0  
ppr 0 0 0 1 0 0 0 0 0 0  
ppu 0 0 0 0 1 0 0 0 0 0  
eer.1 0 0 0 0 0 1 0 0 0 0  
 [ reached getOption("max.print") -- omitted 4 rows ]  
attr(,"term.labels")  
 [1] "eer" "eeu" "ep" "ppr" "ppu" "eer.1" "eeu.1" "ep.1"   
 [9] "ppr.1" "ppu.1"  
attr(,"order")  
 [1] 1 1 1 1 1 1 1 1 1 1  
attr(,"intercept")  
[1] 1  
attr(,"response")  
[1] 1  
attr(,".Environment")  
<environment: R\_GlobalEnv>  
attr(,"predvars")  
list(eng, eer, eeu, ep, ppr, ppu, eer.1, eeu.1, ep.1, ppr.1,   
 ppu.1)  
attr(,"dataClasses")  
 eng eer eeu ep ppr ppu eer.1   
"numeric" "numeric" "numeric" "numeric" "numeric" "numeric" "numeric"   
 eeu.1 ep.1 ppr.1 ppu.1   
"numeric" "numeric" "numeric" "numeric"   
  
$call  
qda(formula = eng ~ ., data = joint, CV = TRUE)  
  
$xlevels  
named list()

confusion(q$class, joint$eng)

Overall accuracy = 0.566   
  
Confusion matrix   
 Predicted (cv)  
Actual 0 1  
 0 0.556 0.444  
 1 0.419 0.581

sum(diag(table(q$class, joint$eng)))

[1] 43

Now, the qda model is a reasonable improvement over the LDA model–even with Cross-validation. We were at 46% accuracy with cross-validation, and now we are at 57%. This increased cross-validation accuracy from 35 to 43 accurate cases.

##Variable Selection in LDA We now have a good measure of how well this model is doing. But we suspect that–at least for LDA, the predictors might be over-fitting. We’d like to try removing variables to see if we get a better cross-validation performance. We could do this by hand, or using some tools built for this. The stepclass function within klaR package will do this:

library(klaR)  
modelstepL <- stepclass(eng ~ ., "lda", direction = "both", data = joint)

correctness rate: 0.55; in: "ppr"; variables (1): ppr   
correctness rate: 0.61071; in: "ep.1"; variables (2): ppr, ep.1   
  
 hr.elapsed min.elapsed sec.elapsed   
 0.00 0.00 0.53

modelstepL

method : lda   
final model : eng ~ ppr + ep.1  
<environment: 0x55f515a63258>  
  
correctness rate = 0.6107

modelstepQ <- stepclass(eng ~ ., "qda", direction = "both", data = joint)

correctness rate: 0.57857; in: "ppr"; variables (1): ppr   
  
 hr.elapsed min.elapsed sec.elapsed   
 0.000 0.000 0.334

modelstepQ

method : qda   
final model : eng ~ ppr  
<environment: 0x55f517c1b1c0>  
  
correctness rate = 0.5786

If you run this several times, you will find that you get a slightly different model each time. The best models have 1 to 2 predictors, and vary in accuracy from 55 to 65%. This is happening because the cross-validation the method uses is somewhat random, so the best model will depend on how the cross-validation is initialized. Perhaps if we reduce the improvement required, and use a higher cross-validation value, we will end up at a more stable result. Using fold=76 should be similar to doing leave-one-out cross-validation, and using a smaller improvement criterion will avoid stopping early.

