1. (30 pts) For the feedback system shown in Figure 1, with the transfer functions given by

\[
G_p = \frac{5}{2s + 1}
\]

\[
G_c = 2 \frac{s + 1}{s}
\]

obtain the magnitude ratio of \( G_{cl} \), the closed loop transfer function from \( y_{set} \) to \( y \), as a function of frequency, \( \omega \) (rads/sec).

\[\text{Figure 1.}\]

(Hint/Check: Magnitude ratio of \( G_{cl} \) at \( \omega = 1 \) rad/sec is 1.04)

2. (10 pts) Consider the same feedback structure shown in Figure 1, but with a different process transfer function, \( G_p \), whose nyquist plot is shown in Figure 2. Using a proportional control, \( G_c = K_c \), determine the value of \( K_c \) so that the resulting gain margin of \( G_cG_p \) is 1.75.
3. (30 pts) Consider again the feedback system shown in Figure 1, but this time the Bode plot of $G_p$ is given in Figure 3. Obtain the PI control tuning based on the Tyreus-Luyben rules.
4. (30 pts) Determine which of the transfer functions given in Table 1 matches the Bode plots shown in Figures 4, 5 and 6.

**Table 1.**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td><strong>G1</strong></td>
<td>( \frac{(10s+1)(s+0.01)}{s(s+1000)} )</td>
<td><strong>G5</strong></td>
</tr>
<tr>
<td><strong>G2</strong></td>
<td>( \frac{10}{100s^2+2s+1} )</td>
<td><strong>G6</strong></td>
</tr>
<tr>
<td><strong>G3</strong></td>
<td>( \frac{s+100}{s+0.01} )</td>
<td><strong>G7</strong></td>
</tr>
<tr>
<td><strong>G4</strong></td>
<td>( 1+\frac{-s+1}{0.01s+1} )</td>
<td><strong>G8</strong></td>
</tr>
</tbody>
</table>

**Figure 4.** Bode plot for Case 1.
Figure 5. Bode plot for Case 2.

Figure 6. Bode plot for Case 3.
5. Bonus (10 pts). Consider a circle contour $\Gamma$ of radius 1 and centered at (-1,0) in the $s$-plane as shown in Figure 7. Find the number of clockwise encirclements of the origin that the map of

$$G = \frac{s + 1}{(s+1)^2 + 0.5^2}$$

will have as $s$ traverses $\Gamma$ in the clockwise manner.