1. (10 pts) The model for the pressure in a vessel was determined to be first order and given by

$$\frac{dP}{dt} = 10 - 5P$$

What is the time constant for this process?

2. (15 pts) The temperature for a process was found to follow a second order process given by

$$4 \frac{d^2T}{dt^2} + 8 \frac{dT}{dt} = 5 - 2T$$

Determine whether the process is underdamped or overdamped.

3. (20 pts) Find the values of parameter $\alpha$ such that the process described by the following equations is stable

$$\frac{dx}{dt} = (\alpha + 1)x + 2y$$

$$\frac{dy}{dt} = -3x - \alpha y$$

4. (25 pts) Implementing a proportional control given by: $u = k_c (C_{set} - C)$ on the system described by

$$2 \frac{d^2C}{dt^2} + \frac{dC}{dt} = \frac{1}{3} (2u - C)$$

Find the value of proportional control gain, $k_c$, that would yield a stable process with a decay ratio of 1/4.
5. (30 pts) You know that the nonlinear model of your CSTR process follows the equation given by

\[
\frac{dC}{dt} = \frac{1}{\tau} (C_{in} - C) - k_r C^2
\]

The engineering consultants you hired a few years ago showed in a report of their control study of your process that they used a model linearized around a steady state and is given by

\[
\frac{dC}{dt} = 0.048 - 0.34C + 0.1C_{in}
\]

However, they forgot to mention the values of \(\tau\) and \(k_r\) that they found, nor did they mention the steady state they used in their study. From the given equations, recover the values of \(\tau\) and \(k_r\), plus the steady state values they used, i.e. \(C_{ss}\) and \(C_{in,ss}\).

6. (Bonus: 10 pts) Given the process described by

\[
\frac{dx}{dt} + 2x = 0.5u
\]

Using a PD controller given by

\[
u = k_c \left( e + \tau_D \frac{de}{dt} \right)
\]

where \(e = x_{set} - x\)

with \(k_c = 2\) and \(\tau_D = 5\), and \(x_{set} = 1\), determine whether the offset will be zero.