1. Tanks in Series.

For a liquid flowing through a continuously stirred tank under perfect level control, the model is given by

\[ \frac{dT}{dt} = \frac{F}{V}(T_{in} - T) \]

If one decides instead to flow it through two tanks in series whose volume is half the size of \( V \), the model is given by

\[ \frac{dT_1}{dt} = \frac{F}{(V/2)}(T_{in} - T_1) \]
\[ \frac{dT}{dt} = \frac{F}{(V/2)}(T_1 - T) \]

where \( T_1 \) is the temperature coming out of the first and entering the second tank.

We could continue this division for \( n \) tanks in which the volume of each tank becomes \( V/n \), and the model is given by

\[ \frac{dT_1}{dt} = \frac{F}{(V/n)}(T_{in} - T_1) \]
\[ \vdots \]
\[ \frac{dT_{n-1}}{dt} = \frac{F}{(V/n)}(T_{n-2} - T_{n-1}) \]
\[ \frac{dT}{dt} = \frac{F}{(V/n)}(T_{n-1} - T) \]

Let all the temperatures be in deviation variable form, and thus we can let all the initial conditions be zero.

Using Simulink, create a Model window that contains the cases for \( n=1 \) to \( n=5 \). Use \( V=10 \), \( F=1 \). Simulate the system responding to a step change in \( T_{in} \) as follows:

\[ T_{in} = \begin{cases} 
0 & \text{if } t \leq 1 \\
1 & \text{if } t > 1 
\end{cases} \]
A sample window is shown in Figure 1 for the cases n=1 and n=2.

![Figure 1](image)

Plot all the temperature responses in one plot and discuss what happens when the number of tanks is increased.

2. **Optimization of Tuning Parameters.**

   (Note: for an tutorial including example files in using Matlab and Simulink, go to the link: [http://www.chem.mtu.edu/~tbco/cm416/OptTune_2k4.zip](http://www.chem.mtu.edu/~tbco/cm416/OptTune_2k4.zip))

   a) Use the model of System B obtained from Project 4, part 2, to get $G_p(s)$, the transfer function of the process.
   
   b) Next, construct the Simulink model that implements the feedback control system shown in Figure 2, where $G_c(s)$ is a PI Controller.

   ![Figure 2](image)

   c) Using the Matlab function, *fminsearch*, find the values of proportional gain, $k_c$, and integral time, $\tau_I$, that minimizes the IAE (integrated absolute error).
   
   d) Now go back to the java simulator through the link:

   [http://www.chem.mtu.edu/~tbco/cm416/newpidb.html](http://www.chem.mtu.edu/~tbco/cm416/newpidb.html)

   and implement the values found from part c) for a PI controller. Compare the result with that using Ziegler-Nichols tuning.
3. **Ultimate gain and ultimate period from Transfer Functions.**

   a) Suppose the process transfer function is given by

   \[ G_p = \frac{s + 1.5}{s^4 + 5s^3 + 9s^2 + 7s + 2} \]

   Use a proportional control, \( G_c = K_c \), in the feedback system given in Figure 2. Using the Routh-Hurwitz method, determine the ultimate gain and the ultimate period.

   b) Using Simulink, simulate the process in Figure 2 and verify that the value predicted in part a) will indeed be the ultimate gain and ultimate period.

4. **Control of Drug Delivery.**

   Do the problems given in M12.3, page 694 of your book (background is given in M12.2). Use Simulink to simulate your feedback process and use a PI controller. (Note: you can either use the Transfer function block or the State Space block for your process.)

5. **Nonminimum Phase Dynamics (additional exercise: not required for submission)**

   Using Simulink, try to reproduce Figure 3-11, on page 109. (You can modify the script given in pages 13-15 of the Matlab Tutorial: Simulink Basics, http://www.chem.ntu.edu/~tbco/cm416/MatlabTutorialPart3.pdf)