1. (20 pts) In terms of design parameters $\alpha$, $\beta$ and $V$, a second order system is modeled by the following equation

$$\frac{d^2 C_a}{dt^2} + 2\alpha \frac{dC_a}{dt} + VC_a = \beta$$

Actual data from a run starting at $C_a(0) = 5$ and $dC_a/dt = 0$ is shown in Figure 1. Obtain the values of $\alpha$, $\beta$ and $V$ using the data.

![Figure 1: Second order system response.](image)

2. (10 pts) Obtain a first order differential equation which would yield the data shown in Figure 2.

3. (10 pts) The pressure of a vessel can be described by

$$8 \frac{d^2 P}{dt^2} + 2 \frac{dP}{dt} + 2P = 6$$

What is the damping coefficient of the system? Is it underdamped or overdamped?

4. Consider a tank whose cross-sectional area depends on height, $A = A(h)$. The differential volume at $h(t)$ is then given by $dV = A(h)dh$
Figure 2: First order system response.

(a) (10 pts) Show that the process model for liquid height in the tank shown in Figure 3 is given by

\[
\frac{dh}{dt} = \frac{F_{\text{in}} - k\sqrt{h}}{A(h)}
\]

where \(F_{\text{in}}\) is the volumetric flow of liquid into the tank and the flow out of the tank is given by \(k\sqrt{h}\).

Figure 3: A tank where cross-sectional area depends on height.

(b) (5 pts) Derive a process model which describes the rate of change in height for a paraboloid tank shown in Figure 4 whose radius \(r\) changes with \(h\) according to \(r = 2h^2\)

5. The temperature in a reactor is described by the following nonlinear equation:

\[
20\frac{dT}{dt} = [0.01T^2 - T + 20] - [0.5T - 30]
\]
Figure 4: Spherical tank system.

(a) (10 pts) What are the steady state temperatures of this system?
(b) (10 pts) Obtain a linearized equation for the system around the temperature \( T = 100^\circ F \).

6. (15 pts) A two stirred tank system is shown in Figure 5 where a fraction \( \alpha \) of the flow out of tank 2 is recycled into tank 1. Assuming that the liquid volumes in both tanks are kept constant, the temperature dynamics for both tanks are described by

\[
\frac{dT_1}{dt} = \frac{F_o}{V_1} \left[ (T_{in} - T_1) + \frac{\alpha}{1-\alpha} (T_2 - T_1) \right] \\
\frac{dT_2}{dt} = \frac{F_o}{V_2} \frac{1}{1-\alpha} (T_1 - T_2)
\]

Let \( F_o = 5 \), \( V_1 = 10 \), \( V_2 = 5 \), \( \alpha = 0.3 \), obtain the characteristic equation and the eigenvalues for the system.

Figure 5: Two tank system with recycle.

7. (10 pts) For a temperature control system, the process is described by

\[
\frac{dT'}{dt} = \frac{1}{\tau} (T' + u')
\]
where $T'$ is the deviation temperature and $u$ is deviation manipulated variable. Show that using the proportional control law

$$u' = K_c(T'_{set} - T')$$

will yield a steady state offset: $T'_{ss} - T'_{set}$ that is not zero, for $K_c < \infty$, where $T'_{ss}$ is the steady state temperature.

8. (Bonus: 5 pts) Describe briefly how the pressure regulator shown in Figure 6 is self-regulating for $P_2$, assuming $P_2 < P_1$ always.

![Figure 6: A pressure regulator system.](image)