modelstepL <- stepclass(eng ~ ., "lda", direction = "both", data = joint, improvement = 0.001,   
 fold = 76)

correctness rate: 0.57895; in: "ppr"; variables (1): ppr   
correctness rate: 0.59211; in: "ppu"; variables (2): ppr, ppu   
correctness rate: 0.60526; in: "ppu.1"; variables (3): ppr, ppu, ppu.1   
  
 hr.elapsed min.elapsed sec.elapsed   
 0.00 0.00 4.75

modelstepL

method : lda   
final model : eng ~ ppr + ppu + ppu.1  
<environment: 0x55f515224408>  
  
correctness rate = 0.6053

modelstepQ <- stepclass(eng ~ ., "qda", direction = "both", data = joint, improvemnet = 0.001,   
 fold = 76)

correctness rate: 0.57895; in: "ppr"; variables (1): ppr   
correctness rate: 0.64474; in: "ep"; variables (2): ppr, ep   
  
 hr.elapsed min.elapsed sec.elapsed   
 0.00 0.00 3.21

modelstepQ

method : qda   
final model : eng ~ ep + ppr  
<environment: 0x55f515b60040>  
  
correctness rate = 0.6447

Now, the each model tends to converge on the same result each time. The variables selected are different for the two models, but that is probably fine. We can refit the best models using lda and qda to get more details about the fit:

l.final <- lda(eng ~ ppr + ppu + ppu.1, data = joint)  
l.final

Call:  
lda(eng ~ ppr + ppu + ppu.1, data = joint)  
  
Prior probabilities of groups:  
 0 1   
0.5 0.5   
  
Group means:  
 ppr ppu ppu.1  
0 0.5789474 0.6052632 3007.688  
1 0.6754386 0.7017544 3432.817  
  
Coefficients of linear discriminants:  
 LD1  
ppr 1.2417704238  
ppu 2.5194768800  
ppu.1 0.0004672492

q.final <- qda(eng ~ ep + ppr, data = joint)  
q.final

Call:  
qda(eng ~ ep + ppr, data = joint)  
  
Prior probabilities of groups:  
 0 1   
0.5 0.5   
  
Group means:  
 ep ppr  
0 0.6951754 0.5789474  
1 0.6973684 0.6754386

# Example: LDA on the iphone data set

The following work through all the steps of LDA and QDA again with the iphone data set. ## Data Preprocessing

phone\_ds <- read.csv("data\_study1.csv")

phone\_type <- phone\_ds[, c(1, 3:13)]

phone\_type[, 2:12] <- scale(phone\_type[, 2:12], center = TRUE, scale = TRUE)

## Loading library for LDA

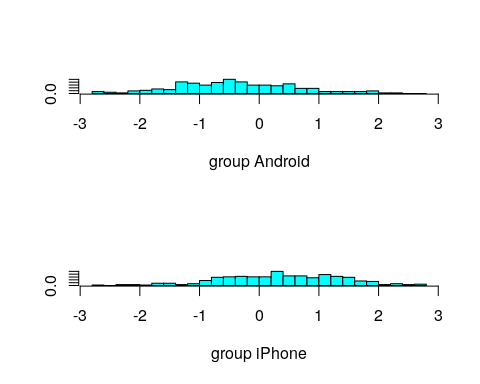
library(MASS)

## Compute LDA without Cross validation

lda\_mod1 <- lda(Smartphone ~ ., data = phone\_type)  
lda\_mod1

Call:  
lda(Smartphone ~ ., data = phone\_type)  
  
Prior probabilities of groups:  
 Android iPhone   
0.4139887 0.5860113   
  
Group means:  
 Age Honesty.Humility Emotionality Extraversion  
Android 0.2071126 0.2304728 -0.1885209 -0.08233383  
iPhone -0.1463150 -0.1628179 0.1331809 0.05816486  
 Agreeableness Conscientiousness Openness Avoidance.Similarity  
Android 0.04975669 -0.02859348 0.11922875 0.1678358  
iPhone -0.03515069 0.02019991 -0.08422934 -0.1185679  
 Phone.as.status.object Social.Economic.Status  
Android -0.2850676 -0.02423018  
iPhone 0.2013865 0.01711745  
 Time.owned.current.phone  
Android 0.06110396  
iPhone -0.04316699  
  
Coefficients of linear discriminants:  
 LD1  
Age -0.24833268  
Honesty.Humility -0.45905620  
Emotionality 0.34275770  
Extraversion 0.32566258  
Agreeableness 0.01021013  
Conscientiousness 0.26431245  
Openness -0.15490974  
Avoidance.Similarity -0.29824541  
Phone.as.status.object 0.38353264  
Social.Economic.Status -0.06839454  
Time.owned.current.phone -0.01885251

plot(lda\_mod1)



