Lecture 20. Bode Plots For PID Tuning

1. Recall

\[ LM(H) = 20 \log (|H(i\omega)|) \]
\[ \phi = \tan^{-1} \frac{\text{Im}(H(i\omega))}{\text{Re}(H(i\omega))} \]

Thus, the critical point (-1,0) of the Nyquist plot of \( H(s) \) occurs when
\[ LM(H) = 20 \log(|-1|) = 0 \text{ dB} \]
\[ \phi = \tan^{-1} \left( \frac{0}{-1} \right) = -180^\circ \]

2. Crossover frequencies: (see page 231-232)

a) The crossover frequency, \( \omega_c \), is the frequency where phase shift, \( \phi \), is equal to \(-180^\circ\).
b) The phase-margin frequency, \( \omega_{pm} \), is the frequency where the amplitude ratio is 1, or when log modulus is equal to 0 dB.

3. Bode Stability Criterion (based on Nyquist Criterion)

If the log modulus of \( H(s) \) at \( \omega = \omega_c \) is less than 0 dB, then the feedback system is stable.

4. Stability Margins

a) Gain Margin
   - Determine the log modulus corresponding at \( \omega = \omega_c \), i.e. \( LM_c \).
   - The gain margin can then be evaluated as
   \[ GM = 10 \left( -\frac{LM_c}{20} \right) \]

b) Phase Margin
   - Simply measure the number of degrees above \(-180^\circ\) for the phase shift \( \phi \) at \( \omega = \omega_{pm} \).
Example 1:

The crossover frequency is at $\omega_c = 1.05$ rad/sec. At this frequency the log modulus is above 0 dB. Thus the feedback process will be unstable. The phase-margin frequency is $\omega_{pm} = 1.6$ rad/sec.
Example 2:

The phase crossover frequency is at 1.43 rad/sec, while the gain crossover frequency is at 1.06 rad/sec. The system is closed loop stable with the following stability margins:

\[ PM = 38^\circ \quad \text{and} \quad GM = \frac{17}{20} = 0.85 \]
Bode Reshaping via PID Control (Ziegler-Nichols Design):

Suppose the process is described by the following transfer function:

\[
G_p(s) := \frac{3}{4 \cdot s^2 + 0.5 \cdot s + 1} \cdot (3 \cdot s + 1)
\]

Using Ziegler-Nichols method, we find that the ultimate gain is given by

\[
\text{Ultimate gain: } K_u = 10^{-14/20} = 0.2
\]
Ultimate period: \( Pu = \frac{2\pi}{0.541} = 11.6 \text{ sec} \)

Based on the Ziegler-Nichols tuning of parameters, the resulting PID is given by

\[
G_c(s) = K_c \left( \frac{\tau_I s + 1}{\tau_I s} \right) \left( \frac{\tau_d s + 1}{\alpha \tau_d s + 1} \right)
\]

where \( K_c = 0.118 \), \( \tau_I = 5.82 \) and \( \tau_d = 1.454 \), with \( \alpha = 0.01 \). The Bode plots of this PID is given in Figure 2.

Upon connecting \( G_c \) in series with \( G_p \), the resulting Bode plots of \( G_cG_p \) are shown in Figure 3.
Note that the result of implementing Ziegler-Nichols yields a gain margin=2 and phase margin approximately $45^\circ$. 

Figure 3.