1. **All-Pass Systems.** A transfer function, $G(s)$, is said to be all-pass if the magnitude $|G(i\omega)|$ is equal to 1 for all values of frequency $\omega$. Let $y = 2x - u$ and 

\[
\frac{1}{\alpha} \frac{dx}{dt} = u - x \quad ; \quad x(0) = 0
\]

(a) (10 pts) Show that the transfer function from $u$ to $y$ is given by 

\[
G(s) = \frac{\alpha - s}{\alpha + s}
\]

**Answer:**

(b) (10 pts) Show that for this transfer function, $|G(i\omega)| = 1$ at all frequencies, and thus this is system is all-pass.

**Answer:**
2. **Non-minimum phase 1st order lead.** (10 pts) A first order lead is nonminimum phase if the zero is in the right half plane. So for a 1st order system, the general nonminimum phase type is given by

\[ G(s) = 1 - \tau_s \]

Determine the value of \( \arg[G(i\omega)] \) as \( \omega \rightarrow \infty \).

**Answer:**

3. **Stability margins using Bode plots.** Suppose the plots in Figure 1 were obtained from the frequency response experiments as

(a) (10 pts) What is the phase margin?

**Answer:**

(b) (10 pts) What is the gain margin?

**Answer:**

4. **Complex Mapping Theorem.** (10 pts) Let \( G(s) \) be given by

\[ G(s) = \frac{s^2 - 2s + 1.16}{s^2 + 2s + 1.36} \]
Figure 1: Bode plots of $G_c G_p$. 
How many encirclements of the origin will the mapping $G(s)$ make as $s$ traverses the contour $\Gamma(s)$ shown in Figure 2.

![Diagram of contour Γ(s)](image)

Figure 2: The contour $\Gamma(s)$.

**Answer:**
5. **Nyquist stability and robustness.** Suppose the Nyquist plot for $G_p$ is shown in Figure 3.

![Figure 3: Nyquist plot of $G_p$.](image)

(a) (10 pts) Using $K_c = 0.5$, will the feedback system, $K_c G_p / (1 + K_c G_p)$ be stable? Why or why not?

**Answer:**

(b) (10 pts) What maximum value of $K_c$ should be used so that Log modulus of $K_c G_p$ is less than -2 dB for all frequencies?

**Answer:**
6. **Proportional Control.** Suppose the plots shown in Figure 4 are Bode plots for the process, \( G_p(s) \). Using proportional control,

(a) (10 pts) What value of proportional control gain is needed such that \( K_c G_p \) will have a gain margin of 2.

**Answer:**

(b) (10 pts) What value of proportional control gain is needed such that \( K_c G_p \) will have a phase margin of 45°?

**Answer:**

(c) (Bonus: 10 pts) Which \( K_c \) value should be used? Why?

**Answer:**
Figure 4: Bode plots of $G_p$. 
**Scratch Space:** (Please still put answers in the spaces provided for each question.)