## Predict on phone\_type

library(DAAG)  
plda1 <- predict(object = lda\_mod1)  
confusion(phone\_type$Smartphone, plda1$class)

Overall accuracy = 0.673   
  
Confusion matrix   
 Predicted (cv)  
Actual Android iPhone  
 Android 0.484 0.516  
 iPhone 0.194 0.806

## Compute LDA with Cross-validation

lda\_mod2 <- lda(Smartphone ~ ., data = phone\_type, CV = TRUE)  
lda\_mod2

$class  
 [1] iPhone iPhone Android iPhone Android iPhone iPhone Android  
 [9] iPhone iPhone Android iPhone iPhone Android iPhone iPhone   
[17] Android iPhone Android Android Android Android iPhone iPhone   
[25] iPhone Android Android iPhone Android iPhone iPhone Android  
[33] iPhone iPhone iPhone Android iPhone iPhone iPhone iPhone   
[41] iPhone iPhone iPhone iPhone iPhone iPhone iPhone iPhone   
[49] iPhone Android iPhone iPhone iPhone Android iPhone iPhone   
[57] Android iPhone Android iPhone iPhone Android Android iPhone   
[65] Android iPhone iPhone iPhone Android iPhone Android Android  
[73] iPhone iPhone iPhone   
 [ reached getOption("max.print") -- omitted 454 entries ]  
Levels: Android iPhone  
  
$posterior  
 Android iPhone  
1 0.44727912 0.5527209  
2 0.44682280 0.5531772  
3 0.77836711 0.2216329  
4 0.36991321 0.6300868  
5 0.80731292 0.1926871  
6 0.28517134 0.7148287  
7 0.26967843 0.7303216  
8 0.62504334 0.3749567  
9 0.30966026 0.6903397  
10 0.38754803 0.6124520  
11 0.50156460 0.4984354  
12 0.29046062 0.7095394  
13 0.43235762 0.5676424  
14 0.69385166 0.3061483  
15 0.35384079 0.6461592  
16 0.32701994 0.6729801  
17 0.51765170 0.4823483  
18 0.26246323 0.7375368  
19 0.56630053 0.4336995  
20 0.74414226 0.2558577  
21 0.59475013 0.4052499  
22 0.58309407 0.4169059  
23 0.41664815 0.5833518  
24 0.24077709 0.7592229  
25 0.31182798 0.6881720  
26 0.73235151 0.2676485  
27 0.55432598 0.4456740  
28 0.41271310 0.5872869  
29 0.56705971 0.4329403  
30 0.22477284 0.7752272  
31 0.39035268 0.6096473  
32 0.53711910 0.4628809  
33 0.27895785 0.7210422  
34 0.18762229 0.8123777  
35 0.13193830 0.8680617  
36 0.74365126 0.2563487  
37 0.11854692 0.8814531  
 [ reached getOption("max.print") -- omitted 492 rows ]  
  
