1. (15 pts) a) Find the equivalent transfer function from \( y_{set} \) to \( y \) for the system shown in Figure 1.

![Figure 1](image)

b) (15 pts) With 
\[
L = \frac{1}{3}, \quad G_c = K_c, \quad G_p = \frac{3}{(20s + 1)(s + 1)(5s + 1)}
\]
the closed-loop transfer function from \( L[y_{set}] \) to \( L[y] \) should be
\[
\frac{1 + 3K_c}{100s^2 + 125s^2 + 26s + 1 + 3K_c}
\]

Determine the maximum range of proportional gain, \( K_c \), allowed for stability of the process.

c) (15 pts) Based on a step change in set point, \( y_{set} = \alpha u(t) \), and using the same values for \( L \), \( G_c \) and \( G_p \) as in question (b) above, what is the steady-state offset, 
\[
\text{offset} = y_{set}(\infty) - y(\infty)
\]

in terms of \( K_c \)?

2. (20 pts) Given the following process model:
\[
\begin{align*}
\frac{d}{dt}x_1 &= -3x_1 + x_2 + 2u \\
\frac{d}{dt}x_2 &= 2x_1 - 2x_2 + u \\
y &= x_1 + x_2
\end{align*}
\]

Find the transfer function from \( L[u] \) to \( L[y] \), assuming all initial conditions are zero.
3. (15 pts) The plot in Figure 2 shows the response of concentration, C, when the process is under proportional feedback control, with $K_c$ as the proportional gain. Using these plots, what are the prescribed values by the Ziegler-Nichols method for $K_c$, $t_I$ and $t_D$ for the PID controller?

![Kc=3.2 Temp Time (min)]

![Kc=6.4 Temp Time (min)]

Figure 2.

4. (20 pts) Suppose the Laplace transform of $x$ is given by

$$\hat{x}(s) = \frac{5s - 1}{(s + 1)[(s + 2)^2 + 4^2]}s$$

determine $x(t)$.

5. (Bonus: 10 pts) Find the Laplace transform of

$$f(t) = 3\sin\left[2t + \left(\frac{\pi}{2}\right)\right]$$