1. a) (30 pts) Obtain the equivalent transfer function, \( G_{cl} \), from \( \hat{T}_{set}(s) \) to \( \hat{T}(s) \) (in terms of transfer functions A, B, C and D) for the block diagram shown in Figure 1.

![Block Diagram](image)

Figure 1.

b) (30 pts) Suppose the transfer functions in Figure 1 are given by:

\[
A = \frac{K}{5s + 1} \quad B = \frac{1}{-3s + 1} \quad C = \frac{2}{s + 1} \quad D = \frac{1-A}{B} \left( \frac{1}{s + 1} \right)
\]

then the equivalent transfer function, \( G_{cl} \), from \( \hat{T}_{set}(s) \) to \( \hat{T}(s) \) will be

\[
\hat{T}(s) = G_{cl}(s) \hat{T}_{set}(s)
\]

\[
G_{cl} = \frac{15s^2 + (-2 - 4K)s - 1}{15s^3 + 13s^2 + (7 - K)s + 1 - K}
\]

Determine the range of \( K \) that would maintain the stability of the transfer function from \( \hat{T}_{set}(s) \) to \( \hat{T}(s) \).
2. (30 pts) Two continuously stirred tank reactors are connected in series as shown in Figure 2.

![Diagram]

The model for the process is described by the following equations:

\[
\frac{dC_{A1}}{dt} = \frac{1}{\tau_1} \left( C_{A,in} - C_{A1} + \alpha C_{A2} \right) - k_1 C_{A1}
\]

\[
\frac{dC_{A2}}{dt} = \frac{1}{\tau_2} \left( C_{A1} - C_{A2} \right) - k_2 C_{A2}
\]

where the feed concentration is given by \( C_{A,in} \).

a) (25 pts) Assuming zero initial conditions, show that the transfer function from \( C_{A,in}(s) \) to \( C_{A2}(s) \), with the following parameter values,

| \( \tau_1 \) | 1 |
| \( \tau_2 \) | 2 |
| \( k_1 \) | 0.2 |
| \( k_2 \) | 0.3 |
| \( \alpha \) | 0.1 |

is given by

\[
\hat{C}_{A2}(s) = \left[ \frac{1}{2s^2 + 4s + 1.82} \right] \hat{C}_{A,in}(s)
\]
b) (15 pts) Suppose $C_{\text{in}}(t)$ is given by

$$C_{\text{in}}(t) = 0.4 + 0.5 e^{-t}$$

Solve for $C_{A2}$ as a function of time.

3. (Bonus: 5 pts) Determine the Laplace transform of

$$f(t) = \left( 5 - 2 \sin \left( \frac{2\pi t}{10} \right) \right) e^{-2t}$$