$terms  
Smartphone ~ Age + Honesty.Humility + Emotionality + Extraversion +   
 Agreeableness + Conscientiousness + Openness + Avoidance.Similarity +   
 Phone.as.status.object + Social.Economic.Status + Time.owned.current.phone  
attr(,"variables")  
list(Smartphone, Age, Honesty.Humility, Emotionality, Extraversion,   
 Agreeableness, Conscientiousness, Openness, Avoidance.Similarity,   
 Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone)  
attr(,"factors")  
 Age Honesty.Humility Emotionality Extraversion  
Smartphone 0 0 0 0  
Age 1 0 0 0  
Honesty.Humility 0 1 0 0  
Emotionality 0 0 1 0  
Extraversion 0 0 0 1  
Agreeableness 0 0 0 0  
 Agreeableness Conscientiousness Openness  
Smartphone 0 0 0  
Age 0 0 0  
Honesty.Humility 0 0 0  
Emotionality 0 0 0  
Extraversion 0 0 0  
Agreeableness 1 0 0  
 Avoidance.Similarity Phone.as.status.object  
Smartphone 0 0  
Age 0 0  
Honesty.Humility 0 0  
Emotionality 0 0  
Extraversion 0 0  
Agreeableness 0 0  
 Social.Economic.Status Time.owned.current.phone  
Smartphone 0 0  
Age 0 0  
Honesty.Humility 0 0  
Emotionality 0 0  
Extraversion 0 0  
Agreeableness 0 0  
 [ reached getOption("max.print") -- omitted 6 rows ]  
attr(,"term.labels")  
 [1] "Age" "Honesty.Humility"   
 [3] "Emotionality" "Extraversion"   
 [5] "Agreeableness" "Conscientiousness"   
 [7] "Openness" "Avoidance.Similarity"   
 [9] "Phone.as.status.object" "Social.Economic.Status"   
[11] "Time.owned.current.phone"  
attr(,"order")  
 [1] 1 1 1 1 1 1 1 1 1 1 1  
attr(,"intercept")  
[1] 1  
attr(,"response")  
[1] 1  
attr(,".Environment")  
<environment: R\_GlobalEnv>  
attr(,"predvars")  
list(Smartphone, Age, Honesty.Humility, Emotionality, Extraversion,   
 Agreeableness, Conscientiousness, Openness, Avoidance.Similarity,   
 Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone)  
attr(,"dataClasses")  
 Smartphone Age Honesty.Humility   
 "factor" "numeric" "numeric"   
 Emotionality Extraversion Agreeableness   
 "numeric" "numeric" "numeric"   
 Conscientiousness Openness Avoidance.Similarity   
 "numeric" "numeric" "numeric"   
 Phone.as.status.object Social.Economic.Status Time.owned.current.phone   
 "numeric" "numeric" "numeric"   
  
$call  
lda(formula = Smartphone ~ ., data = phone\_type, CV = TRUE)  
  
$xlevels  
named list()

## confusion() on lda\_mod2

confusion(phone\_type$Smartphone, lda\_mod2$class)

Overall accuracy = 0.648   
  
Confusion matrix   
 Predicted (cv)  
Actual Android iPhone  
 Android 0.466 0.534  
 iPhone 0.223 0.777

We can see that some of the accuracy comes from overfitting.

## Compute QDA without CV

qda\_mod1 <- qda(Smartphone ~ ., data = phone\_type)  
qda\_mod1

Call:  
qda(Smartphone ~ ., data = phone\_type)  
  
Prior probabilities of groups:  
 Android iPhone   
0.4139887 0.5860113   
  
Group means:  
 Age Honesty.Humility Emotionality Extraversion  
Android 0.2071126 0.2304728 -0.1885209 -0.08233383  
iPhone -0.1463150 -0.1628179 0.1331809 0.05816486  
 Agreeableness Conscientiousness Openness Avoidance.Similarity  
Android 0.04975669 -0.02859348 0.11922875 0.1678358  
iPhone -0.03515069 0.02019991 -0.08422934 -0.1185679  
 Phone.as.status.object Social.Economic.Status  
Android -0.2850676 -0.02423018  
iPhone 0.2013865 0.01711745  
 Time.owned.current.phone  
Android 0.06110396  
iPhone -0.04316699

qlda1 <- predict(object = qda\_mod1)  
confusion(phone\_type$Smartphone, qlda1$class)

Overall accuracy = 0.682   
  
Confusion matrix   
 Predicted (cv)  
Actual Android iPhone  
 Android 0.53 0.47  
 iPhone 0.21 0.79

