1. The height of liquid inside the spherical tank shown in Figure 1 can be modelled to be

\[
\frac{dh}{dt} = \frac{F_o - k_v u \sqrt{h}}{\pi r^2}
\]

where

\[
r = \sqrt{2Rh - h^2}
\]

with parameters fixed at \(R=20\) ft and \(k_v=1.3\) (ft\(^3\)/min)(ft\(^{-1/2}\)).

a) (10 pts) Find the steady state height corresponding to \(F_{o,ss}=2\) ft\(^3\)/min and \(u_{ss}=0.5\).

b) (20 pts) Obtain a linearized model for height, \(h\), around the steady state found in problem (a). (Note: treat \(F_o\) as a disturbance variable instead of a constant.)

2. The dynamic model of for the temperature of two tanks are given by

\[
10 \frac{dT_1}{dt} = (100 - T_1) + 10T_2
\]

\[
5 \frac{dT_2}{dt} = (T_1 - T_2) + 2u
\]
Using a proportional control, with \( u_c = 0.5 \) and \( T_{2, \text{set}} = 30 \),

\[
u = 0.5 + k_c (30 - T_2)\]

a) (20 pts) Find the range of values for proportional control gain, \( k_c \), that would stabilize the process

b) (20 pts) Using a value for \( k_c = 30 \), calculate the steady state offset for \( T_2 \), with \( T_{2, \text{set}} = 30 \).

3. (30 pts) The dynamic model for pressure in tank is given by:

\[
\frac{dP}{dt} = -3P + 2u
\]

Using a PI control, with \( u_c = 10 \),

\[
u = 10 + k_c \left( [P_{\text{set}} - P] + \frac{1}{\tau_I} \int [P_{\text{set}} - P] dt \right)
\]

With \( k_c = 1.0 \), find the range of values for \( \tau_I \) that will make the process underdamped.

4. (Bonus: 10 pts) The nonlinear process dynamics for \( z \) is given by the following:

\[
(1 + z) \frac{d^3 z}{dt^3} + 3 \left( \frac{dz}{dt} \right)^3 = \frac{d^2 z}{dt^2}
\] (1)

By introducing the following variables,

\[
y = \frac{dz}{dt} \quad x = \frac{d^2 z}{dt^2}
\]

and fixing \( \Delta t = 0.1 \) sec, determine the functions \( f(x_k, y_k, z_k) \), \( g(x_k, y_k, z_k) \) and \( h(x_k, y_k, z_k) \) for the recursion equations

\[
\begin{align*}
  z_{k+1} &= z_k + f(x_k, y_k, z_k) \\
  y_{k+1} &= y_k + g(x_k, y_k, z_k) \\
  x_{k+1} &= x_k + h(x_k, y_k, z_k)
\end{align*}
\]

based on Euler's method for numerical solution of equation (1).