Consider the physical situation involving two capacitors connected in series as is illustrated schematically below, where both the switches start open and the capacitors are initially uncharged. It is assumed that there is no resistance of any kind.

Now cycle the switches through the following sequence:

<table>
<thead>
<tr>
<th>Step</th>
<th>Switch S1</th>
<th>Switch S2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>closed</td>
<td>open</td>
</tr>
<tr>
<td>2.</td>
<td>open</td>
<td>open</td>
</tr>
<tr>
<td>3.</td>
<td>open</td>
<td>closed</td>
</tr>
<tr>
<td>4.</td>
<td>open</td>
<td>open</td>
</tr>
</tbody>
</table>

Go back to step 1 and repeat indefinitely.

Each step along the way is held long enough so that electrostatic equilibrium is achieved. What are the potentials $V_1$ and $V_2$ for each step?
To solve this problem correctly you need to recognize there are two equipotential “bodies” plus a conducting ground. One body is the top plate of $C_1$ and the other body is the combination of the bottom plate of $C_1$ and the top plate of $C_2$. Hence, qualitatively, it looks like this:

![Diagram](image)

and it is clear there is also some capacitance from the top plate of $C_1$ (i.e. object 1) to ground. To model that, a third capacitance, $C_3$, is introduced in the schematic diagram. That third capacitance may be small as capacitances go and it will depend on the surroundings. Whether or not it is negligible will, of course, depend on the other values. The circuit to analyze looks like:

![Diagram](image)

Now, denote

- $Q_1 = \text{total net charge on the top plates of } C_1 \text{ and } C_3$ (i.e. the net charge on object 1).
- $Q_2 = \text{total net charge on the bottom plate of } C_1 \text{ and top plate of } C_2$ (i.e. the net charge on object 2). These are not the usual charges one sees used for capacitor problems, but combinations of them. The electric potentials are relative to ground.

The superposition principle will be used to relate the charges and potentials. If $V_1$ is forced to be zero (e.g. by a connection to ground) and $V_2$ is not zero, then

$$Q_2 = (C_1 + C_2) V_2$$
$$Q_1 = -C_1 V_2$$

and if $V_2$ is forced to zero while $V_1$ is not zero, then

$$Q_2 = -C_1 V_1$$

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1 This solution is inspired by the discussion on pages 99 to 101 of “Electricity and Magnetism, Berkeley Physics Course Volume II,” E. M. Purcell, (McGraw-Hill, 1965).
\[ Q_1 = (C_1 + C_3) V_1. \]

For the general case, you add these together. In matrix form, the result is
\[ \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} (C_1 + C_3) - C_1 \\ -C_1 (C_1 + C_2) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = A \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}. \]

For this problem it is useful to rearrange the equations to get them in the form
\[ \begin{pmatrix} Q_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1 \\ C_1 + C_2 \end{pmatrix} \begin{pmatrix} -C_1 \\ (C_1 C_2 + C_1 C_3 + C_2 C_3) \end{pmatrix} \begin{pmatrix} Q_2 \\ V_1 \end{pmatrix} = B \begin{pmatrix} Q_2 \\ V_1 \end{pmatrix}. \]

Thus, at step 1 (the first time) we have \( V_1 = V_r \) and \( Q_2 = 0 \), and so \( Q_1 \) and \( V_2 \) are found from
\[ \begin{pmatrix} Q_1 \\ V_2 \end{pmatrix} = B \begin{pmatrix} 0 \\ V_r \end{pmatrix}. \]

At step 2, the charges are conserved and the new potentials are computed from
\[ \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = A^{-1} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}. \]

At step 3 the potential \( V_2 \) is forced to zero while the charge \( Q_1 \) is left alone.

In matrix notation, that can be accomplished using \( C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and so at step 3 one has
\[ \begin{pmatrix} Q_2 \\ V_1 \end{pmatrix} = B^{-1} C B \begin{pmatrix} 0 \\ V_r \end{pmatrix}. \]

In step 4 the charges are again left alone and the potentials are computed as for step 2. For the repeat of step 1, the charge \( Q_2 \) is left alone and the potential \( V_1 \) is forced to be \( V_r \). Hence, \( \begin{pmatrix} Q_1 \\ V_2 \end{pmatrix} = B \left[ C B^{-1} C B + 1 \right] \begin{pmatrix} 0 \\ V_r \end{pmatrix} \), and so on.

Since the result depends on \( C_3 \), which is unspecified in the initial problem, you cannot answer the problem in terms of what was given. Depending on the situation, one might be able to make some assumptions about \( C_3 \) (i.e. that it is very small compared to the others) to get a working solution. There are many capacitor problems which include such unspoken assumptions as part of their “solution.”