## Compute QDA with LOOC

qda\_mod2 <- qda(Smartphone ~ ., data = phone\_type, CV = TRUE)  
qda\_mod2

$class  
 [1] Android iPhone Android iPhone Android iPhone iPhone Android  
 [9] iPhone iPhone iPhone Android iPhone Android iPhone iPhone   
[17] iPhone Android Android Android Android iPhone iPhone iPhone   
[25] iPhone Android Android iPhone Android iPhone iPhone iPhone   
[33] iPhone iPhone iPhone Android iPhone iPhone iPhone iPhone   
[41] iPhone Android iPhone iPhone iPhone iPhone iPhone iPhone   
[49] iPhone iPhone iPhone Android iPhone Android iPhone iPhone   
[57] Android Android Android iPhone iPhone iPhone Android iPhone   
[65] iPhone Android iPhone iPhone Android Android Android Android  
[73] iPhone Android iPhone   
 [ reached getOption("max.print") -- omitted 454 entries ]  
Levels: Android iPhone  
  
$posterior  
 Android iPhone  
1 0.550069334 4.499307e-01  
2 0.459988979 5.400110e-01  
3 0.758731421 2.412686e-01  
4 0.373753872 6.262461e-01  
5 0.509395253 4.906047e-01  
6 0.223179114 7.768209e-01  
7 0.046547074 9.534529e-01  
8 0.629604274 3.703957e-01  
9 0.155903266 8.440967e-01  
10 0.404783055 5.952169e-01  
11 0.390001107 6.099989e-01  
12 0.527163938 4.728361e-01  
13 0.277148516 7.228515e-01  
14 0.668604563 3.313954e-01  
15 0.352077947 6.479221e-01  
16 0.241226583 7.587734e-01  
17 0.442156078 5.578439e-01  
18 0.712822007 2.871780e-01  
19 0.629898021 3.701020e-01  
20 0.653601501 3.463985e-01  
21 0.537587845 4.624122e-01  
22 0.365360338 6.346397e-01  
23 0.486570165 5.134298e-01  
24 0.053057264 9.469427e-01  
25 0.223667837 7.763322e-01  
26 0.791062415 2.089376e-01  
27 0.682312320 3.176877e-01  
28 0.338804289 6.611957e-01  
29 0.541435101 4.585649e-01  
30 0.159967108 8.400329e-01  
31 0.283987762 7.160122e-01  
32 0.251433765 7.485662e-01  
33 0.188424179 8.115758e-01  
34 0.203354196 7.966458e-01  
35 0.149893921 8.501061e-01  
36 0.504465263 4.955347e-01  
37 0.057381722 9.426183e-01  
 [ reached getOption("max.print") -- omitted 492 rows ]  
  
$terms  
Smartphone ~ Age + Honesty.Humility + Emotionality + Extraversion +   
 Agreeableness + Conscientiousness + Openness + Avoidance.Similarity +   
 Phone.as.status.object + Social.Economic.Status + Time.owned.current.phone  
attr(,"variables")  
list(Smartphone, Age, Honesty.Humility, Emotionality, Extraversion,   
 Agreeableness, Conscientiousness, Openness, Avoidance.Similarity,   
 Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone)  
attr(,"factors")  
 Age Honesty.Humility Emotionality Extraversion  
Smartphone 0 0 0 0  
Age 1 0 0 0  
Honesty.Humility 0 1 0 0  
Emotionality 0 0 1 0  
Extraversion 0 0 0 1  
Agreeableness 0 0 0 0  
 Agreeableness Conscientiousness Openness  
Smartphone 0 0 0  
Age 0 0 0  
Honesty.Humility 0 0 0  
Emotionality 0 0 0  
Extraversion 0 0 0  
Agreeableness 1 0 0  
 Avoidance.Similarity Phone.as.status.object  
Smartphone 0 0  
Age 0 0  
Honesty.Humility 0 0  
Emotionality 0 0  
Extraversion 0 0  
Agreeableness 0 0  
 Social.Economic.Status Time.owned.current.phone  
Smartphone 0 0  
Age 0 0  
Honesty.Humility 0 0  
Emotionality 0 0  
Extraversion 0 0  
Agreeableness 0 0  
 [ reached getOption("max.print") -- omitted 6 rows ]  
attr(,"term.labels")  
 [1] "Age" "Honesty.Humility"   
 [3] "Emotionality" "Extraversion"   
 [5] "Agreeableness" "Conscientiousness"   
 [7] "Openness" "Avoidance.Similarity"   
 [9] "Phone.as.status.object" "Social.Economic.Status"   
[11] "Time.owned.current.phone"  
attr(,"order")  
 [1] 1 1 1 1 1 1 1 1 1 1 1  
attr(,"intercept")  
[1] 1  
attr(,"response")  
[1] 1  
attr(,".Environment")  
<environment: R\_GlobalEnv>  
attr(,"predvars")  
list(Smartphone, Age, Honesty.Humility, Emotionality, Extraversion,   
 Agreeableness, Conscientiousness, Openness, Avoidance.Similarity,   
 Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone)  
attr(,"dataClasses")  
 Smartphone Age Honesty.Humility   
 "factor" "numeric" "numeric"   
 Emotionality Extraversion Agreeableness   
 "numeric" "numeric" "numeric"   
 Conscientiousness Openness Avoidance.Similarity   
 "numeric" "numeric" "numeric"   
 Phone.as.status.object Social.Economic.Status Time.owned.current.phone   
 "numeric" "numeric" "numeric"   
  
