1. Reversible processes for ideal gases in closed system (section 2.7)

a) Isothermal Expansion and Compression ($\Delta u = 0$ because $u = u(T)$ for ideal gas)

- Path in $P$ vs $v$ diagram
  i. From Ideal gas law, for isothermal process, $Pv = RT$ = constant
  ii. It is a polytropic process (i.e. $Pv^\gamma = \text{constant}$, with $\gamma = 1$)

- Reversible work of isothermal process
  
  \[
  w_{by,1 \rightarrow 2} = \int P \, dv = RT \int \frac{1}{v} \, dv = RT \ln \frac{v_2}{v_1}
  \]

  or
  
  \[
  w_{by,1 \rightarrow 2} = RT \ln \frac{P_1}{P_2}
  \]

- Heat in: $q_{in} = w_{by,1 \rightarrow 2} = RT \ln(P_1/P_2)$

b) Adiabatic Expansion and Compression

For ideal gases, $\, dv = d \left( \frac{RT}{P} \right) = R \left( \frac{1}{P} \, dT - \frac{T}{P^2} \, dP \right)$

- Path in $P$ vs $v$ diagram
  i. Adiabatic $\Rightarrow q_{in} = 0$, thus

  \[
  (du = c_v dT) = -\delta w_{by} = -P \, dv
  \]

  \[
  c_v dT = -R \left( dT - \frac{T}{P} \, dP \right)
  \]

  Rearranging,

  \[
  \frac{c_v + R}{R} \, \frac{dT}{T} = \frac{dP}{P}
  \]
Assume $c_v$ constant (not true for all ideal gases), then integrating,

$$
\left( \frac{c_v + R}{R} \right) \ln \frac{T_2}{T_1} = \ln \frac{P_2}{P_1} \quad \rightarrow \quad \left( \frac{T_2}{T_1} \right)^{\left( \frac{c_v + R}{R} \right)} = \left( \frac{P_2}{P_1} \right).
$$

Since $c_p = c_v + R$, we have $(c_v + R)/R = c_p'/(c_p - c_v) = k/(k - 1)$, where $k = c_p/c_v$. Thus,

$$
\left( \frac{T_2}{T_1} \right)^{k/(k-1)} = \left( \frac{P_2}{P_1} \right).
$$

To obtain $P$ vs $v$ relationship, use ideal gas law to replace $T$,

$$
\left( \frac{P_2 v_2}{P_1 v_1} \right)^{k/(k-1)} = \left( \frac{P_2}{P_1} \right) \quad \rightarrow \quad \left( \frac{P_2}{P_1} \right)^{1/(k-1)} \left( \frac{v_2}{v_1} \right)^{k/(k-1)} = 1
$$

or

$$
P_1 v_1^k = P_2 v_2^k \quad \rightarrow \quad P v^k = \text{constant}
$$

i.e. another polytropic path, $P v^\gamma = \text{constant}$, with $\gamma = k = c_p/c_v$.

Remarks:

a) Same trend as isothermal but steeper because $k > 1$

b) True if $c_v$ (or $c_p$) is constant (i.e. not a function of $T$), and true for ideal gases

c) $c_p$ for monatomic ideal gas is approximately $(3R/2)$
• Work done by ideal gas under adiabatic expansion (with $c_p$ constant)

Then using $Pv^k = \text{constant} = P_i v_i^k = P_f v_f^k$

$$w_{by} = \int P \, dv = \int \frac{\text{constant}}{v^k} \, dv = \frac{1}{1-k} \left( \frac{P_f v_f^k}{v_f^{k-1}} - \frac{P_i v_i^k}{v_i^{k-1}} \right) = \frac{1}{1-k} (P_f v_f - P_i v_i)$$

Or in terms of $T$,

$$w_{by} = \frac{R}{k-1} (T_i - T_f) = c_v (T_i - T_f) = -\Delta u$$

2. Open-system balances on process equipment (pp. 95-102)

a) Nozzles and diffusers

• Main function is to change velocities (i.e. $\Delta E_k \neq 0$)

• $Q_{in} = 0, W_{by,shaft} = 0, \Delta E_p \approx 0$

• $h_{out} - h_{in} = e_{k,in} - e_{k,out}$

b) Turbines and pumps

(check out video on steam turbines [https://www.youtube.com/watch?v=MuWTBx3szc]

• Main function is to convert energy to work (turbines) or convert work to increase energy (pumps and compressors)

• $Q_{in} = 0, W_{by,shaft} \neq 0, \Delta E_p \approx 0$

• $(h_{out} - h_{in}) + (e_{k,out} - e_{k,in}) = -w_{by,shaft}$

c) Heat exchangers

• Main function is to transfer energy to change the temperature of feed

• $Q_{in} \neq 0, W_{by,shaft} = 0, \Delta E_p \approx 0, \Delta E_k \approx 0$

• $(h_{out} - h_{in}) = q_{in}$

d) Throttling devices

• Main function is to reduce pressure of feed

• $Q_{in} = 0, W_{by,shaft} = 0, \Delta E_p \approx 0, \Delta E_k \approx 0$

• $h_{out} = h_{in}$ (also known as “isenthalpic process”)
Example: Throttle Calorimeter

Purpose: To estimate steam quality of main steam line

Available measurements: \( P_{\text{main}}, T_{\text{cal}}, P_{\text{atm}} \)

Data from steam tables:
- \( h_{\text{vap,main}}(P_{\text{main}}), h_{\text{liq,main}}(P_{\text{main}}), h_{\text{vap,sat}}(P_{\text{atm}}), T_{\text{sat}}(P_{\text{atm}}), c_{p,\text{vap}} \)

Energy balance:

\[
x \hat{h}_{\text{vap,main}} + (1 - x) \hat{h}_{\text{liq,main}} = \hat{h}_{\text{vap,sat}} + c_{p,\text{vap}}(T_{\text{cal}} - T_{\text{sat}})
\]

Remarks:
1. Care should be taken to not siphon condensed liquid in main line
2. Calculation might be improved by using steam table (functions) instead of \( c_p \).

Assignment 2:

2.19, 2.24, 2.25, 2.30, 2.41, 2.51