1. (20 pts) The response of the production of A in a reactor is known to follow a first order system with a time constant of 10 hours. Data obtained for this process is shown in Figure 1 where the initial concentration is given by \( C_a(0) = 5 \text{ g/l} \). What will be the steady state concentration of \( C_a \) for this first order system?

![Figure 1. Response of a first order system.](image)

2. Given a second order system described by the following,

\[
\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + x = u
\]

Use a proportional feedback control,

\[
u = k_c (x_{set} - x)
\]

a) (20 pts) With the proportional control in place, what is the steady state as a function of \( k_c \) and \( x_{set} \)?
b) (20 pts) With the proportional control in place, determine the damping coefficient, \( \zeta \), and the natural period, \( \tau_n \), as a function of \( k_c \).

3. (20 pts) The following equations describe the concentration and temperature changes in a reactor:

\[
\frac{dC}{dt} = \alpha (C_{in} - C) - k \exp\left(\frac{\beta}{T}\right) C
\]

\[
\frac{dT}{dt} = \alpha (T_{in} - T) + \gamma \exp\left(\frac{\beta}{T}\right) C
\]
Using the following set of parameters:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.01</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>600</td>
</tr>
</tbody>
</table>

linearize both equations with respect to $C$, $C_{in}$, $T$, $T_{in}$ at the following operating conditions:

<p>| | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$C$</td>
<td>0.60</td>
</tr>
<tr>
<td>$T$</td>
<td>680</td>
</tr>
<tr>
<td>$C_{in}$</td>
<td>0.9</td>
</tr>
<tr>
<td>$T_{in}$</td>
<td>500</td>
</tr>
</tbody>
</table>

4. (20 pts) Determine the minimum value of $K$ such that the system described by the equations below is stable.

\[
\frac{dx}{dt} = (4 - K)y - x
\]

\[
\frac{dy}{dt} = 2(x - y)
\]

(Bonus: 5 pts) Explain how the system shown in Figure 2 is self-regulating for the liquid level in the tank.

![Figure 2. Level Regulating System](image-url)