$call  
qda(formula = Smartphone ~ ., data = phone\_type, CV = TRUE)  
  
$xlevels  
named list()

## confusion() on qda\_mod2

confusion(phone\_type$Smartphone, qda\_mod2$class)

Overall accuracy = 0.594   
  
Confusion matrix   
 Predicted (cv)  
Actual Android iPhone  
 Android 0.420 0.580  
 iPhone 0.284 0.716

## Split-half Cross Validation

This function will compute an lda and qda model, randomly picking half each time. The model fits on 1/2 of the data, and then predicts the other half.

cv.lda <- function(class, predictors) {  
 selection <- rep(c(T, F), length.out = length(class))[order(runif(length(class)))]  
 out1 <- class[selection]  
 pred1 <- predictors[selection, ]  
 joint1 <- data.frame(out = out1, pred1)  
   
 out2 <- class[!selection]  
 pred2 <- predictors[!selection, ]  
 joint2 <- data.frame(out = NA, pred2)  
   
 ll <- lda(out ~ ., data = joint1)  
 table(predict(ll, newdata = joint2)$class, out2)  
 out.ll <- sum(diag(table(predict(ll, newdata = joint2)$class, out2)))/length(out2)  
 qq <- qda(out ~ ., data = joint1)  
 table(predict(qq, newdata = joint2)$class, out2)  
 out.qq <- sum(diag(table(predict(qq, newdata = joint2)$class, out2)))/length(out2)  
   
 c(out.ll, out.qq)  
}  
  
cv.lda(phone\_type$Smartphone, phone\_type[, 2:12])

[1] 0.6287879 0.5833333

There is no advantage for the qda here, and the overall split-half cross-validation numbers are only 64%.

## Step from klaR

We can automate variable selection with klaR

library(klaR)  
model <- stepclass(Smartphone ~ ., data = phone\_type, method = "lda", fold = 2,   
 start.vars = 1:11, direction = "both", output = T)

correctness rate: 0.65216; starting variables (11): Age, Honesty.Humility, Emotionality, Extraversion, Agreeableness, Conscientiousness, Openness, Avoidance.Similarity, Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone   
correctness rate: 0.6616; out: "Age"; variables (10): Honesty.Humility, Emotionality, Extraversion, Agreeableness, Conscientiousness, Openness, Avoidance.Similarity, Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone   
correctness rate: 0.66537; out: "Openness"; variables (9): Honesty.Humility, Emotionality, Extraversion, Agreeableness, Conscientiousness, Avoidance.Similarity, Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone   
correctness rate: 0.66538; out: "Agreeableness"; variables (8): Honesty.Humility, Emotionality, Extraversion, Conscientiousness, Avoidance.Similarity, Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone   
  
 hr.elapsed min.elapsed sec.elapsed   
 0.000 0.000 0.891

modell2 <- lda(model$formula, data = phone\_type)  
  
confusion(phone\_type$Smartphone, predict(modell2)$class)

