Short Tutorial on Matlab
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Part 6. Finding Steady States and Linearization via Simulink

1. Consider the biochemical system,


\[
\begin{align*}
\frac{dx_1}{dt} &= (\mu - D)x_1 \\
\frac{dx_2}{dt} &= D(x_{2f} - x_2) - \frac{\mu x_1}{Y} \\
\mu &= \frac{\mu_{\text{max}} x_2}{k_m + x_2 + k_1 x_2^2}
\end{align*}
\]

2. Build a Simulink model

```matlab
function dx = bioreactor(t,x,D,x2f)
%
% model for bioreactor process
% where,  x1 = biomass concentration
%         x2 = substrate concentration
%         D = dilution rate
%         x2f = substrate feed
%
% x1 = x(1)                           ;
% x2 = x(2)                           ;

% mumax   = 0.53          ;   % hr^(-1)
% km      = 0.12          ;   % g/liter
% k1      = 0.454         ;   % liter/g
% Y       = 0.4           ;   %

% mu  = mumax*x2/(km+x2+k1*x2^2)      ;
% dx1 = (mu - D)*x1                   ;
% dx2 = D*(x2f-x2)-mu*x1/Y            ;

dx = [dx1;dx2]                      ;
```

Figure 1. m-File for Bioprocess System
function [sys,x0,str,ts]= ...  
   bioreactor_sfcn(t,x,u,flag, ...  
   x1_init,x2_init)

switch flag
    case 0  % initialize
      str=[]                     ;
      ts = [0 0]                 ;
      s = size(sizes)            ;
      s.NumContStates  = 2   ;
      s.NumDiscStates  = 0   ;
      s.NumOutputs     = 2   ;
      s.NumInputs      = 2   ;
      s.DirFeedthrough = 0   ;
      s.NumSampleTimes = 1   ;
      sys = size(sizes(s))      ;
      x0 = [x1_init, x2_init]    ;
    case 1  % derivatives
      D   = u(1)             ;
      x2f = u(2)             ;
      sys = bioreactor(t,x,D,x2f);
    case 3  % output
      sys = x                 ;
    case {2 4 9}    % 2:discrete,
                   % 4:calcTimeHit,
                   % 9:termination
      sys =[]                    ;
    otherwise
      error(...
        ['unhandled flag =',...
        num2str(flag)])        ;
end

Figure 2. Code for S-function
3. Use the `trim` function to get steady state.

   a) set some more parameters: i.e. we want to fix the input values of \( D=0.3 \) and \( x2f=4.0 \)

   ```matlab
   >> X0=[ ];
   >> U0=[0.3;4];
   >> Y0=[ ];
   >> IX=[ ];
   >> IU=[1;2];
   >> IY=[ ];
   ```
Remarks:

i) $X_0$, $U_0$ and $Y_0$ are the state vector, input vector and output vector that will be fixed while the function attempts to find the steady state. Note that since we will not constrain the states and outputs, we can set these to null.

ii) $IX$, $IU$ and $IY$ indicates which values of $X_0$, $U_0$ and $Y_0$ will be held fixed. In our case, we want both input vector to be fixed, so we set $IU = [1;2]$.

b) run the trim function:

```matlab
>> [X,U,Y,DX]=TRIM('bioreactor_sys',X0,U0,Y0,IX,IU,IY);
>> X
X =
   0.9943
   1.5141
```

Thus, the function found $X=(0.9943,1.5141)$ as the steady state.

4. Linearization via the `linmod` function.

```matlab
>> [A,B,C,D]=linmod('bioreactor_sys',X,U0);
```

which results in the following:

```
A =
   0.0000  -0.0678
   -0.7500  -0.1304
B =
  -0.9943     0
   2.4859  0.3000
C =
   1.0000     0
   0    1.0000
D =
   0     0
   0     0
```