1. (20pts) The process reaction curves of the system to be controlled are shown in Figure 1 where flow rate is the manipulated variable and temperature is the output variable. Obtain the PI control parameters using Cohen-Coon prescribed tuning rules. (Note: use the method discussed in class for obtaining the process parameters)

![Flow Rate Graph](image)

![Temperature Graph](image)

Figure 1: Process reaction curve for $y$ in response to a step change in input $u$.

2. Suppose the transfer function of a process is given by

$$G_p = \frac{-s + a}{(s + b)(s + c)}$$

where $a$, $b$ and $c$ are all positive real numbers.
(a) (15 pts) Using a proportional controller having gain, $K_c$, show that the characteristic polynomial of the closed loop transfer function is given by

$$s^2 + (b + c - K_c)s + (bc + aK_c)$$

(Assume the sensor and actuator transfer functions to be 1.)

(b) (15 pts) What are the controller tunings for a PI controller for this process as prescribed by the Ziegler-Nichols method? (Hint: the ultimate period is $2\pi/\omega$, where $\omega$ is the frequency in radians per second.)

3. (20pts) Given the following set of equations,

$$\begin{align*}
\frac{dx_0}{dt} &= -\frac{3}{5}x_0 - \frac{4}{5}x_1 + u \\
\frac{dx_1}{dt} &= x_0 \\
y &= \frac{1}{5}x_0 + \frac{2}{5}x_1
\end{align*}$$

with initial conditions $x_0(0) = x_1(0) = 0$. Obtain the transfer function from $\bar{u}(s)$ to $\bar{y}(s)$.

4. (15pts) The internal model control structure is shown in Figure 2. Obtain the transfer functions from setpoint $\bar{y}_p(s)$ and disturbance $\bar{d}(s)$ to output $\bar{y}(s)$.

5. (15pts) Use the Routh-Hurwitz method to determine the range of $k$ such that the following characteristic polynomial will have no roots having positive real parts:

$$s^4 + 2s^3 + (6 - k)s^2 + (k - 2)$$

6. (Bonus: 10pts) Find the Laplace transform of

$$f(t) = \frac{k}{(\alpha \beta)^{\beta t}}$$

where $\alpha$, $\beta$ and $k$ are all positive real constants.