Overall accuracy = 0.671   
  
Confusion matrix   
 Predicted (cv)  
Actual Android iPhone  
 Android 0.484 0.516  
 iPhone 0.197 0.803

modelq <- stepclass(Smartphone ~ ., data = phone\_type, method = "qda", fold = 2,   
 start.vars = 1:11, direction = "both", output = T)

correctness rate: 0.59546; starting variables (11): Age, Honesty.Humility, Emotionality, Extraversion, Agreeableness, Conscientiousness, Openness, Avoidance.Similarity, Phone.as.status.object, Social.Economic.Status, Time.owned.current.phone   
correctness rate: 0.61246; out: "Social.Economic.Status"; variables (10): Age, Honesty.Humility, Emotionality, Extraversion, Agreeableness, Conscientiousness, Openness, Avoidance.Similarity, Phone.as.status.object, Time.owned.current.phone   
correctness rate: 0.62194; out: "Honesty.Humility"; variables (9): Age, Emotionality, Extraversion, Agreeableness, Conscientiousness, Openness, Avoidance.Similarity, Phone.as.status.object, Time.owned.current.phone   
correctness rate: 0.62382; out: "Conscientiousness"; variables (8): Age, Emotionality, Extraversion, Agreeableness, Openness, Avoidance.Similarity, Phone.as.status.object, Time.owned.current.phone   
correctness rate: 0.63136; out: "Age"; variables (7): Emotionality, Extraversion, Agreeableness, Openness, Avoidance.Similarity, Phone.as.status.object, Time.owned.current.phone   
  
 hr.elapsed min.elapsed sec.elapsed   
 0.000 0.000 1.028

modelq2 <- qda(modelq$formula, data = phone\_type)  
  
confusion(phone\_type$Smartphone, predict(modelq2)$class)

Overall accuracy = 0.665   
  
Confusion matrix   
 Predicted (cv)  
Actual Android iPhone  
 Android 0.457 0.543  
 iPhone 0.187 0.813

This appears to improve things; by fitting a smaller model we actually do better.

# Applications of LDA

Although the performance of LDA can often be surpassed by more modern machine learning methods, there are several reasons it still sees widespread use.

* It is simple to use and understand. Like logistic regression, it can be used to make a simple model or decision tool that is both easy to implement and transparent.
* It is sufficient for many situations. Many times, the benefit you might get from using a more complex model is negligible, at the cost of complexity or (worse yet) the possibility of making large mistakes because of strange interactions that you might not be able to predict.

Some of the most widely-used LDA models are within finance. For example, Altman’s (1968) bankruptcy model is based on LDA, predicting bankruptcy of firms within the next two years based on a handful of publicly-available statistics (see Altman, 1968, Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. The Journal of Finance, 23(4), 589-609.) This is nice because the model can be implemented in a spreadsheet and decisions can be made by individuals evaluating stock purchases.

# Alternatives and extensions in Machine Classification

There are hundreds of special-purpose methods available for machine classification, many of which are developed for special kinds of situations or that work under different assumptions. We will cover several of these in this class, and here is a partial listing of methods you might want to be familiar with:

Within the klaR library, there are several implementations of related methosd

* rda: Regularized discriminant Analysis. Attempts to build a discriminat model that is more robust to correlation between predictors (multi-colinearity)
* Probabilistic LDA. This frames the LDA problem in a Bayesian and/or maximum likelihood format, and is increasingly used as part of deep neural nets as a ‘fair’ final decision that does not hide complexity.
* loclda: Makes a local lda for each point, based on its nearby neighbors.
* sknn: simple k-nearest-neighbors classification. Makes classificition based on a vote of the nearest observations
* NaiveBayes: A common and simple classifier based on bayes rule
* svmlight: a lightweight ‘support vector machine’, which generalizes lda, focusing especially on identifying a good decision rule that separates the two groups

The klaR library also has a lot of functions to help with variable selection and cross-validation.

Within the nnet library:

* nnet: a neural net classifier–essentially a network of LDA classifiers or logistic regressions.
* multinom: an extension of generalized linear regression for multiple groups